Lecture 9 (Interference, Beats, and Doppler Effect)

Physics 2310-01 Spring 2020

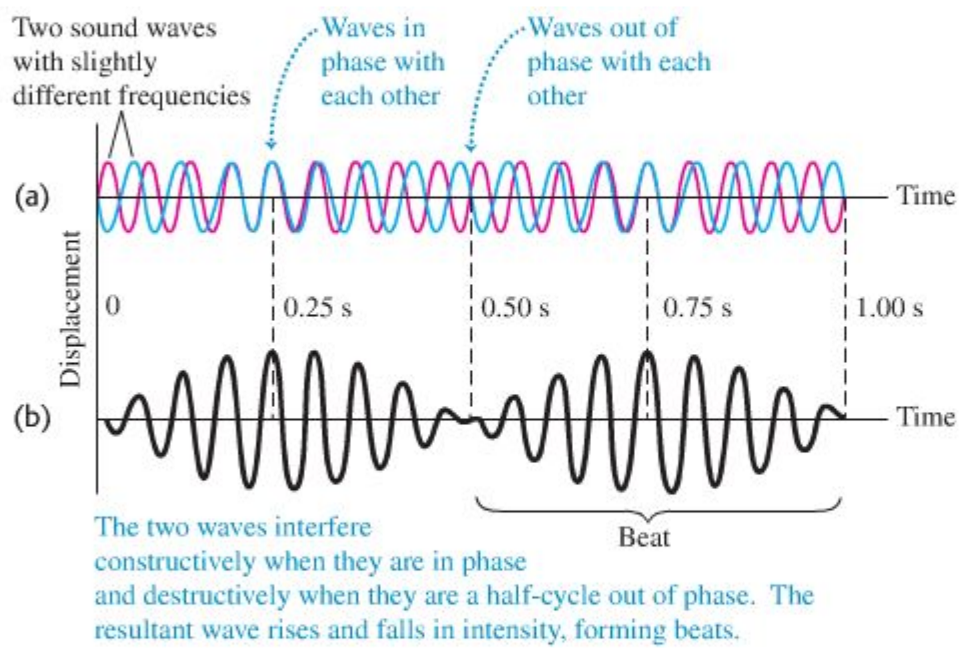
Douglas Fields

Beats

- If the two waves have slightly different wavelengths, they can interfere with a time dependence:

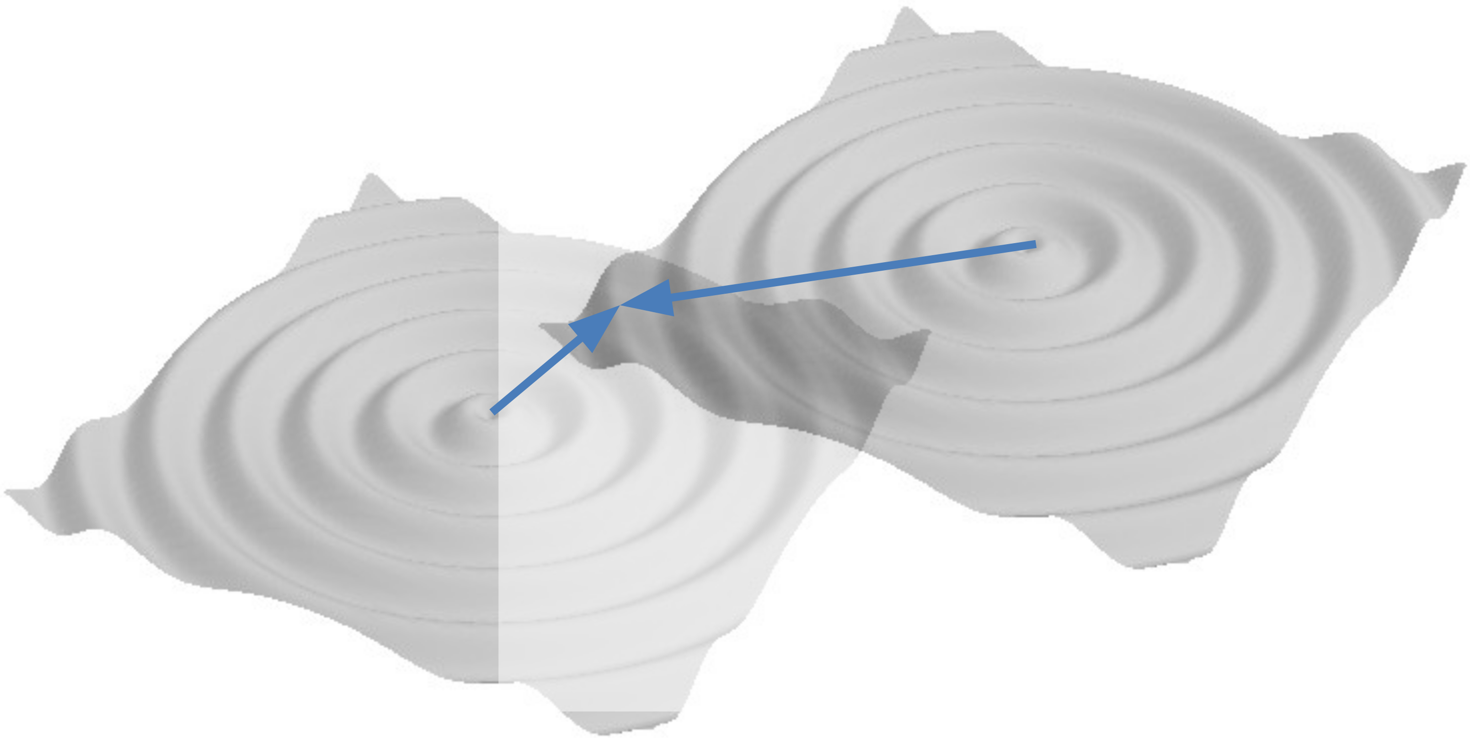
$$f_a = 16Hz, \quad f_b = 18Hz$$

$$f_{beat} = f_a - f_b$$

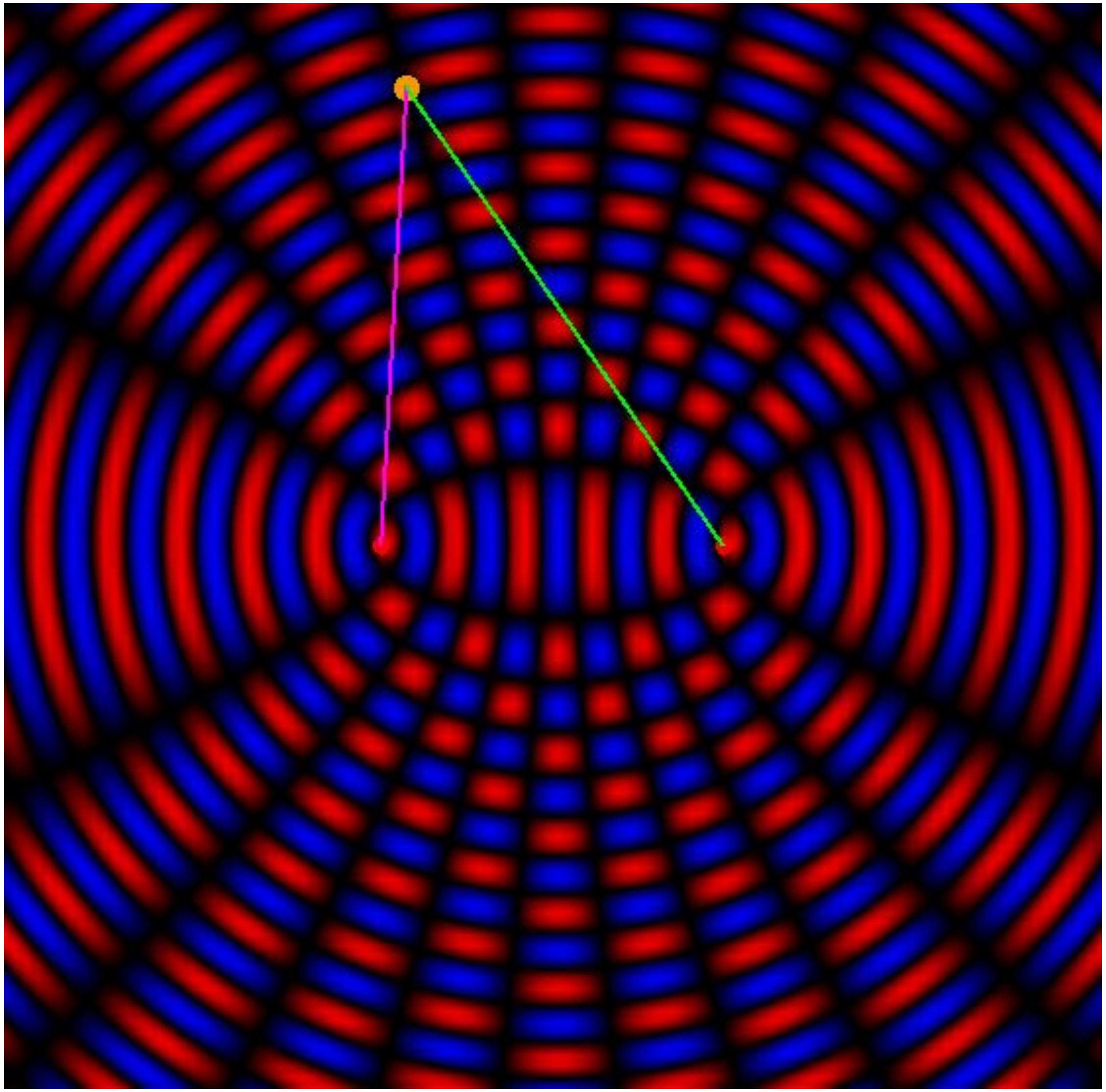


https://phet.colorado.edu/sims/html/wave-interference/latest/wave-interference_en.html

2D Waves

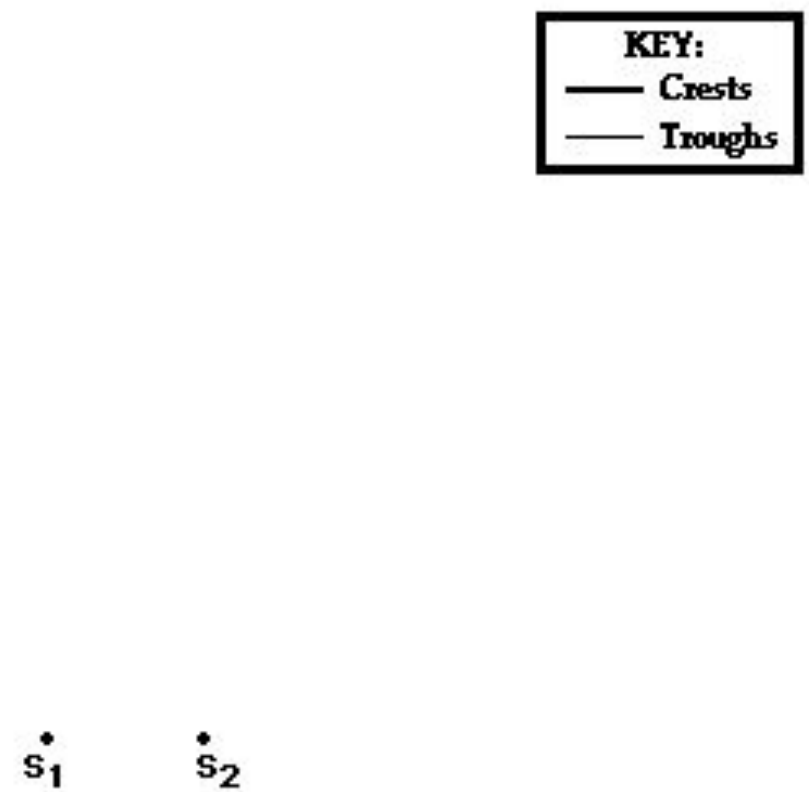


Interference

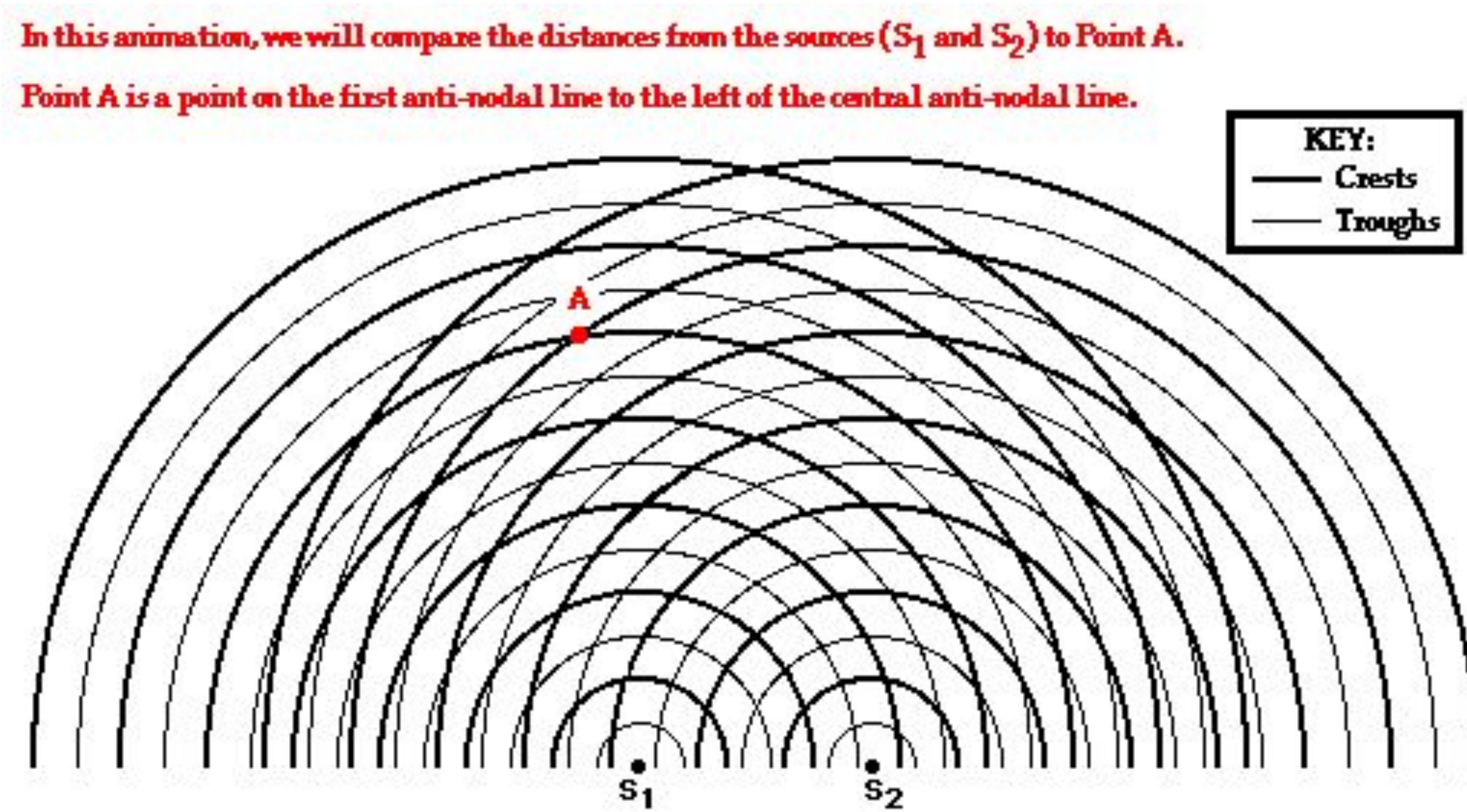


Interference

- Let’s look at two source interference:



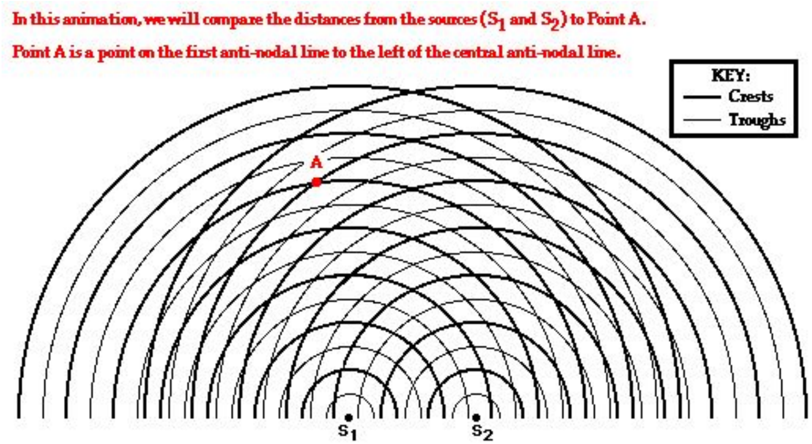
Interference and path length



$$y_1(A,t) = sin(k[s_1 \to A] - \omega t)$$
$$= sin(kx_1 - \omega t)$$

$$y_2(A,t) = sin(k[s_2 \to A] - \omega t)$$
$$= sin(kx_2 - \omega t)$$

Interference and path length



$$y_1(A,t) = \sin(kx_1 - \omega t) \qquad y_1(A,t) = \sin(kx_2 - \omega t)$$

$$\delta\phi = (kx_1 - \omega t) - (kx_2 - \omega t) = kx_1 - kx_2 = k(x_1 - x_2)$$

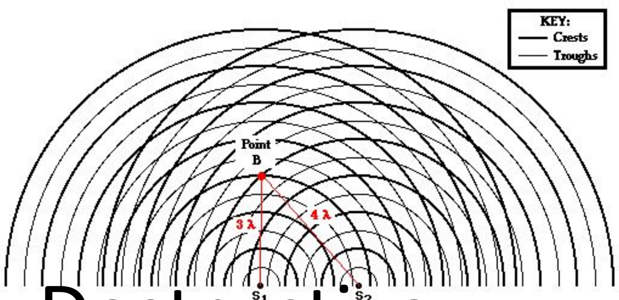
$$= \frac{2\pi}{\lambda}(x_1 - x_2)$$

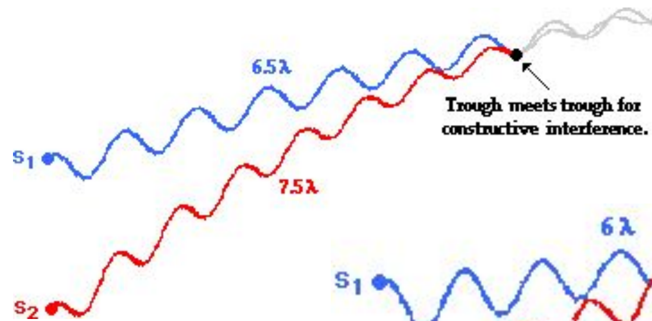
Interference and path length

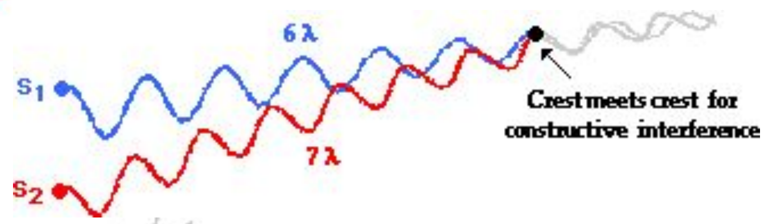
$$\delta\phi = \frac{2\pi}{\lambda}(x_1 - x_2) \rightarrow$$

$$\delta\phi = \begin{cases} 2n\pi & \text{if } (x_1 - x_2) = \text{whole number of } \lambda \text{ then, constructive interference.} \\ (n + \frac{1}{2})\pi & \text{if } (x_1 - x_2) = \text{half (odd) number of} \lambda \text{ then, destructive interference.} \end{cases}$$

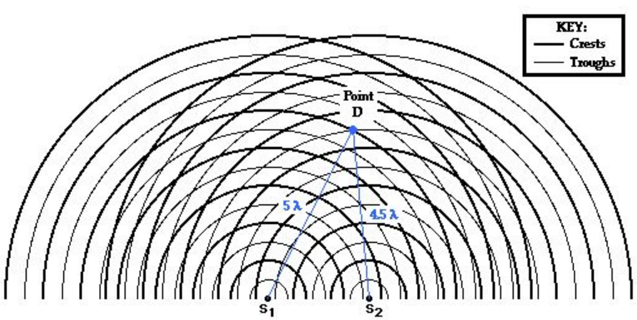
- Constructive

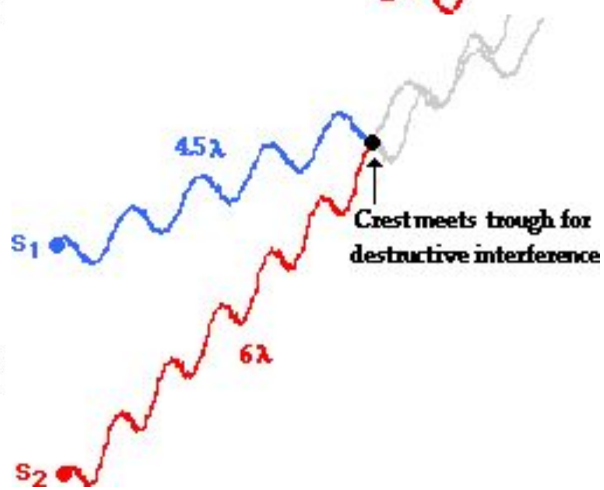


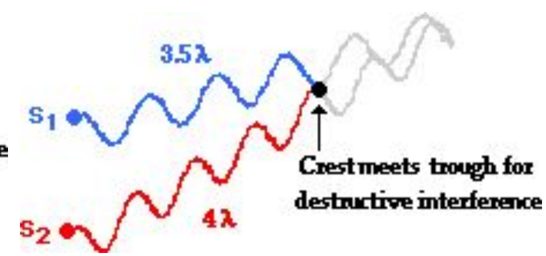




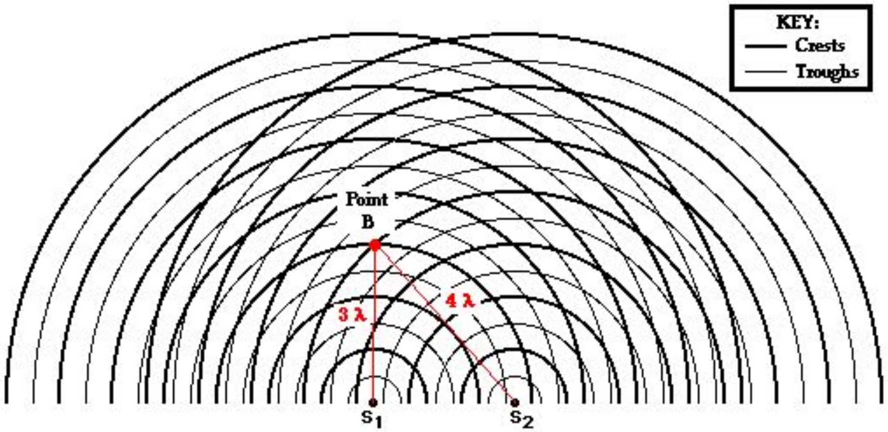
- Destructive

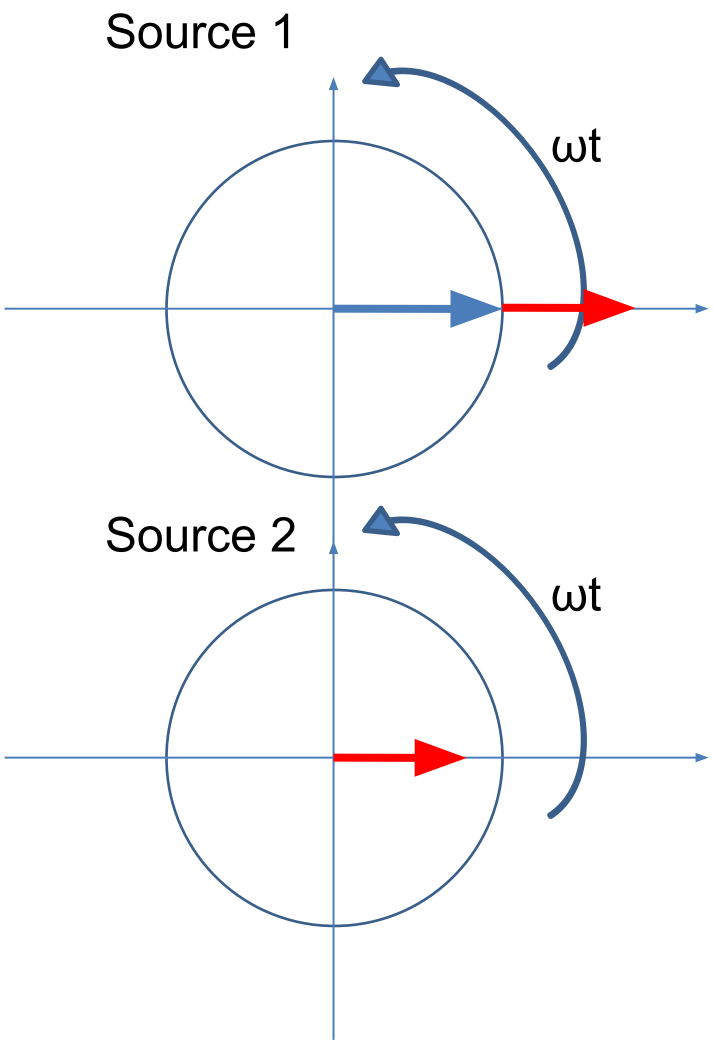




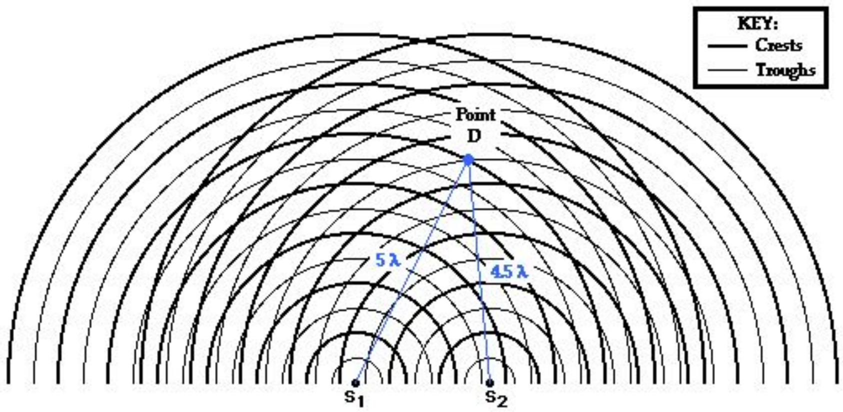


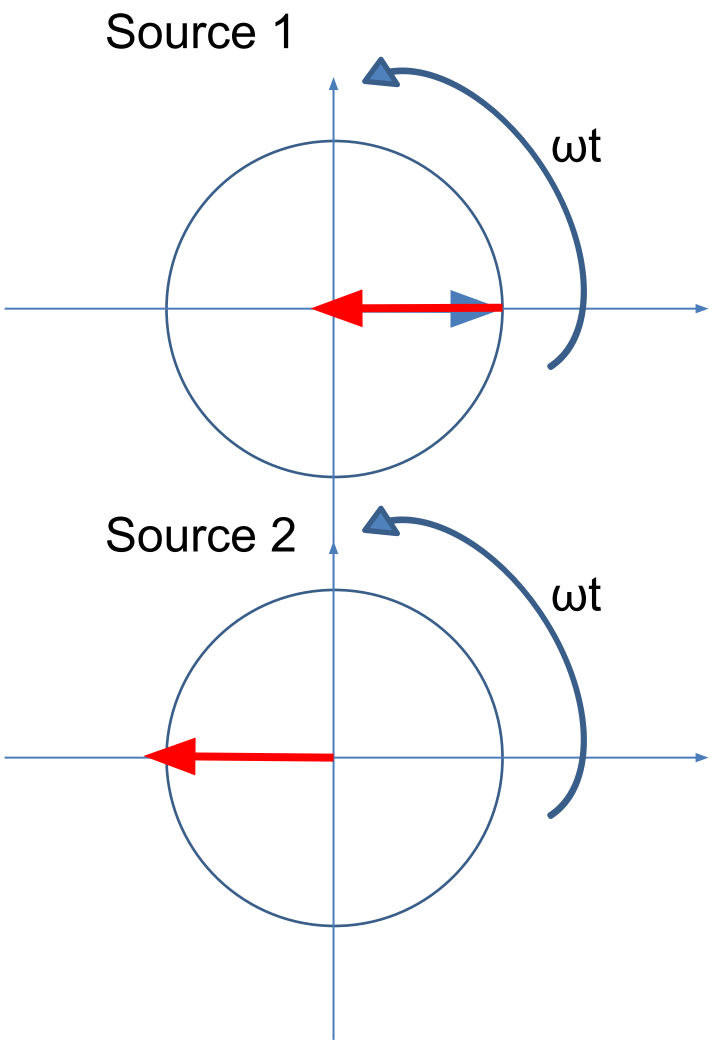
Phasor Analysis





Phasor Analysis





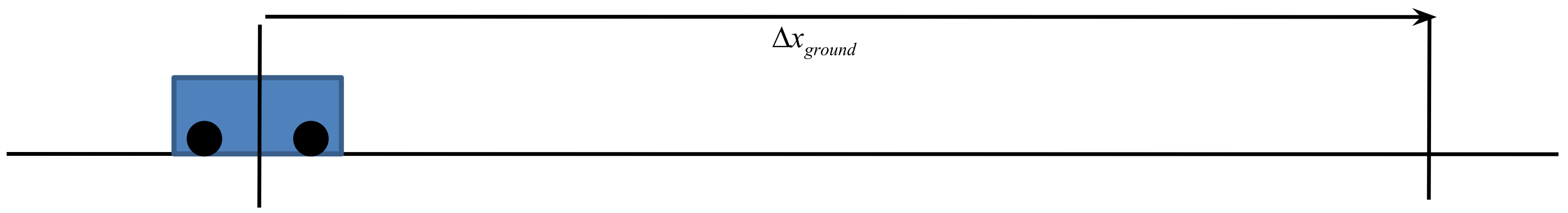
What else can cause a phase difference???

- Even if the distance between two sources and a detector are the same, you can still get destructive interference…
  - The sources can be out of phase.
  - The waves might travel through media with different wave speeds.
  - One wave might undergo a reflection (flipping the phase).

Relative Velocities

- We need to review what we know about relative velocities:

$$v_{car-ground} = \frac{\Delta x_{car-ground}}{\Delta t}$$



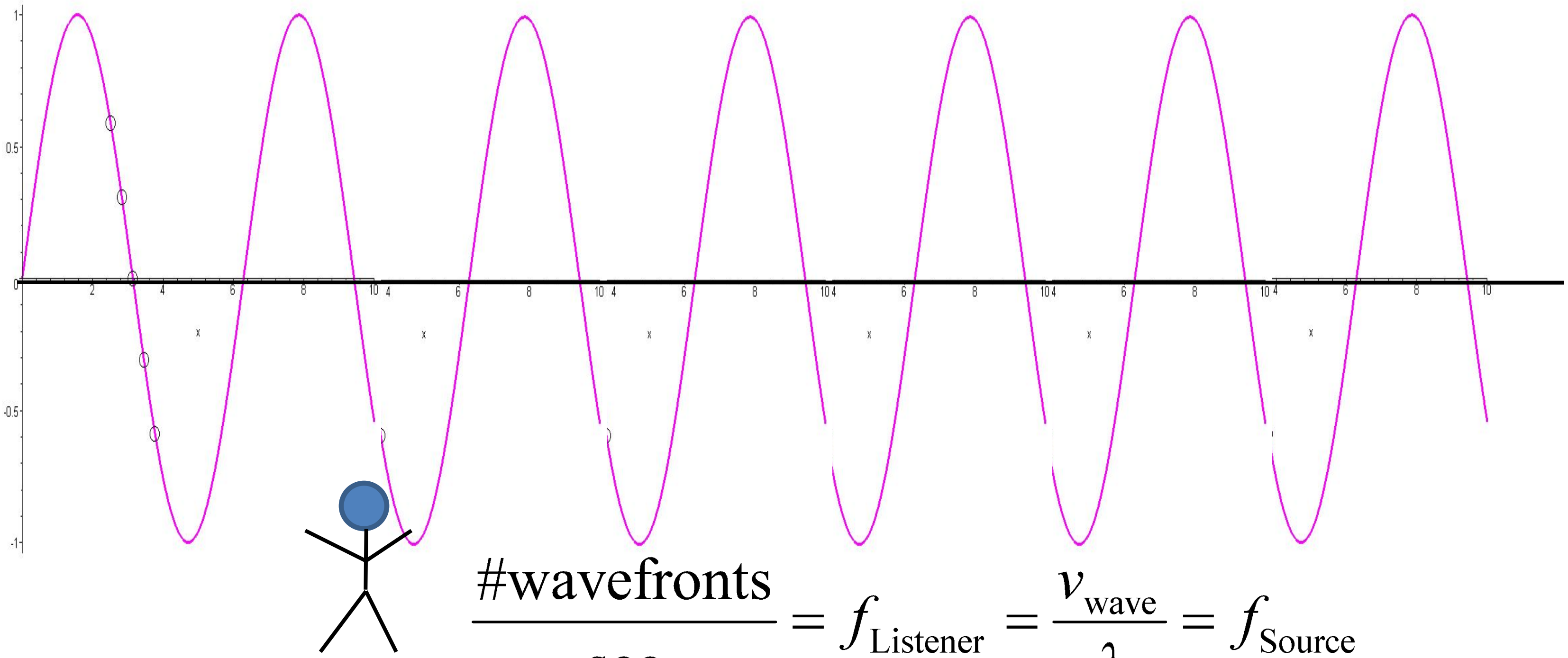
Relative Velocities

- We need to review what we know about relative velocities:

$$v_{car-person} = \frac{\Delta x_{car-ground} - \Delta x_{person-ground}}{\Delta t} = v_{car-ground} - v_{person-ground}$$

Relative Velocities & Waves

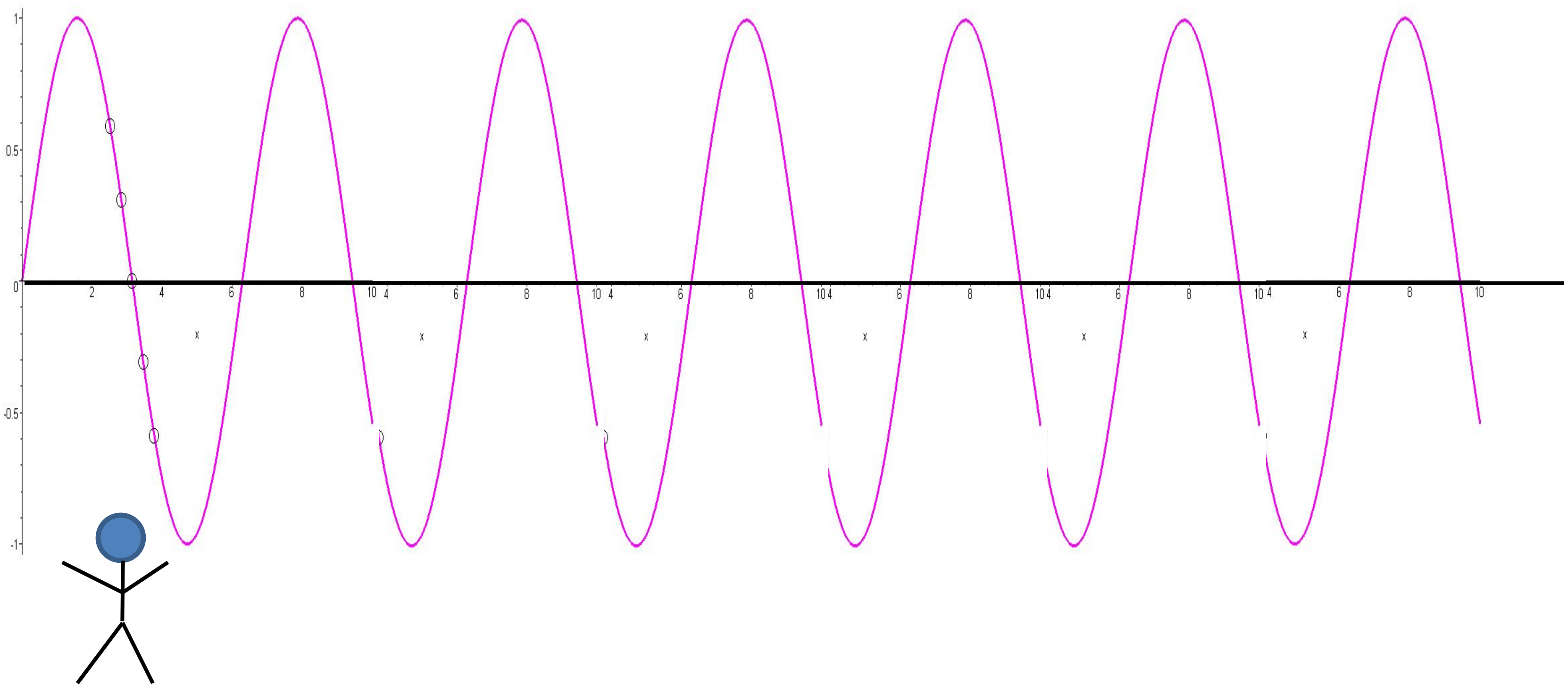
- Now, instead of the car, what if we are viewing a wave?
- We perceive the wave by measuring the number of wave-fronts that arrive at our location (we sense the changes in pressure in our ears, or detect the changes in electric field).



$$\frac{\#\text{wavefronts}}{\text{sec}} = f_{\text{Listener}} = \frac{v_{\text{wave}}}{\lambda} = f_{\text{Source}}$$

Relative Velocities & Waves

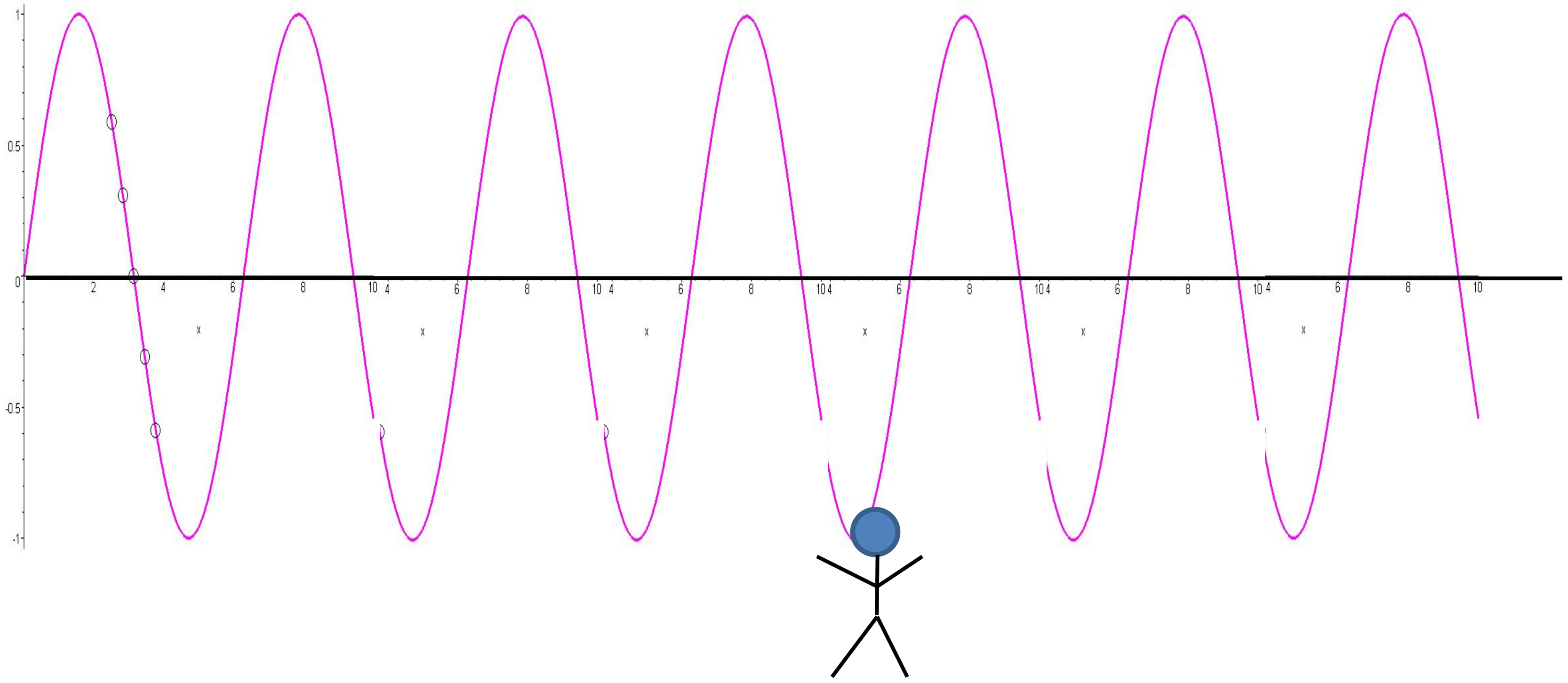
- What if we are moving too?



$$f_{\text{Listener}} = \frac{v_{\text{wave-person}}}{\lambda} = \frac{v_{\text{wave}} - v_{\text{person}}}{\lambda} = \frac{v_{\text{wave}} - v_{\text{person}}}{v_{\text{wave}} / f_{\text{Source}}} = \left(\frac{v_{\text{wave}} - v_{\text{person}}}{v_{\text{wave}}} \right) f_{\text{Source}}$$

Relative Velocities & Waves

- Notice that the directions matter:



Doppler Effect

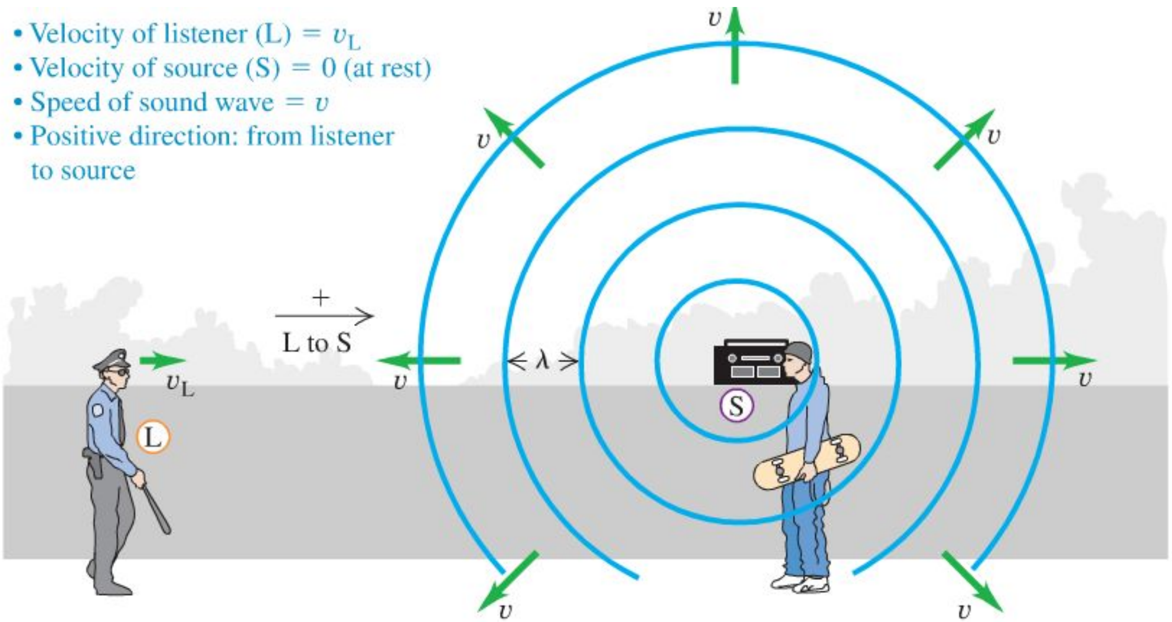
- This change in the perception of the frequency of sound (or light) emitted from a source is known as the Doppler effect.
- You have heard this in sound waves emitted from sirens, train whistles, etc.
- But the same principle is used to determine car speeds by a policeman with a radar gun, or the relative velocity and rotation speeds of galaxies as viewed by astronomers.

Book Definitions

- The book uses a little different formalism.
- They define Doppler shift as:

$$f_{\text{Listener}} = \left(\frac{v + v_{\text{Listener}}}{v} \right) f_{\text{Source}} = \left(1 + \frac{v_{\text{Listener}}}{v} \right) f_{\text{Source}}$$

- Where they have defined a positive listener velocity as in the direction *toward the source*.



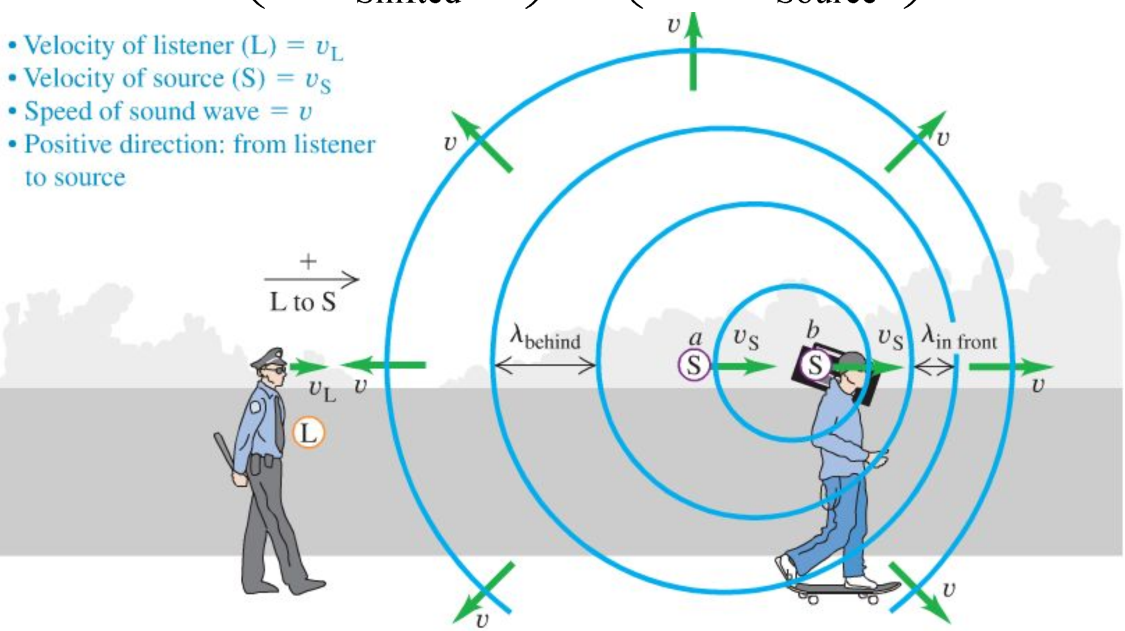
Moving Source

- If the source is moving, then the wavelength is also changed:

$$\lambda_{\text{Shifted}} = \left(\frac{v}{f_{\text{Source}}} + \frac{v_{\text{Source}}}{f_{\text{Source}}} \right) = \frac{v + v_{\text{Source}}}{f_{\text{Source}}}$$

- Then, the frequency heard by the listener is:

$$f_{\text{Listener}} = \left(\frac{v + v_{\text{Listener}}}{\lambda_{\text{Shifted}}} \right) = \left(\frac{v + v_{\text{Listener}}}{v + v_{\text{Source}}} \right) f_{\text{Source}}$$



Keeping track of signs…

- The only hard thing is to keep track of the signs of the velocities.
- Just put a listener where you need the information, and then the positive direction is towards the source

Example 1

$$\lambda_{\text{Shifted}} = \left(\frac{v}{f_{\text{Source}}} + \frac{v_{\text{Source}}}{f_{\text{Source}}} \right) = \frac{v + v_{\text{Source}}}{f_{\text{Source}}}$$

$$f_{\text{Listener}} = \left(\frac{v + v_{\text{Listener}}}{\lambda_{\text{Shifted}}} \right) = \left(\frac{v + v_{\text{Listener}}}{v + v_{\text{Source}}} \right) f_{\text{Source}}$$

Example 16.15 Doppler effect I: Wavelengths

A police siren emits a sinusoidal wave with frequency $f_S = 300$ Hz. The speed of sound is 340 m/s. (a) Find the wavelength of the waves if the siren is at rest in the air. (b) If the siren is moving at 30 m/s (108 km/h, or 67 mi/h), find the wavelengths of the waves in front of and behind the source.

SOLUTION

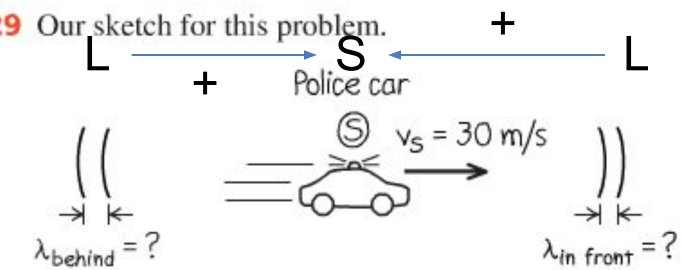
IDENTIFY: The Doppler effect is not involved in part (a), since neither the source nor the listener is moving. In part (b), the source is in motion and we must invoke the Doppler effect.

SET UP: Figure 16.29 shows the situation. We use the relationship $v = \lambda f$ to determine the wavelength when the police siren is at rest. When it is in motion, we find the wavelength on either side of the siren using Eqs. (16.27) and (16.28).

EXECUTE: (a) When the source is at rest,

$$\lambda = \frac{v}{f_S} = \frac{340 \text{ m/s}}{300 \text{ Hz}} = 1.13 \text{ m}$$

16.29 Our sketch for this problem.



Example 2

Example 16.16 Doppler effect II: Frequencies

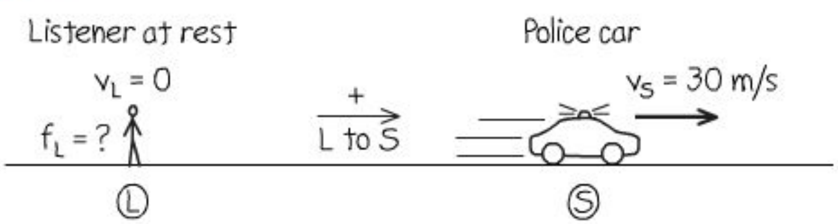
If a listener L is at rest and the siren in Example 16.15 is moving away from L at 30 m/s, what frequency does the listener hear?

SOLUTION

IDENTIFY: Our target variable is the frequency f_L heard by the listener, who is behind the moving source.

SET UP: Figure 16.30 shows the situation. We know $f_S = 300$ Hz from Example 16.15, and we have $v_L = 0$ and $v_S = 30$ m/s. (The source velocity v_S is positive because the siren is moving in the same direction as the direction from listener to source.)

16.30 Our sketch for this problem.



EXECUTE: From Eq. (16.29),

$$f_L = \frac{v}{v + v_S} f_S = \frac{340 \text{ m/s}}{340 \text{ m/s} + 30 \text{ m/s}} (300 \text{ Hz}) = 276 \text{ Hz}$$

EVALUATE: The source and listener are moving apart, so the frequency f_L heard by the listener is less than the frequency f_S emitted by the source.

Here's an alternative approach we can use to check our result. From Example 16.15, the wavelength behind the source (which is where the listener in Fig. 16.30 is located) is 1.23 m, so

$$f_L = \frac{v}{\lambda} = \frac{340 \text{ m/s}}{1.23 \text{ m}} = 276 \text{ Hz}$$

Even though the source is moving, the wave speed v relative to the stationary listener is unchanged.

Example 3

Example 16.17 **Doppler effect III: A moving listener**

If the siren is at rest and the listener is moving away from the siren at 30 m/s, what frequency does the listener hear?

SOLUTION

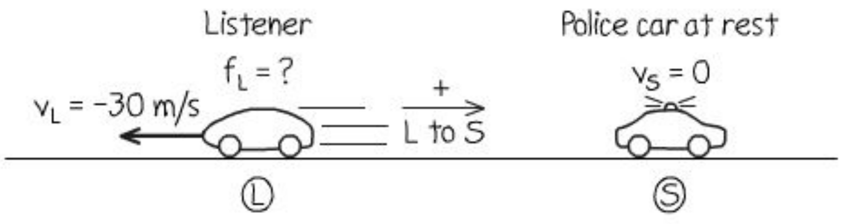
IDENTIFY: Again our target variable is the frequency f_L heard by the listener, but now the listener is in motion and the source is at rest.

SET UP: Figure 16.31 shows the situation. The positive direction (from listener to source) is still from left to right, so $v_L = -30\text{ m/s}$.

EXECUTE: From Eq. (16.29),

$$f_L = \frac{v + v_L}{v} f_S = \frac{340\text{ m/s} + (-30\text{ m/s})}{340\text{ m/s}}(300\text{ Hz}) = 274\text{ Hz}$$

16.31 Our sketch for this problem.



EVALUATE: Again the frequency heard by the listener is less than the source frequency. Note that the *relative velocity* of source and listener is the same as in Example 16.16, but the Doppler shift is different because the velocities relative to the *air* are different.

Example 4

Example 16.18 Doppler effect IV: Moving source, moving listener

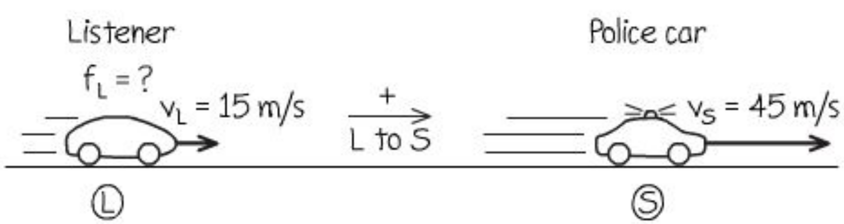
If the siren is moving away from the listener with a speed of 45 m/s relative to the air and the listener is moving toward the siren with a speed of 15 m/s relative to the air, what frequency does the listener hear?

SOLUTION

IDENTIFY: Now *both* the listener and the source are in motion. Once again our target variable is the frequency f_L heard by the listener.

SET UP: Figure 16.32 shows the situation. Both the source velocity $v_S = 45$ m/s and the listener's velocity $v_L = 15$ m/s are positive because both velocity vectors point in the direction from listener to source.

16.32 Our sketch for this problem.



EXECUTE: Once again using Eq. (16.29), we find

$$f_L = \frac{v + v_L}{v + v_S} f_S = \frac{340 \text{ m/s} + 15 \text{ m/s}}{340 \text{ m/s} + 45 \text{ m/s}} (300 \text{ Hz})$$
$$= 277 \text{ Hz}$$

EVALUATE: The frequency heard by the listener is again less than the source frequency, but the value is different than in the preceding two examples, even though the source and listener move away from each other at 30 m/s in all three cases. The *sign* of the Doppler shift of frequency (that is, whether f_L is less than or greater than f_S) depends on how the source and the listener are moving relative to each other; to determine the *value* of the Doppler shift of frequency, you must know the velocities of source and listener relative to the air.