Physics 480/581

Problem Session No. 3 Monday, 17 September, 2018

1. Let $\underset{\sim}{Z} = Z_x\,dx + Z_y\,dy + Z_z\,dz + Z_t\,dt$ be a 1-form, where the components depend on all of the usual Cartesian coordinates in spacetime, i.e., $\{x, y, z, t\}$. Create the Hodge dual $*\underset{\sim}{Z}$, which then lives in the vector space Λ^3. Then calculate the exterior derivative of this 3-form, which is then a 4-form. Lastly, calculate its Hodge dual, which is simply a scalar function.

2. $T^{\mu}{}_{\lambda}$ are the components of a tensor of type [1,1], as, perhaps, can be seen from the location of the indices, and is currently presented, as such a tensor, relative to the basis of that vector space, as

$$T = T^{\mu}{}_{\lambda}\,dx^{\lambda} \otimes \frac{\partial}{x^{\mu}}\ .$$

Please use the metric tensor, $\boldsymbol{\eta}_{\mu\nu}$ and/or its inverse to find matrix presentations of $T^{\alpha\beta}$ and $T_{\rho\sigma}$.

3. Begin with the usual form of the Faraday, as a 2-form over spacetime, in special relativity, namely $F_{\mu\nu}\,dx^{\mu} \wedge dx^{\nu}$. Determine the Lorentz invariant quantity $F_{\mu\nu}F^{\mu\nu}$. Then show that if we use the skew-symmetric matrix $\mathbf{F}$ to present the components of the original 2-form, that

a. the quantities $F^{\mu\nu}$ are presented via the matrix $\mathbf{W} \equiv H^T\mathbf{F}H$. Lastly, find the relation between our invariant and the trace of the matrix product of $\mathbf{F}$ and $\mathbf{W}$.

4. Work out the 3-dimensional plus 1-dimensional forms of the proper-time derivative of the 4-momentum, which should involve the 3-dimensional force and power, perhaps?