Physics 406

Problem Sessions: Questions

1. 29 August, 2006

a. Problem 8.3

b. Problem 8.7, along with Example 8.4

c. Problems 8.8a, b, and also 8.10

2. 5 September, 2006

a. as it was not completed last time, I outlined how to do 8.8b, which I thought was useful

b. Take two electric field vectors, one left-circularly polarized and the other right-circularly polarized, and this second one 90 degrees ahead in phase relative to the first one. Add them together to determine the total electric field in question. [It is best to first write everything as complex-valued and complete the calculation in that manner before, at the end, taking the real part.]

c. Calculate the Maxwell stress tensor for a plane electromagnetic, plane-polarized monochromatic wave travelling in the $\hat{z}$-direction

d. A plane-polaried, monochromatic plane wave, of frequency ω, moves through glass, with index of refraction $n = 1.5$. It passes through the two points $(1, 1, 1)$ and $(0, 2, 0)$. Write down the electric and magnetic field for this wave.

e. Let the complex form of an electric field be $\widetilde{\vec{E}} = \widetilde{E}_0 \hat{y} e^{iu}$, where, as usual, $u \equiv \vec{k} \cdot \vec{r} - \omega t$, but, distinct from our practice so far, take the wave vector to be complex, of the form $\vec{k} = k\hat{x} + i\kappa\hat{z}$. Find the real form for the electric and magnetic fields, and their associated Poynting vector and energy densities.

f. Consider the approach used in class that shows that if $\psi(\vec{r}, t)$ is a solution of the scalar wave equation then $\vec{r} \times \nabla\, \psi$ is a solution of the vector wave equation, and consider the very special solution of the scalar wave equation, in spherical coordinates, $\psi = \psi(r, t) = (1/r)e^{i(kr-\omega t)}$. Determine the associated solution of the vector wave equation. Generalize this to an arbitrary solution of the scalar wave equation in spherical coordinates, namely $\psi = f(r - \omega t)/r$.

3. 12 September, 2006

a. Problem 9.17, concerning properties of diamond for parallel-electric-field waves

b. Derive, from the (4) boundary conditions for electric and magnetic fields at plane interfaces between two different media, the relations between the incoming electric field and the outgoing reflected and transmitted electric fields, in the case when the incoming electric field polarization is normal to the plane of incidence. Given the incoming field, including its wave vector, this requires determining the relations between it and the other two wave vectors, as well as ratios of the (complex) amplitudes.

c. A long solenoid of radius a, carrying steady current I, and having n turns per unit length, is looped by a closed piece of wire. A particular portion of that loop has a total resistance of amount R. During some short time the solenoid is pulled out of the loop, turned around, and re-inserted. What total charge pases through the resistor, R, during this time? Ignore any fringing fields at the ends of the solenoid.

d. A particular travelling electromagnetic wave has an electric field given in complex form by

$$\tilde{\vec{E}}(\vec{r},t) = E_0(-i\hat{x} + \hat{y})e^{i\omega(2z/c-t)} \ .$$

a. Taking E_0 as a real constant, what is the physical form for this electric field?

b. What is the wave vector, $\vec{k}$ for this field? Please include correct dimensions?

c. What is the physical form for the magnetic field associated with this wave?

d. What is the index of refraction of the material through which the wave is moving?

e. Determine the scalar product of the electric and magnetic fields, $\vec{E} \cdot \vec{B}$.

f. Determine the cross product of these two fields, which is of course proportional to $\vec{S}$.

4. 19 September, 2006

a. We had the first exam during this time.

5. 26 September, 2006

a. Prob. 9.36, involving the minimum and maximum of the transmissivity of light from water, through glass, and into air, where it is the width of the glass that is being varied.

b. Prob. 9.18a, to consider the rate at which charge moves to the edges in a good conductor.

c. Prob. 9.20, to determine the ratio of the energy densities due to the magnetic and the electric fields in a conductor.

d. To consider the time variation of an electromagnetic wavepacket which started out as a Gaussian, but was moving in a dispersive medium with a non-zero cutoff frequency. This problem was somewhat complicated, but very important; therefore, I have posted a solution for it at http://panda.unm.edu/courses/finley/p466/DispersiveGaussian.pdf .

6. 3 October, 2006

a. A particular monochromatic wave has the complex form of its electric field given by

$$\vec{E}(z,t) = E_0(\hat{x} + i\hat{y})\, e^{i(kz-\omega t)} \ .$$

Find the associated magnetic field, the real forms of both the electric and magnetic field. Also find the time-dependent forms of the two different energy densities, u_e and u_m, and the Poynting vector.

b. A monochromatic, plane wave is incident normally on a conductor, with conductivity g. Determine the relative phase of the outgoing, transmitted wave to the incoming wave.

c. Problems 12.8 and 12.13.

7. 10 October, 2006

a. Problem 12.27, concerning a particle in hyperbolic motion.

b. Problem 12.35, concerning the annihilation of a moving electron by a positron at rest, and the subsequent emission of two, not one, photons.

c. Problem 12.39, concerning the form of the invariant “length” [or magnitude] of the 4-force.

8. 17 October, 2006 Instead of quite the usual approach where there were some 3 or 4 problems considered, each by separate groups, I posed several questions/problems, after dividing the attendees into 3 groups, and solicited responses.

1. What follows is a list of at least most of those questions:

   i.) What are the Fresnel coefficients, for both reflection and transmission, of an incoming, plane, plane-polarized electromagnetic wave, incident at an interface between two media, at an angle θ relative to the normal to the interface, for both possible cases of the direction of the plane polarization?

ii.) Suppose that a given 4-vector, $\widetilde{A}$, has components $(\beta\cosh(\alpha), 0, 0, \beta\sinh(\alpha))$. Is it space-like, lightlike, or timelike? Suppose we switch the cosh and sinh; does this change the answer?

iii.) Given a particular 4×4 Lorentz transformation, with relatively few zeroes among its elements, what is the 3-velocity to which it would "boost" an object which was originally at rest?

iv.) Given the 4-velocity, $\widetilde{u} = \left(3, -3, 0, \sqrt{19}\right)$, what is the associated 3-velocity, and what is the boost matrix that would boost a particle from rest to have this 4-velocity?

2. Then I asked that the 3 groups work, separately, on the questions, what are $\Lambda(v_1\hat{x})$, $\Lambda(v_2\hat{y})$, and their products, in both possible orders. Then I asked that each one of these two products should be allowed to act on a particle at rest, in order to determine the resultant 4-velocity, and then the 3-velocity associated with each one. Lastly, they showed that although the two resultant 3-velocities were different, they did have the same magnitude (as 3-vectors).

9. 24 October, 2006, will be taken up by the second examination.

10. 31 October, 2006, was cancelled because of feared goblin attacks, on All Hallow's Eve.

11. 7 November, 2006

a. Using the standard form for the Faraday tensor and the current/charge density 4-vector, determine the 4-vector $f^\mu = F^{\mu\nu} J_\nu$. Explain the physical significance of this 4-vector.

b. Problem 10.23

c. Problems 11.1 and 11.3

12. 14 November, 2006

a. Problems 10.25

b. Problmes 11.22, and 11.25

13. 21 November, 2006

a. In the frame $\mathcal{O}'$ an electric dipole with moment $\vec{p}$ is observed to be at rest, and so has an electric potential given by $V' = k(\vec{p}\cdot\hat{r}')/r'^2$. The different frame $\mathcal{O}$ measures $\mathcal{O}'$ to be moving with (arbitrary, but constant) velocity $\vec{v}$. What are the measurements that $\mathcal{O}$ makes for both $V(\vec{r})$ and $\vec{A}(\vec{r})$?

b. Problems 11.13 and 11.23

c. Rewrite Maxwell's equations in terms of the complex-valued vector field $\vec{C} \equiv \vec{B} + i\vec{E}/c$. As well determine, in terms of $\vec{C}$, the 4-tensor $\omega \equiv F + i^*F$, where F is the usual Faraday tensor and $^*F$ is its dual tensor.

14. 28 November, 2006

15. 5 December, 2006

16. on Thursday, 7 December, 2006, we will have the Third Examination.