Possible short-Projects for Physics 495

due 25 November, 2009

by Daniel Finley, 28 October, 2009

The idea of this project is to consider some interesting question relating to the application of special relativity and/or differential forms to an interesting problem of physics or astronomy. It is supposed to require approximately 4 weeks of moderately-concentrated effort to

a. first do some reading about the problem, and

b. then perform some calculations guided by that reading.

I anticipate that the result will be approximately 10 pages of **typed** copy, which explains what you read, what you learned from it, and, of course, the associated calculations, including a complete list of references.

The list given below are some suggestions that occurred to me, for reasonable and acceptable projects for this purpose, and should be considered as guidelines toward your own selection. Therefore, you may either choose one of those below, or, if you have something of more special interest to yourself we should discuss that a bit, before I am willing to approve them as appropriate for this purpose.

I insist that you tell me what your selection is, and have it approved by Monday, 2 November, after classtime, so that you will have sufficient time to complete the work.

1. Look at the product of two general, non-collinear Lorentz boosts. Determine the rotation and pure boost that this product is equivalent to, both in the order $R_1\Lambda_1$ and $\Lambda_2 R_2$. Then determine how the two sets of rotation parameters above, and the two sets of boost parameters, are related to the two input velocities, for the two original boosts.

2. Consider the two 3-dimensional representations of the Lorentz group, $D(1,0)$ and $D(0,1)$, and determine the mappings between each of them and our standard 4-dimensional presentation of the elements of that group. Do the same thing for the 4-dimensional, so-called Dirac representation, $D(1/2,0) \oplus D(0,1/2)$.

3. Consider the total stress-energy tensor for a perfect, charged fluid, moving through some electromagnetic fields, which will involve both the stress-energy for the fluid itself and also the associated electromagnetic fields. Use this tensor to determine the modifications to the usual equations of fluid flow that result from the generalizations described above, and also from the fact that we are assuming that the motion is allowed to be quite fast.

4. The source-free Maxwell equations and the Lorentz gauge choice, which allows us to require that the 4-potential satisfy the constraint equation, $*d*\underset{\sim}{A} = 0$. Therefore, one may determine a 2-form, $\underset{\approx}{\Pi} = \underset{\approx}{\Pi}(\widetilde{x})$, such that $\underset{\sim}{A} = *d\underset{\approx}{\Pi}$. This is the tensor version of what is usually referred to as either the Hertz potential or the Debye potential. First determine how these 3 different potentials are related, and then consider some interesting special case—I recommend the case of magnetic dipoles—and determine the associated Debye potential, and the resultant potential, $\underset{\approx}{\Pi}$, and show how it does in fact create the associated electric and magnetic fields.

5. Consider the 4-dimensional rates of change of the 4-velocity of a fluid, i.e., the tensor which is the covariant derivative of the 4-velocity of that fluid: $\nabla_\alpha u_\beta$. Decompose this tensor into physically-relevant parts, using the tensor that projects any vector onto a surface perpendicular to $\widetilde{u}$, namely $P_{\alpha\beta} \equiv g_{\alpha\beta} + u_\alpha u_\beta$. These various parts have standard names: acceleration, expansion, rotation, and shear. Therefore, first explain why these names are reasonable, and then determine how they are constructed from the tensor above, and then show how each of these quantities transforms under standard Lorentz boosts. These quantities are actually quite important in modern understandings of the way the expansion of the universe is used to create cosmology.

6. Consider a rotating coordinate system, in oblate-ellipsoidal coordinates, $\{r, \phi, z, t\}$ with constant rotation rate, ω. This is appropriate, say, for discussion of the electromagnetic field generated by a rotating star such as our sun, or, if you want one that rotates rather faster, a neutron star. Determine the form of the wave equation for both a scalar, such as a scalar potential, Φ, but also for a 4-vector, such as the electromagnetic potential, $\widetilde{A}$. This is best done in its differential form mode, by calculating $(d * d * + * d * d)\underset{\sim}{A}$.

7. Study a special-relativistic generalization of thermodynamics, including especially the Lorentz-transformation behavior for temperature, entropy, etc. Write down the first and second laws of thermodynamics for a relativistically-moving fluid. Use this to answer some simple questions that might be relevant to you. An example of one would be: a thermally-conducting black sphere with an attached thermometer moves with a velocity v through a black-body radiation field of temperature T_0—such as the CMB radiation in the universe. What does the thermometer read, as a function of v? The discussions in Schutz would be, at least, a good place to begin, perhaps along with some of the problems he outlines on this subject.

8. Learn more about the momentum-space representations of the Poincaré group for a particle of fixed mass and spin. Consider both the cases of spin 0 and spin 1/2, determining the action of Lorentz boosts and rotations on scalar functions, for spin 0, and 2-spinor-valued functions, for spin 1/2.

9. Consider the generalization of angular momentum, and of spin, to 4 dimensions. Discuss the relation of the center-of-mass motion to the orbital angular momentum, for a not-too-large body. Determine an understanding of the Pauli-Ljubanski vector that generalizes spin to 4 dimensions. Since, quantum-mechanically, one may only determine the value of the component of the spin in one direction, explain how one's choice of that direction changes as its velocity changes. Also explain how this relates to the concept called "helicity."