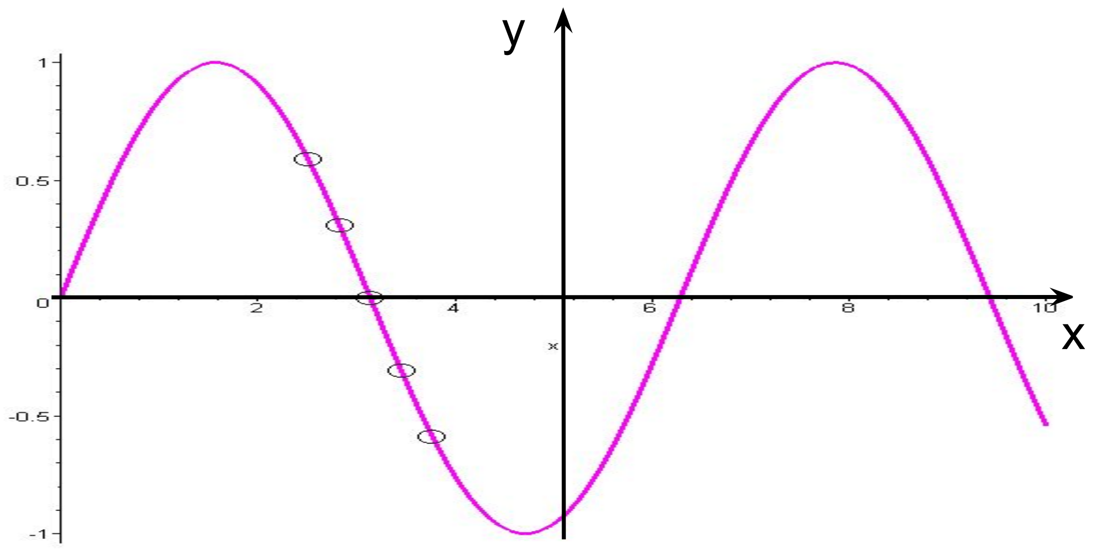
Lecture 6
(Reflection, Interference and Standing Waves)

Physics 2310-01 Spring 2020

Douglas Fields

Un-bounded Periodic Wave Description

- We can put these together:



$$y(x,t) = A\cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}, \quad \omega = \frac{2\pi}{T}$$

Wave travelling to the right

Energy and Power

- I want to cover a little of energy transfer in a wave, although I find the discussion in the book mostly boring.
- Please don't memorize the equation in the book, but let's do go through the derivation.
- The work done on a piece of a string by a wave is just as usual: $dW = \vec{F} \cdot d\vec{y}$

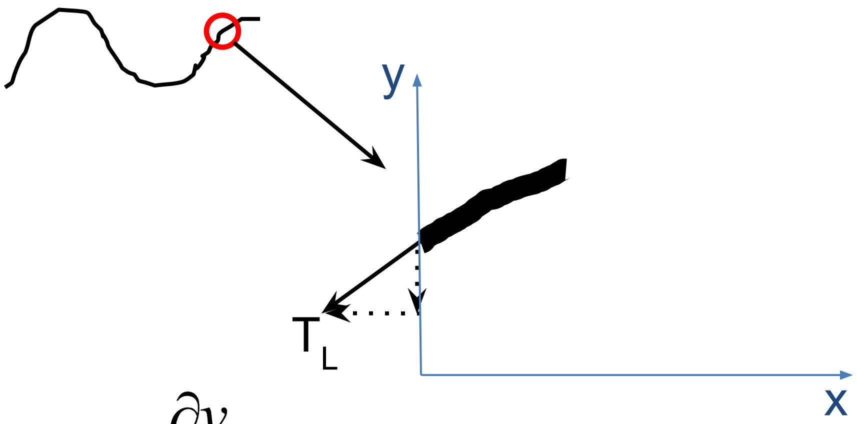
where, the force F is the total force on the piece, and dy is the little displacement it undergoes.

Power transmitted by a mechanical wave

- As before, but now just examining what happens at one point,

For small slopes, $T_x \approx T$, so

$$T_{Ly} = -T\frac{\partial y}{\partial x}$$



- So, the work is just:

$$dW = -T\frac{\partial y}{\partial x}dy$$

- And the power (work per time) is:

$$P(x,t) = \frac{dW}{dt} = -T\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}$$

Power transmitted by a mechanical wave

$$P(x,t) = \frac{dW}{dt} = -T\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}$$

- But,

$$y(x,t) = A\cos(kx-\omega t) \Rightarrow$$

$$\frac{\partial y(x,t)}{\partial x} = -Ak\sin(kx-\omega t)$$

$$\frac{\partial y(x,t)}{\partial t} = A\omega\sin(kx-\omega t)$$

- So,

$$P(x,t) = TA^2k\omega\sin^2(kx-\omega t)$$

$$= \sqrt{\mu T}A^2\omega^2\sin^2(kx-\omega t)$$

$$v = \frac{\omega}{k} = \sqrt{\frac{T}{\mu}} \Rightarrow$$

$$\frac{k}{\omega} = \sqrt{\frac{\mu}{T}}$$

Average Power

$$P(x,t) = \sqrt{\mu T}\, A^2 \omega^2 \sin^2(kx - \omega t)$$

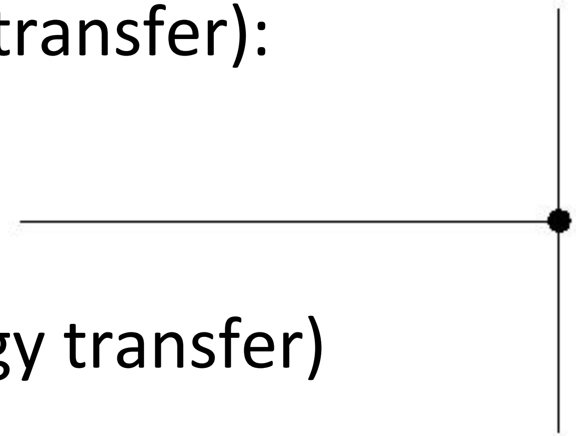
- And the average of $\sin^2$ is just 1/2, so:

$$P_{avg} = \frac{1}{2}\sqrt{\mu T}\, A^2 \omega^2$$

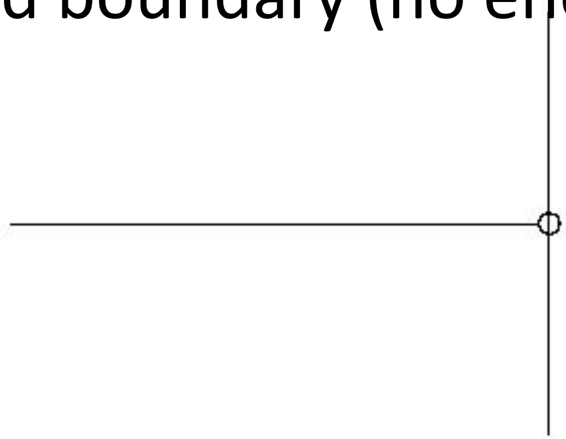
- What to get from this?
- For mechanical waves, the power is a function of both the amplitude and the frequency squared.
- For E&M waves, it is independent of the frequency (as we will discover).

Waves at Boundaries

- What happens to a wave when it reaches a boundary depends upon the boundary:
  - Fixed boundary (no energy transfer):

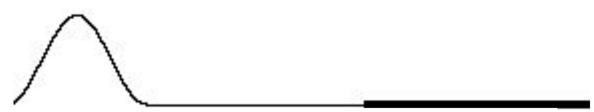


  - Unfixed boundary (no energy transfer)



Waves at Boundaries

- What happens when there ***can*** be energy transfer at a boundary?
  - Wave from low mass/length to high mass/length:



  - Wave from high mass/length to low mass/length:

  - Remember by taking the limit as heavier string becomes a wall…

Waves at Boundaries

- What happens when there the wave is bounded on two sides?

Superposition

- The wave equation, because it is linear, lends itself to superposition of solutions:
  - If y_1 is a solution of the wave equation:

$$y_1(x,t) = \cos(k_1 x - \omega_1 t)$$

  - And y_2 is also a solution of the wave equation:

$$y_2(x,t) = \cos(k_2 x - \omega_2 t)$$

  - Then $(Ay_1 + By_2)$ is also a solution of the wave equation.

Superposition

- Superposition also means that waves “interfere” with each other.
  - If two different waves both exist on a string (or any other medium), then the resultant wave is just the superposition (linear sum) of the individual waves.
  - This can mean the amplitude gets bigger,
  - Or smaller…
- But each wave exists on the string independently!

Standing Waves

- Let's take y_1 as a wave moving to the right:

$$y_1(x,t) = A\cos(kx - \omega t)$$

- And y_2 as a wave moving to the left:

$$y_2(x,t) = A\cos(kx + \omega t)$$

- Then $(y_1 + y_2)$ is also a solution of the wave equation:

$$y_1(x,t) + y_2(x,t) = A\cos(kx - \omega t) + A\cos(kx + \omega t)$$

Standing Waves

- Lets simplify this:

$$y_1(x,t)+y_2(x,t)=A\cos(kx-\omega t)+A\cos(kx+\omega t)$$

But, $\cos(A)+\cos(B)=2\sin\frac{1}{2}[A+B]\cos\frac{1}{2}[A-B]$

So,

$$y_1(x,t)+y_2(x,t)=2A\sin\frac{1}{2}\left[(kx-\omega t)+(kx+\omega t)\right]\cos\frac{1}{2}\left[(kx-\omega t)-(kx+\omega t)\right]$$

$$=2A\sin(kx)\cos(\omega t)$$

Standing Waves

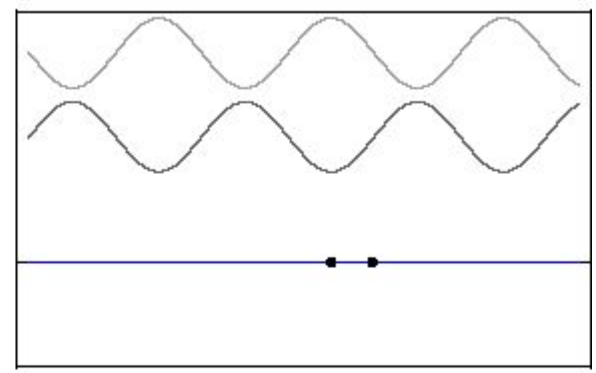
- Examine this solution:

$$\begin{aligned} y_1(x,t) + y_2(x,t) &= 2Asin(kx)cos(\omega t) \\ &= [2Acos(\omega t)][sin(kx)] \\ &= [\text{time dep. amplitude}][\text{position dep.}] \end{aligned}$$

- In position, there is a sin dependence whose amplitude depends upon time.

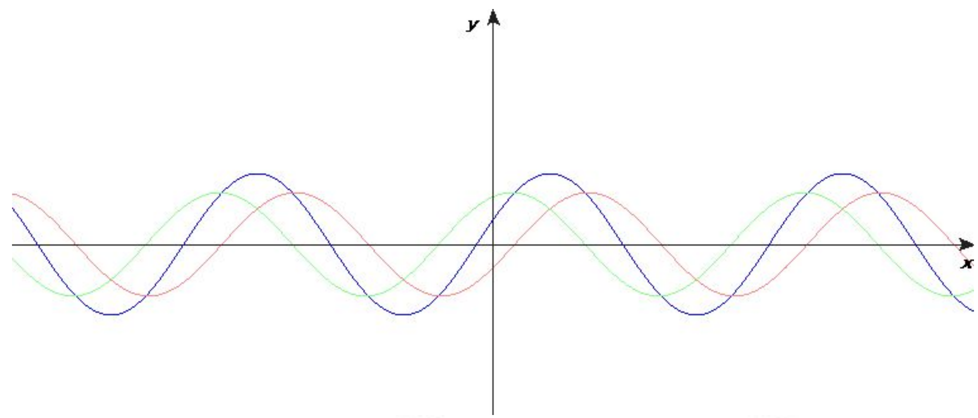
Standing Waves

$$
\begin{aligned}
y_1(x,t) + y_2(x,t) &= 2Asin(kx)cos(\omega t) \\
&= [2Acos(\omega t)][sin(kx)] \\
&= [\text{time dep. amplitude}][\text{position dep.}]
\end{aligned}
$$



$y_1(x,t)$
$y_2(x,t)$

$y_1(x,t)$
$y_2(x,t)$



$y_1(x,t)$
$y_2(x,t)$