

Physics 495

Homework No. 3

due Wednesday, 16 September, 2009

1. An incoming light ray is being observed in both \mathcal{O} and \mathcal{O}' . The direction from which it is coming is measured by \mathcal{O} as α and by \mathcal{O}' as α' . Show that

$$\tan(\alpha'/2) = \sqrt{\frac{1-\beta}{1+\beta}} \tan(\alpha/2).$$

Now consider a source of light that emits isotropically in its own rest frame. When considered from a different frame in which it is moving with velocity v , show that half of the light, i.e., half of the total number of photons emitted, are emitted into a cone with half angle $\sin \theta = 1/\gamma$, so that they are somewhat narrowly focused into the forward direction.

2. Show that the following is the transformation equation between frames \mathcal{O} and \mathcal{O}' for the usual 3-acceleration, \vec{a} , i.e., the time derivative of the velocity \vec{v} :

$$\vec{a} = \left(\frac{\sqrt{1-\beta^2}}{1+\vec{\beta}\cdot\vec{v}'} \right)^3 \left[\vec{a}'_{\parallel} + \gamma_{\beta} \vec{a}'_{\perp} + \gamma_{\beta} \vec{\beta} \times (\vec{a}' \times \vec{v}') \right].$$

3. Please do problem 2.22 in Schutz' text.
4. A very long limousine has a proper length of 20 meters. It is to be driven through a garage whose proper length is only 15 meters. Given that the front door will be open when the car begins to drive through, and that the back door will not open until the rear bumper is completely inside and the door closed, this should be possible—without destroying the garage—if the limousine is driven sufficiently fast, via the perceived Lorentz contraction of the car. For example, suppose the limousine is driven at a forward speed of $v = 0.8$; then the Lorentz contraction factor should be $1/\gamma = 3/5$. Let us consider the four world lines of the front and rear of the garage and of the front and rear of the limousine. As well consider the following two events. Event 1 is when the rear bumper just clears the front door of the garage and that door is then closed. [For simplicity, put the origin of the reference frames at this event.] Event 2 is when the front bumper of the limousine reaches the back door, when that door opens (immediately) to prevent it from being

destroyed. Please draw a Minkowski diagram, from the point of view of the stationary observer standing next to the stationary garage, showing all 4 world lines and these two events. Be sure to arrange things so that Event 2 occurs after Event 1.

However, we now want to consider what this entire situation looks like from the point of view of the chauffeur inside the limousine. Therefore, please create another Minkowski diagram from the chauffeur's point of view, showing all 4 world lines and the two events. In this frame, in what order do the two events occur?