Physics 495

Homework No. 4 due Wednesday, 30 September, 2009

1. Let $f = f(x, y, z, t)$ be a function of the four (independent) variables shown, which are being measured by observers in the reference frame $\mathcal{O}$. Set the differential of f in the standard way:

$$df \equiv \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz + \frac{\partial f}{\partial t}dt \,.$$

Now let $\mathcal{O}'$ be a second reference frame, who $\mathcal{O}$ measures as moving with velocity $\vec{\beta} = \beta\hat{z}$. Using the Lorentz transformation between the two frames, determine the components of the differential df that would be measured by $\mathcal{O}'$; i.e., how is $\partial f/\partial x'$, and the other partial derivatives, related to those derivatives measured by $\mathcal{O}$?

2. Using the matrices $\mathcal{K}_i$ that are the generators for pure Lorentz transformations, and also the matrices $\mathcal{J}_j$ that are the generators for rotations, calculate the following matrices:

$$(\vec{\beta}\cdot\vec{\mathcal{K}})^3 \,, \qquad (\vec{\theta}\cdot\vec{\mathcal{J}})^3 \,, \qquad [\vec{\theta}\cdot\vec{\mathcal{J}}, \vec{\beta}\cdot\vec{\mathcal{J}}] \,, \quad [\vec{\theta}\cdot\vec{\mathcal{J}}, \vec{\beta}\cdot\vec{\mathcal{K}}] \,, \quad [\vec{\theta}\cdot\vec{\mathcal{K}}, \vec{\beta}\cdot\vec{\mathcal{K}}] \,.$$

We note that this "dot product" of an ordinary 3-dimensional vector and a *matrix-valued* 3-dimensional vector should be thought of as follows:

$$\begin{aligned}
\vec{\beta}\cdot\vec{\mathcal{K}} &= \beta^x\mathcal{K}_x + \beta^y\mathcal{K}_y + \beta^z\mathcal{K}_z \\
&= \begin{pmatrix} 0 & 0 & 0 & -\beta^x \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\beta^x & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta^y \\ 0 & 0 & 0 & 0 \\ 0 & -\beta^y & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta^z \\ 0 & 0 & -\beta^z & 0 \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 & 0 & -\beta^x \\ 0 & 0 & 0 & -\beta^y \\ 0 & 0 & 0 & -\beta^z \\ -\beta^x & -\beta^y & -\beta^z & 0 \end{pmatrix} \,.
\end{aligned}$$

3. Please do problem 3.4 in Schutz' text.

4. Please do problem 3.24 in Schutz' text.

5. Please do problem 3.33 in Schutz' text. However, please append an additional section into his list of 4 parts, perhaps, between his section (c) and section (d), as follows:

Show that the $(4,4)$-element in an arbitrary matrix representing the Lorentz group, $\boldsymbol{O}(3,1)$, i.e., $L^4{}_4$, has the property that $(L^4{}_4)^2 \geq +1$.

6. Please do problem 3.34 in Schutz' text.

7. Consider a particle of mass m moving with a velocity $\vec{v}(t)$, that is not too much below the speed of light, and an acceleration $d\vec{v}/dt \equiv \vec{a}(t)$. The acceleration is due to the action on the particle by a force $\vec{F}(t)$. Divide the force and the acceleration into their components that are either parallel or perpendicular to the particle's velocity, at any particular instant, and determine the scalar proportionality factors between the force and the acceleration, for each of the two sorts, i.e., for the parallel components and the perpendicular components. As these two proportionality components are different, this assures us that the force 3-vector is not parallel to the acceleration 3-vector.