Physics 495

Homework No. 6 due Wednesday, 14 October, 2009

1. A charge q is released from rest at the origin, in the presence of a uniform electric field $\vec{E} = E_0\hat{z}$ and a uniform magnetic field, $\vec{B} = B_0\hat{x}$, where we assume that $E_0 < B_0$. Determine the trajectory of the particle by first transforming to a reference frame in which $\vec{E} = 0$, finding the path in that frame, and then transforming back to the original frame.

2. Consider the following 4×4 matrix:

$$L = \begin{pmatrix} -1 & 0 & a & -a \\ 0 & 1 & 0 & 0 \\ -a & 0 & -1 + a^2/2 & -a^2/2 \\ a & 0 & -a^2/2 & 1 + a^2/2 \end{pmatrix} .$$

a. Please show that L is in fact a special, orthochronous Lorentz transformation, for all real values of the constant parameter a. However, explain why we are sure that it is neither a pure rotation nor a pure Lorentz boost.

b. Even though L is in the connected part of the Lorentz group that contains the identity, it is nevertheless not sufficiently near the identity that it can be written as a single exponential. However, it can be written as the product of a rotation $R(\theta; \hat{y})$ multiplied on the right by a pure boost. What is the direction of the velocity associated with this boost?

3. Please calculate the matrix presentation, in terms of the electric and magnetic fields, $\vec{E}$ and $\vec{B}$, for the energy-momentum tensor or the electromagnetic field:

$$4\pi M^{\mu\nu} \equiv \mathcal{F}^{\mu}{}_{\lambda}\mathcal{F}^{\nu\lambda} - \tfrac{1}{4} g^{\mu\nu}\mathcal{F}^{\alpha\beta}\mathcal{F}_{\alpha\beta} .$$

Try to present it in the following form:

$$\begin{pmatrix} 3 \times 3 \text{ dyadic matrix} & 3\text{-vector} \\ 3\text{-vector}^T & 3\text{-scalar} \end{pmatrix} ,$$

and then a physical interpretation/name for these different portions.

(Note: a *dyadic matrix* is one created from the matrix product of a column vector multiplied by a row vector on the right, i.e., of the form $\vec{A}\vec{B}^T$.)