Physics 303 Fall 2022 Exam 1 NAME: Solutions

1) Find the angle between the two vectors **a** = (3, 4, 2) and **b** = (4, 3, 3).

$$\vec{a}\cdot\vec{b} = ab\cos\theta_{ab} = a_1b_1 + a_2b_2 + a_3b_3$$

$$a = \sqrt{a_1^2+a_2^2+a_3^2} \quad \text{and} \quad b = \sqrt{b_1^2+b_2^2+b_3^2}$$

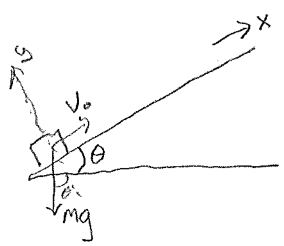
$$a = \sqrt{29} \qquad b = \sqrt{34}$$

$$\therefore \theta_{ab} = \cos^{-1}\left[\frac{3\cdot 4 + 4\cdot 3 + 2\cdot 3}{\sqrt{29}\sqrt{34}}\right]$$

$$= \cos^{-1}\left[\frac{30}{\sqrt{29}\cdot\sqrt{34}}\right] = \cos^{-1}\left[0.9554\right]$$

$$= 0.2998 \text{ rad} = 17.177^\circ$$

2) A frictionless puck is launched up a plane, inclined at angle θ with respect to the horizontal, with initial velocity v_0. How long will it take to return to its starting point, and what was the total distance traveled? **Ignore air resistance.**



$$F_x = -mg\sin\theta = m\dot{V}_x$$

$$\therefore V_x = V_0 - g\sin\theta\cdot t$$

$$x = V_0t - \tfrac{1}{2}g\sin\theta\cdot t^2 \quad (\text{With } x_0 = 0)$$

when $V_x = 0$, $V_0 = g\sin\theta\cdot t \Rightarrow t = \dfrac{V_0}{g\sin\theta}$

so, total trip is twice this.

at the top,

$$x = V_0\left(\frac{V_0}{g\sin\theta}\right) - \frac{1}{2}\cdot g\sin\theta\left(\frac{V_0}{g\sin\theta}\right)^2$$

$$= \frac{V_0^2}{2g\sin\theta}$$

so, total trip is twice that.

3) A mass m on the end of a rod of length R is rotating in a vertical circle with a constant angular velocity ω. Write down Newton's second law for the mass in polar coordinates, and find the force of the rod on the mass when it is at its lowest point. **Ignore air resistance.**

$\sum F_r = m(\ddot{r} - r\dot{\phi}^2)$ but, $r = R$ = constant

$\sum F_\phi = m(r\ddot{\phi} + 2\dot{r}\dot{\phi})$ and $\dot{\phi} = \omega$ = constant $\Rightarrow$

$\sum F_r = -mr\dot{\phi}^2 = mr\omega^2$

$\sum F_\phi = 0$

Let's label the force on the mass from the rod f_r and f_ϕ

and the force on the mass from gravity $\vec{F}_g = -mg\sin\phi\,\hat{r} - mg\cos\phi\,\hat{\phi}$

So, $\sum F_r = f_r - mg\sin\phi\,\hat{r} = -mr\omega^2$

$\sum F_\phi = f_\phi - mg\cos\phi = 0$

at the bottom, $\phi = 270^\circ \Rightarrow$

$f_r = mg\sin\phi - mr\omega^2$

$= -m(g + r\omega^2)$

and $f_\phi = 0$

4) A mass m has velocity v_0 at time t=0, and coasts along the x axis in a medium where the drag force is $F(v) = -cv^{3/2}$. Use the method of separation of variables to find v in terms of time t and the other given parameters.

$$F = m\dot{v} = -cv^{3/2} \Rightarrow$$

$$m\frac{dv}{dt} = -cv^{3/2} \Rightarrow \int_{v_0}^{v} \frac{dv'}{v'^{3/2}} = \frac{c}{m}\int_0^t dt'$$

$$-2v'^{-1/2}\Big|_{v_0}^{v} = \frac{-c}{m}t$$

$$v^{-1/2} - v_0^{-1/2} = \frac{c}{2m}t$$

$$v = \left(\frac{c}{2m}t + v_0^{-1/2}\right)^{-2}$$

$$v = \frac{v_0}{\left(1 + \frac{ct\sqrt{v_0}}{2m}\right)^2}$$

5) A motionless moth of mass 3 grams is struck by the windshield of a car moving at 32 m/s. The collision takes place over the span of one microsecond. What is the average force that the moth puts on the windshield?

$$\text{Force on moth} = F_{Avg} = \frac{\Delta p}{\Delta t} = \frac{p_i - p_f}{\Delta t} = \frac{0 - mv}{\Delta t} = -\frac{0.003\,kg \cdot 32\,m/s}{1\times 10^{-6}\,s}$$

$$= -96{,}000\,N$$

$\therefore$ From Newton's 3rd law

$$F_{avg\ on\ windshield} = 96{,}000\,N$$

6) The transverse velocity of a particle in a magnetic field is given by $\eta = v_x + iv_y$, which can also be written as $\eta = ae^{i\delta}e^{-i\omega t}$. Use Euler's formula to describe the time dependence of the transverse velocity.

$$\eta = V_x + iV_y = ae^{i\delta}e^{-i\omega t}$$

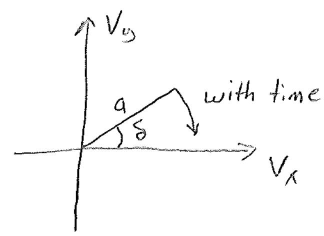
and $V_x = Re(\eta)$, $V_y = Im(\eta)$

$$\eta = ae^{i\delta}e^{-i\omega t} = ae^{i(\delta - \omega t)} = a\left[\cos(\delta - \omega t) + i\sin(\delta - \omega t)\right]$$ Euler's Formula

$\therefore V_x = a\cos(\delta - \omega t)$, $V_y = a\sin(\delta - \omega t)$

The magnitude of $V = (V_x^2 + V_y^2)^{1/2} = \left(a^2\cos^2(\delta - \omega t) + a^2\sin^2(\delta - \omega t)\right)^{1/2} = a$

at $t = 0$, $V_x = a\cos\delta$, $V_y = a\sin\delta$



7) Write z = 4 + 2i in the form $z = re^{i\theta}$.

$$r = \sqrt{4^2 + 2^2} = \sqrt{20}$$

$$\theta = \tan^{-1}\left(\frac{Im(z)}{Re(z)}\right) = 0{,}46 \text{ rad}$$

$$\therefore z = \sqrt{20}\, e^{i(0{,}46)}$$