Physics 303

Bonus Homework No. 1 due Tuesday, 28 October, 2008

60 Homework points

Let us consider a more realistic version of some of the air-resistance complications that actually arise when sending projectiles through the air. In particular, one particular complication arises when projectiles are sent high into the air, because the air resistance decreases as the density of the atmosphere decreases. When, for instance, a cannonball is sent into the air it suffers a quadratic air resistance, with magnitude $c\,v^2$, and a direction opposite to the motion at that instant. However, since the "constant" c depends on the density of the air, and that density now varies as one increases the height—in particular it decreases exponentially with height—so that we should, instead, write the magnitude as $c(y)\,[v(t)]^2$, with $c(y) = \gamma D^2\, e^{-y/\lambda}$, where λ is the exponential scale factor for the air, $\gamma = 1/4$ for a spherical shape, and D is the diameter of the cannonball. The diameter is of course constant, but is related to the mass, which is involved in the process as well; therefore, choose a 15 centimeter diameter cannonball with the density of iron as 7.8 g/cm^3. The scale factor for air varies with the temperature, which we do NOT want to also include, so just take it as 7.6 kilometers, for sort of an average appropriate at temperatures like 260 Kelvin.

The purpose of this problem, then, is to create some graphs of cannonball trajectories that rise relatively high above the surface of the earth—assumed flat, and with a constant (downward) gravitational acceleration. As a reasonable, straightforward approach, consider that the cannonball is given an initial speed of $v_0 = 400$ meters/second, and is sent upward with an initial angle of $\theta_0 = 60°$.

As discussed in class, already the equations, even without this extra complication, for trajectories that move in both the horizontal and vertical directions and experience quadratic drag cannot be solved analytically, but will require computer calculations. Therefore, let us define the problem as follows.

1. First write down explicitly the $\hat{x}$-direction and $\hat{y}$-direction pieces of the equations of motion, i.e., Newton's Second Law, for this cannonball. Then use an appropriate computer program to solve those equations under the initial conditions described above, creating graphs that show the trajectory, with some decent tick-marks on the graphs so that we can calibrate our understanding of the results. As well, do the same thing for two other cases, namely the one where we ignore the changes of c with respect to height, and also where we ignore air resistance altogether. Then plot all three of these trajectories on the same graph, so that we can compare the results. You should of course plot the trajectories for however much time is necessary until the cannonball returns again to the ground.

2. Then do the same computer calculations again, but with $\theta_0 = 30°$, so that the trajectory will not go nearly as high, so that we see how important this effect was. Recall that with no air resistance these two angles should have exactly the same range, i.e., the same total horizontal distance of travel.