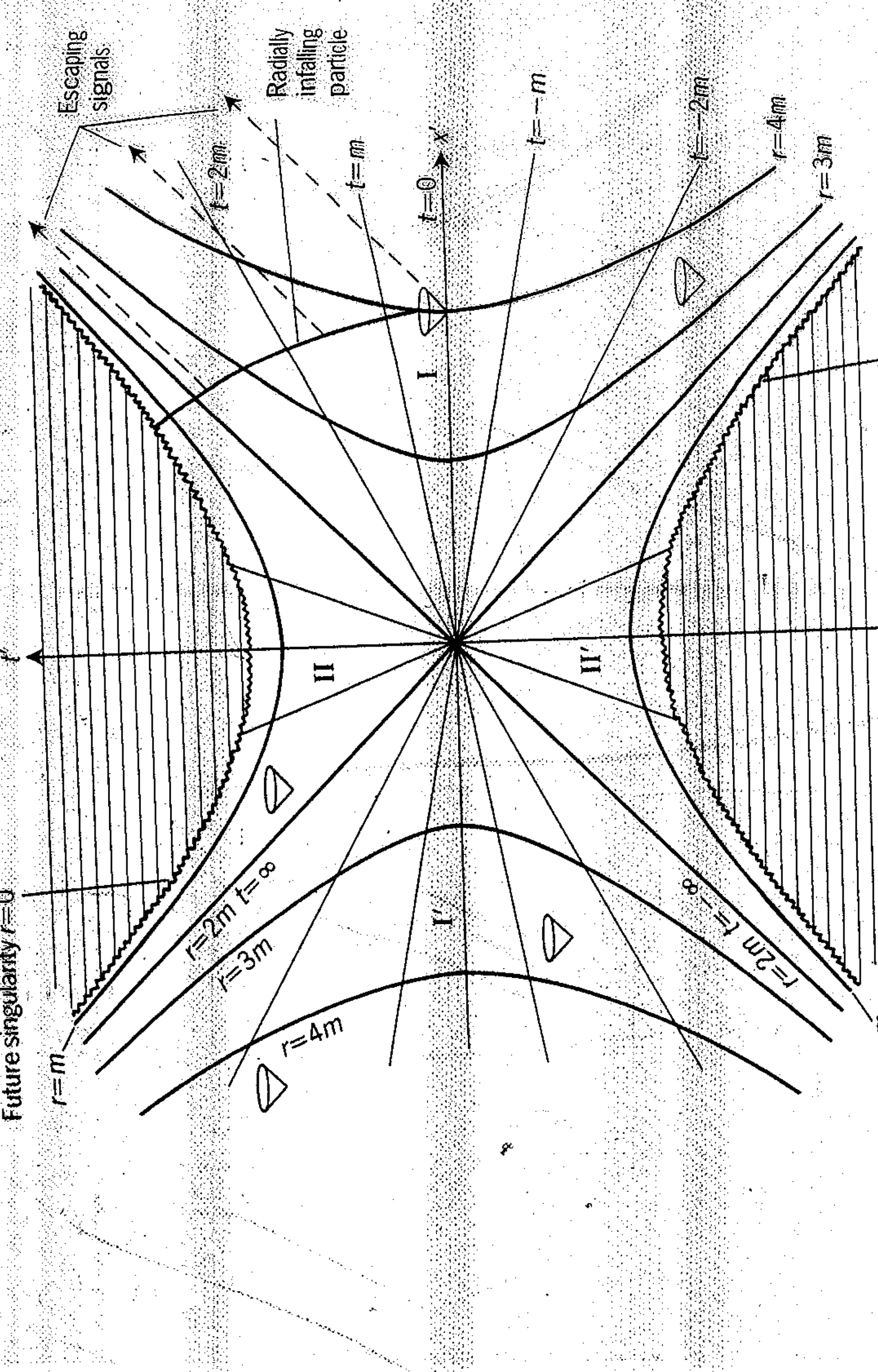


iagram of the

type called a coaxial cable and gives its radii (a, b, c). Equal but opposite currents i are uniformly distributed in the two conductors. Derive expressions for $B(r)$ in the ranges (a) $r < c$, (b) $c < r < b$, (c) $b < r < a$, and (d) $r > a$. (e) Test these expressions for all the special cases that occur to you. (f) Assume that $a = 2.0$ cm, $b = 1.8$ cm, $c = 0.40$ cm, and $i = 120$ A and plot the function $B(r)$ over the range $0 < r < 3$ cm.

48P. The current density inside a long, solid, cylindrical wire of radius a is in the direction of the central axis and varies linearly with radial distance r from the axis according to $J = J_0 r/a$. Find the magnetic field inside the wire.

49P. A long circular pipe with outside radius R carries a (uniformly distributed) current i into the page as shown in Fig. 30-63. A wire runs parallel to the pipe at a distance of $3R$ from center to center. Find the magnitude and direction of the current in the wire such that the resultant magnetic field at point P has the same magnitude as the resultant field at the center of the pipe but is in the opposite direction.

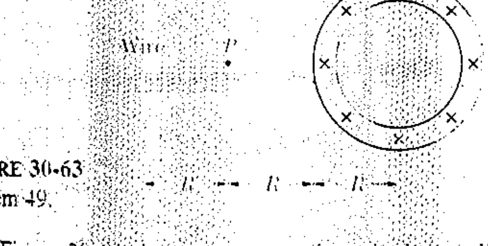


FIGURE 30-63 Problem 49.

50P. Figure 30-64 shows a cross section of a long cylindrical conductor of radius a containing a long cylindrical hole of radius b. The axes of the cylinder and hole are parallel and are a distance d apart; a current i is uniformly distributed over the tinted area. (a) Use superposition to show that the magnetic field at the center of the hole is

$$B = \frac{\mu_0 i d}{2\pi(a^2 - b^2)}.$$

(b) Discuss the two special cases $b = 0$ and $d = 0$. (c) Use Ampère's law to show that the magnetic field in the hole is uniform. (Hint: Regard the cylindrical hole as filled with two equal currents moving in opposite directions, thus canceling each other. Assume that each of these currents has the same current density as that in the actual conductor. Then superimpose the fields due to two complete cylinders of current, of radii a and b, each cylinder having the same current density.)

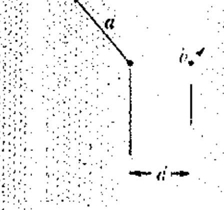


FIGURE 30-64 Problem 50.

to show that for a
below it, the magn
as shown. (b) Use
points P and P'.

FIGURE 30-65 Problem 51.

52P. The magne
las by $B = 3.0i$ −
constant with the
region to cause th
$\oint B \cdot ds$ along the
$d = 0.50$ m in the
dete mine what cu
side length 0.5 m
with one corner at
direction?

SECTION 30-4

53E. A 200-turn s
ter of 10 cm carrie
of the magnetic fie

54E. A solenoid
winding of 1200 tu
magnitude of the

55E. A solenoid
current of 18.0 A
23.0 mT. Find the

56E. A toroid ha
and an inner radius
of 0.800 A. (It is n
as in Fig. 30-17 —
netic field inside th
radius of the toroid

57E. Show that i
pared to its radius
30-26 for the field
field inside a solen

58P. Treat an id
whose current per
axis, is λ. By doin
field inside an idea
the value of the cir
inside the solenoid
that the same chan
current sheet such
this equality surpri

59P. In Section 3

increment.

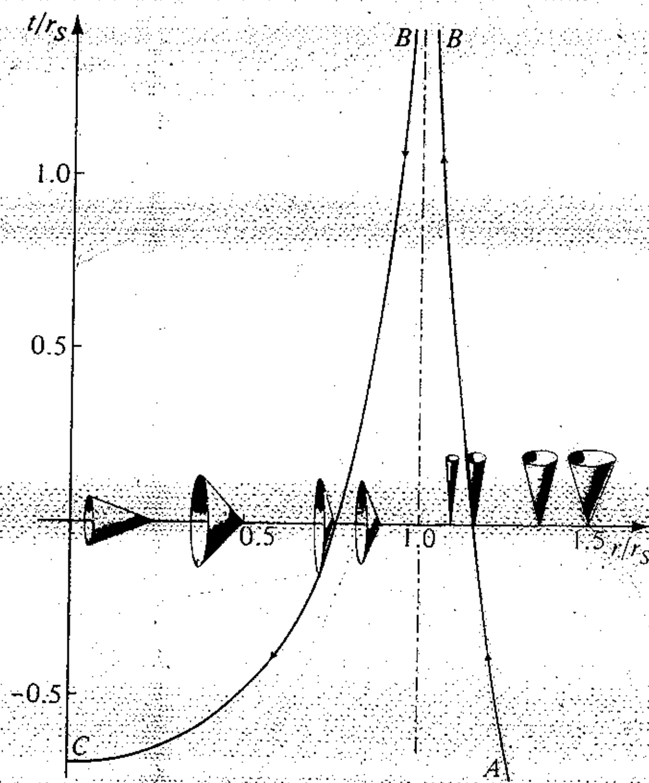
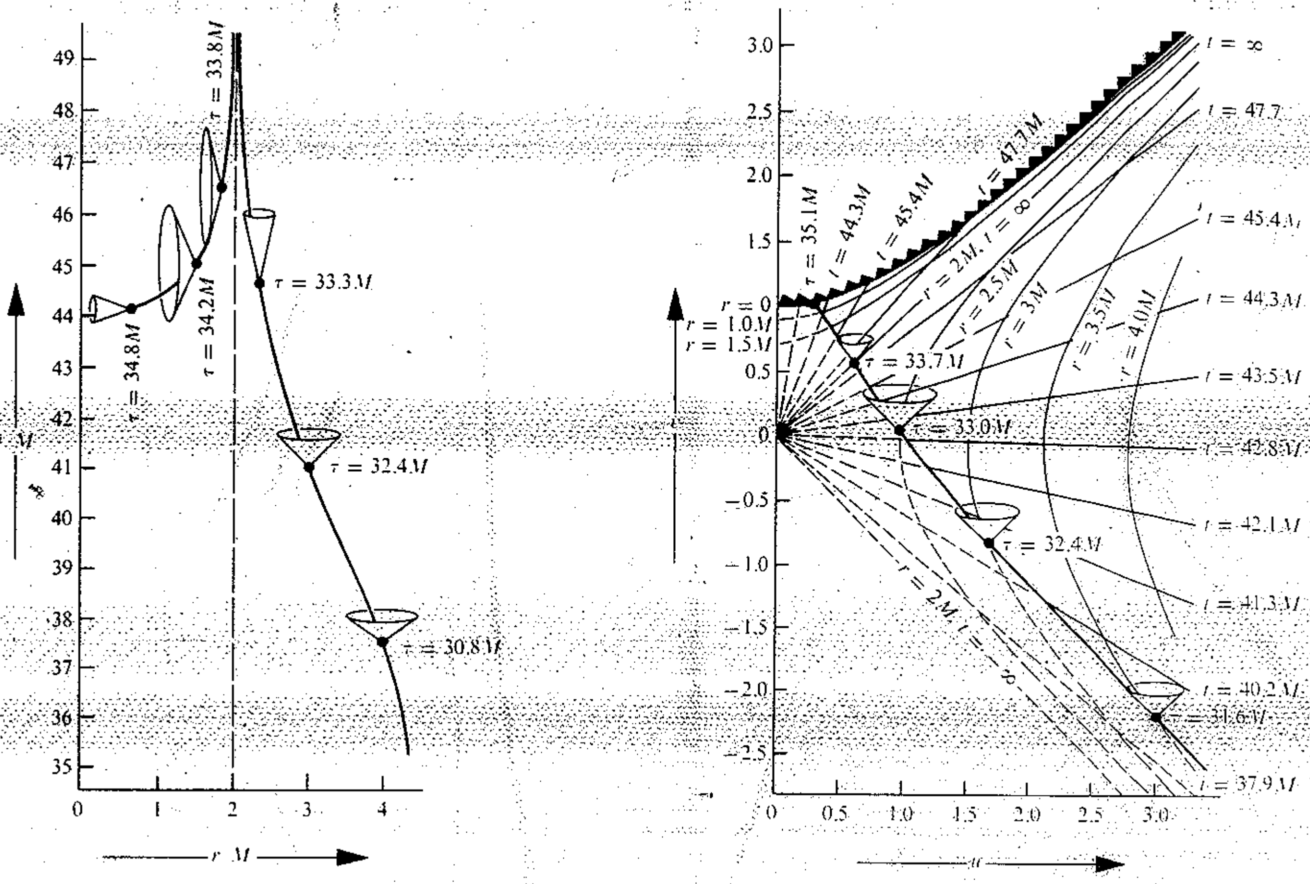


Fig. 8.3 The forward light cones near and inside a black hole. As $r \to \infty$, the light cone assumes its usual shape and direction, that is, $dr/dt = \pm 1$. The curve $ABBC$ is the worldline of an ingoing light signal.

The existence of an event horizon at $r = r_S$ is obvious from an inspection of the light cones in Fig. 8.3. Any kind of signal must necessarily travel in a spacetime direction that lies within a light cone. Since the light cones in the black-hole region are oriented toward $r = 0$, any signal in this region is unavoidably pulled toward decreasing values of r and can never leave the black hole.

Note that the light cones are tangent to the surface $r = r_S$ (indicated by the dashed line in Fig. 8.3); this means that, viewed in spacetime, the horizon is a null surface. This is a general property of event horizons, since a light signal that starts exactly on a horizon and



(a) Schwarzschild

(b) Kruskal-Szekeres

Figure 32.1.

The free-fall collapse of a star of initial radius $R_i = 10\ M$ as depicted alternatively in (a) Schwarzschild coordinates, (b) Kruskal-Szekeres coordinates, and (c) ingoing Eddington-Finkelstein coordinates (see Box 31.2). The region of spacetime inside the collapsing star is grey, that outside it is white. Only the geometry of the exterior region is that of Schwarzschild. The curve separating the grey and white regions is the geodesic world line of the surface of the collapsing star (equations [31.10] or [32.10] with $r_{max} = R_i = 10\ M$). This world line is parameterized by proper time, τ, as measured by an observer who sits on the surface of the star; the radial light cones, as calculated from $ds^2 = 0$, are attached to it.

Notice that, although the shapes of the light cones are not all the same relative to Schwarzschild coordinates or relative to Eddington-Finkelstein coordinates, they are all the same relative to Kruskal-Szekeres coordinates. This is because light rays travel along 45-degree lines in the u,v-plane ($dv = \pm du$), but they travel along curved paths in the r,t-plane and $r,\tilde{V}$-plane.

The Kruskal-Szekeres spacetime diagram shown here is related to the Schwarzschild diagram by equations (31.13) plus a translation of Schwarzschild time: $t \longrightarrow t + 42.8\ M$. The Eddington-Finklestein diagram is related to the Schwarzschild diagram by

$$\tilde{V} = t + r^* = t + r + 2\ M \ln |r/2\ M - 1|$$

(see Box 31.2).

It is evident from these diagrams that the free-fall collapse is characterized by a constantly diminishing radius, which drops from $R = 10\ M$ to $R = 0$ in a finite and short comoving proper time interval, $\Delta\tau = 35.1\ M$. The point $R = 0$ and the entire region $r = 0$ outside the star make up a physical "singularity" at which infinite tidal gravitational forces—according to classical, unquantized general relativity—can and do crush matter to infinite density (see end of §31.2; also §32.6).