Physics 303

Useful "Trigonometric" Integrals

Note: all integrals given below are indefinite, and I have ignored a constant of integration. However, in physical situations, one should always treat them as definite integrals, inserting upper and lower limits appropriate to the problem.

1. $$\int \frac{dx}{a+bx} = \frac{1}{b}\,\ln(a+bx)\ ,$$

2. $$\int \frac{dx}{a^2+b^2x^2} = \frac{1}{ab}\,\mathrm{Tan}^{-1}(bx/a)\ ,$$

3. $$\int \frac{dx}{a^2-b^2x^2} = \begin{cases} \dfrac{1}{ab}\,\mathrm{Tanh}^{-1}(bx/a)\ , & (bx)^2 \le a^2\ , \\ \dfrac{1}{ab}\,\mathrm{Coth}^{-1}(bx/a)\ , & (bx)^2 \ge a^2\ , \end{cases}$$

4. $$\int \frac{dx}{\sqrt{x^2+a^2}} = \mathrm{Sinh}^{-1}(x/a)\ ,$$

5. $$\int \frac{dx}{\sqrt{x^2-a^2}} = \mathrm{Cosh}^{-1}(x/a)\ , \quad x^2 \ge a^2\ ,$$

6. $$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\,\mathrm{Cos}^{-1}(a/x)\ , \quad x^2 \ge a^2\ ,$$

7. $$\int \frac{dx}{\sqrt{a^2-x^2}} = \mathrm{Sin}^{-1}(x/a)\ , \quad x^2 \le a^2\ .$$

The inverse functions shown are to be taken as the "principal values," which are the regions that contain zero.