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On the issue of gravitons

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Abstract
We investigate the problem of whether one can anticipate any features of the graviton without a detailed knowledge of a full quantum theory of gravity. Assuming that in linearized gravity the graviton is in a sense similar to the photon, we derive a curious large number coincidence between the number of gravitons emitted by a solar planet during its orbital period and the number of its constituent nucleons (the coincidence is less exact for extra solar planets since their sample is observationally biased). The coincidence raises a conceptual problem of quantum mechanism of graviton emission, and we show that the problem has no intuitive solution and there is no physical picture of quantum emission from a macroscopic body. In Einstein's general relativity the analogy between the graviton and the photon turns out to be ill founded. A generic relationship between quanta of a quantum field and plane waves of the corresponding classical field is broken in the case of full GR. The graviton cannot be classically approximated by a generic pp wave nor by its special case, the exact plane wave. Furthermore and most important, the ADM energy is a zero frequency characteristic of any asymptotically flat gravitational field, this means that any general relationship between energy and frequency is *a priori* impossible. In particular, the formula $E = \hbar\omega$ does not hold. The graviton must have features different from those of the photon and these cannot be predicted from classical general relativity.

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1. Introduction

The notion of the graviton is popular in modern physics even though any version of quantum gravity (e.g. loop quantum gravity) is still far from providing a well-grounded derivation of the concept. The notion is based on pure analogy with quantum theory of other fields, notably electromagnetic fields. According to the general rules of QFT, quantum fields are the basic ingredients of any matter and particles are the quanta, i.e. grains (or bundles) of the 4-momentum of the fields. The graviton, like the photon, is a fully quantum object. In the case of an electromagnetic field the photon may be approximated, in the low energy limit, by a classical plane monochromatic wave, and the photon's energy E is related to the wave frequency ω by the Planck–Einstein formula $E = \hbar\omega$ or relativistically, $p^\mu = \hbar k^\mu$, with k^μ being the wave 4-vector. Historically, Einstein and others went in the opposite direction and associated a quantum of the electromagnetic field with a monochromatic plane wave. Their conjecture was then fully confirmed in the framework of QED and later in other quantum field theories. The conjecture, though being a basis for the transition to the quantum theory, is compatible with classical electrodynamics and in conjunction with the very concept of a quantum may be anticipated in the classical theory. Namely, a general radiation field may be decomposed into plane waves and then the total 4-momentum is an integral over the momentum space of the product of a relativistic scalar and k^μ; the scalar may be interpreted as the number of photons, each with $p^\mu = \hbar k^\mu$. In this sense, the wave–particle duality is already grounded in the classical field theory.

The concept of the graviton has been based on this analogy and it works in quantum linearized gravity, though not without reservation, see section 4. The problem is whether it may also work in full nonlinear general relativity, the latter being not a field theory in Minkowski spacetime but a theory of the spacetime geometry itself. The well-known difficulties with constructing a quantum theory of gravity clearly indicate that great caution is needed whenever one assigns *a priori* any property to the quantum gravitational field. It is rather expected that the graviton is drastically different from the photon and the quanta of other matter fields.

In section 2, we assume that the concept of graviton is legitimate in quantum-linearized gravity and we apply it to the classical quadrupole formula in the case of gravitationally radiating planets orbiting around the Sun. We find then a large number coincidence (numbers of order 10^{50} for the Earth and 10^{54} for Jupiter) between the number of radiated away gravitons and the number of nucleons in the emitting planet. In the case of eight extrasolar planets with all the known orbital parameters necessary to compute the radiated power, the large number coincidence is less conspicuous (the two numbers may differ by a factor 10^4) and the divergence is likely to be due to an observational bias. The coincidence seems to be rather accidental (at present there are no hints that it may arise from the first principles), nevertheless it is interesting in its own right, and moreover it raises a problem of a physical mechanism responsible for the emission of gravitons by a macroscopic body. We discuss the problem of the mechanism in section 3 and argue, by analogy with QED, that no physical picture of the quantum emission of gravitons (by the individual nucleons? by the whole planet?) may be found and that one's belief in the correctness of this and other predictions of quantized linear gravity is founded solely on making calculations in the framework of this theory.

Section 4 is the heart of the work. We show there how the concept of the photon is anticipated in CED and how the analogous reasoning in linearized general relativity is plagued with troubles with the notion of the gravitational energy density. In full GR the situation is much worse: if the hypothetical graviton carries energy E and momentum $\mathbf{p}$ then it cannot be classically approximated by the pp waves since the only value of the ADM energy and momentum that may be associated with the waves is zero. Furthermore, the ADM energy of any asymptotically flat spacetime is effectively a charge (computed at the spatial infinity) and thus it cannot be classically related to a frequency different from zero. The fundamental relation between energy and wave frequency in the quantum theory of any matter, $E = \hbar\omega$, is broken in quantum gravity. If the gravitational field has a quantum nature (what is not so obvious in the light of recent ideas of emergent gravity) and its quantum does exist, no properties of the graviton can be anticipated from classical general relativity (applying standard quantum mechanics) and the analogy with the photon is false. In other terms, if quantum gravity effects do exist, the correspondence between quantum and classical gravity is more sophisticated than in other quantum fields and any signs of the quantum effects appear in Einstein's general relativity in places where we cannot imagine them at present.

2. The quadrupole formula, gravitons and a large number coincidence

In the linearized version of general relativity (the background spacetime is Minkowski space), the gravitational radiation emitted by any isolated system of masses is dominated by the quadrupole component. The power of this radiation is given by the quadrupole formula. We consider radiation from planets on circular orbits of radius r around the Sun. Let m be a planet's mass and M the solar mass. Then, the emitted power P of gravitational radiation is expressed in terms of the reduced mass μ of the system and the angular velocity of the rotating quadrupole ω:

$$P = \frac{32G}{5c^5}\mu^2\omega^6 r^4. \qquad (1)$$

The angular velocity of any planet is determined by the third Kepler's law,

$$T = 2\pi\sqrt{\frac{r^3}{G(M+m)}}, \qquad (2)$$

where T is the planet's period of revolution around the Sun. Then, equation (1) takes the form

$$P = \frac{32G^4}{5c^5}\frac{M^2(M+m)m^2}{r^5}.$$

The angular velocity ω of the rotating quadrupole is equal to the frequency of the emitted monochromatic gravitational waves.

If the gravitational interaction is fundamentally of quantum nature, then according to our interpretation of quantum field theory we expect that a classical gravitational wave is actually a bundle of gravitons. Here one makes two crucial assumptions. First, that any quantum theory of gravity should reduce, in the weak-field approximation and under some other assumptions (which are presently unknown), to linearized general relativity, i.e. the quadrupole formula should be valid in an appropriate limiting case. Second, at least in some approximation, the general picture of a quantized field as a collection of particles being bundles of energy and momentum applies to quantum gravity and gravitons are quanta of the gravitational field. If the two assumptions are valid, one may view the radiation emitted by each planet as a flux of gravitons with frequency ω and wavelength $\lambda = cT$. Each graviton carries energy $\hbar\omega$. For the Earth $\lambda = 1$ light year $\approx 1\times 10^{13}$ km and graviton energy is $\hbar\omega \approx 2\times 10^{-41}$ J $= 1.3\times 10^{-22}$ eV.

In quantum theory, the power emitted by a planet is $P = n\hbar\omega$, where n is the number of gravitons emitted within 1 s. However, computing of how many gravitons are emitted by the planet within a second or in a shorter time interval makes no sense. The instant of graviton emission may be determined with accuracy Δt not exceeding the wave period T, $\Delta t \geqslant T$, and for planets the periods are years (or at least days for extrasolar planets). Yet a physical meaning may be attributed to the energy PT radiated away by the planet in the time interval equal to the wave period; this amount of energy is carried away by N_g gravitons:

$$N_g = \frac{PT}{\hbar\omega}.$$

From (2) we get

$$N_g = \frac{64\pi G^3}{5c^5\hbar}\frac{m^2M^2}{r^2}. \qquad (3)$$

In the table below we show for four planets: the emitted power P (in W), the common period T of the waves and of the orbital motion and the number of gravitons N_g emitted in the time interval T. If the radiation is viewed as a quantum process, the planets also cannot be treated as solid lumps with continuous mass density, they should be viewed as discrete systems of many nucleons. Let $N = m/m_p$ be the number of nucleons in a planet, where m_p is proton mass. In the last row of the table, the ratio N_g/N of the two large numbers is given.

Planet	Mercury	Earth	Jupiter	Neptune
P (W)	70	200	5400	2.5×10^{-3}
T (years)	0.24	1	11.9	165
N_g	6×10^{48}	3×10^{50}	1×10^{54}	1×10^{50}
N_g/N	0.04	0.1	1	2×10^{-3}

The coincidence of the large numbers is evident and is particularly conspicuous in the case of Jupiter. The origin of the coincidence is unclear. It seems at present that the coincidence is rather accidental and there is no deeper reason for it to occur. Nevertheless, it is amusing to find such a bizarre large number coincidence showing that the solar system reveals some strange regularities (another one is the Titius–Bode law). It is interesting to see if something similar occurs for planets orbiting other stars. The 'Extrasolar Planets Catalog' [1] contains 181 planets and out of them only 10 planets have all the relevant parameters measured: the star's and the planet's mass, the inclination angle i, the eccentricity and the orbital period (or the semi-major axis). Among these, eight planets move on almost circular orbits (the eccentricity is small). All these eight planets have masses of the order of Jupiter's mass, 1.9×10^{27} kg, while their stars are of masses comparable to that of the Sun. This sample of planets is strongly observationally biased for obvious reasons: all the planets are very close to their stars, the radius of the largest orbit is only 0.207 83 AU, in all other cases it is smaller than 0.05 AU (less than 7×10^6 km). As a consequence, the ratio N_g/N for the eight extrasolar planets varies from 100 for Gliese 876 b (period 60.9 days) to 8×10^4 for OGLE-TR-56 b (period 1.21 days); in most cases it is of order 10^4. The selection bias prevents one from inferring whether the large number coincidence found in the solar system is common among planetary systems or is exceptional.

3. Is anything strange in the concept of the graviton?

From equation (3) one derives

$$\frac{N_g}{N} = \frac{64\pi G^3}{5c^5\hbar} m_p \frac{mM^2}{r^2} \tag{4}$$

and it is worth noting that the ratio N_g/N is a linear function of the moving mass m. If for solar planets it is of order 1, for light bodies $N_g/N \ll 1$. For a single hydrogen atom which orbits around the Sun alone following the Earth's orbit the radiated power is $P \approx 2 \times 10^{-101}$ W and $N_g \approx 3 \times 10^{-53}$ or this atom emits one graviton with $\lambda = 1$ light year in 3×10^{52} years. This means that according to quantum physics a hydrogen atom in this state of motion does not radiate at all. Yet if this atom is captured by the gravitational force of the Earth and falls into its atmosphere, the capability of the atom to radiate will grow many times though its macroscopic state of motion (which determines the emission power) has remained unchanged. In fact, when the hydrogen atom enters the atmosphere there appears a correlation among all the atoms of the planet since, according to equation (2), $P \propto N_g \propto m^2$. The puzzle of the correlation lies in the fact that the mere effect of being bound to the Earth by its gravity makes the atom to radiate one graviton per ten years. A problem arises which—as far as we know—up to now has not been clearly solved in the context of quantum gravity: what is the physical mechanism of creating quantum gravity correlations between all atoms (or nucleons) in a macroscopic body which causes that the number of gravitons emitted by the body consisting of N particles to grow as N^2?

We emphasize that in classical linearized gravitational radiation theory the quadrupole formula and following from it equation (2) for P raise no doubts. Classically, gravitational radiation is by definition a macroscopic effect and a whole planet acts as a single emitter, thus there is no problem of correlation between its atoms. The case of a planet orbiting around a star is slightly misleading because then all the formulae depend on the two masses and these enter equations (2) and (3) in different powers. It would be more clear to consider the case of two equal masses m moving in a circular orbit around the centre of mass with constant angular velocity under the influence of a non-gravitational force or even one mass on a circular orbit. In the classical theory, all particles of a given body give almost equal contributions to the amplitude of the wave and this implies that the emitted power is $P \propto m^2$. The problem appears only when one attempts to describe this macroscopic process as a combination of independent microscopic effects.

In the case of electromagnetic radiation (or quantum mechanical emission of any other elementary particles) the situation is different: each atom (molecule or nucleus) emits a photon independently of other atoms. As a result, the number of photons emitted by a system of N microscopic objects is proportional to N. It is exactly this property of electromagnetic interactions (as well as weak and strong interactions) that makes the quantum physics a physics of the microworld. Particles interact with other particles individually and not as collective systems, e.g. a neutrino coming from space is captured by an individual nucleon in one atom in the Earth rather than by the entire planet. In contrast, both emission and absorption of gravitons by a body are a kind of collective process arising due to the correlation between all particles of the body. This raises a question of whether in quantum gravity are there two-particle processes at all, such as elastic or inelastic scattering of (high energy) graviton on electron or proton or rather should one take into account a whole system of gravitationally bounded particles (i.e. is it the system that is subject to a quantum interaction with a single graviton)?

To solve the problem we first compare the rotating quadrupole of two equal masses with the case of an electric dipole rotating with constant angular velocity. Let the dipole consist of N charges +e (grouped together) and N charges −e at a distance l. Each charge +e interacts with each of N negative charges and in this sense the dipole may be decomposed into N^2 pairs +e−e and each such elementary dipole radiates independently of all others. One then expects that the total radiation power of the dipole is proportional to N^2. And in fact the power of classical electromagnetic dipole radiation is proportional to $((\mathrm{d}^2/\mathrm{d}t^2)\mathbf{d})^2$ where $\mathbf{d} = Ne\mathbf{l}$ is the electric dipole momentum, i.e. to N^2. Thus, the classical pictures of electromagnetic and gravitational radiation from rotating systems are very similar.

The issue then is whether the quantum picture generates a real problem for gravitational radiation: the appearance of quantum gravity correlations between nucleons in two bodies at large macroscopic distances. Are there similar effects for other interactions? In quantum field theory one usually investigates interactions between microscopic objects, and these interactions form our concepts about the quantum world. One can, however, also study interactions of classical systems with quantum fields, thus the problem is not specific to gravitation. Consider a macroscopic electric dipole, rotating or oscillating, coupled to quantum electromagnetic field. In QED, a classical macroscopic current may be a source of the quantum field and in this case the number of generated photons is proportional to N^2, i.e. the classical and quantum computations do agree [2]. This means that the problem of macroscopic quantum correlations—if it exists at all—does appear already in QED. Yet in QED no one would admit that a radiating macroscopic electric dipole generates a conceptual problem.

May be the correct answer is that there is no physical problem at all and it is rather of psychological origin. QED is a well developed and fully reliable theory, and if one does not get a satisfactory answer regarding the nature of quantum correlations at macroscopic distances in an electric dipole one does not interpret it as a defect of the theory. In contrast, modern comprehension of QFT implies that search for a detailed physical picture of the process of quantum emission of radiation from macroscopic bodies is groundless. Such 'physical pictures of quantum effects' do not correspond to anything real in the physical world and may be misleading. There is no deeper understanding of quantum processes beyond the outcomes of QFT calculations. There are no hidden parameters nor deeper insights in quantum world. The quantum theory of gravity is still in its initial stage and we do not know which questions should be put forward and which should not.

4. The issue of gravitons

The conclusion that the classical quadrupole formula does not create paradoxes with the concept of the graviton does not ensure that the concept itself is well grounded. The graviton is a fully quantum object, nevertheless its notion should be compatible with classical general relativity.

In QED, the quantum electromagnetic field appears as a bundle of photons carrying the energy and momentum of the field [3]. In the low energy limit, the photon may be classically approximated by a plane electromagnetic wave. There is a clear one-to-one correspondence between photons and plane waves. In other terms, one may precisely point to where in classical electrodynamics one makes the de Broglie conjecture providing transition to the quantum theory. By this we do not mean the standard generic procedure of quantizing a classical field theory. We do mean that in the classical electromagnetic field the plane waves are singled out and already in the classical theory one finds unambiguous hints that plane waves are related to quantum objects. In fact, consider the general explicitly relativistically invariant decomposition of the electromagnetic potential in plane waves with the wave vector k^μ,

$$A_\mu = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \mathrm{d}^4k \big[a_\mu(k)\, \mathrm{e}^{-\mathrm{i}k_\nu x^\nu} + \overline{a}_\mu(k)\, \mathrm{e}^{\mathrm{i}k_\nu x^\nu}\big] \delta(k_\alpha k^\alpha) \theta(k^0), \tag{5}$$

where $\overline{a}_\mu$ is the complex conjugate wave amplitude and $\theta(k^0)$ is the Heaviside step function (and the signature is $+---$). Performing integration over k^0 one gets the standard expression

$$A_\mu = \frac{1}{2} \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \frac{\mathrm{d}^3k}{k^0} \big[a_\mu(\mathbf{k})\, \mathrm{e}^{-\mathrm{i}k_\nu x^\nu} + \overline{a}_\mu(\mathbf{k})\, \mathrm{e}^{\mathrm{i}k_\nu x^\nu}\big] \tag{6}$$

with $k^0 = |\mathbf{k}| > 0$. Here, the Lorentz invariant three-dimensional integration measure is d^3k/k^0. The total energy and momentum of the field is

$$P^\mu = \frac{1}{c} \int_{\mathbf{R}^3} T^{0\mu}\, \mathrm{d}^3x,$$

where $T^{\mu\nu}$ is the symmetric (gauge invariant) energy–momentum tensor (we use conventions of [4]). Inserting (6) one replaces the integral over the whole physical space with an integral over the momentum 3-space[3] [5]:

$$P^\mu = \frac{-1}{8\pi c}\int_{-\infty}^{\infty}\frac{\mathrm{d}^3k}{k^0}a_\nu(\mathbf{k})\overline{a}^\nu(\mathbf{k})k^\mu. \tag{7}$$

The Lorentz gauge condition requires $a_\nu k^\nu = 0$ which implies, since k^μ is null, that both the real and imaginary parts of the amplitude vector are spacelike vectors (if they are null they are gradients), then $a_\nu\overline{a}^\nu < 0$. Therefore, the quantity

$$\frac{-1}{8\pi c}\frac{\mathrm{d}^3k}{k^0}a_\nu(\mathbf{k})\overline{a}^\nu(\mathbf{k})$$

is a Lorentz scalar which is positive and vanishes only for $A_\mu = 0$ and has dimension $\mathrm{ML^2T^{-1}}$, i.e. the dimension of Planck constant. It is here that one makes the Planck–Einstein–de Broglie conjecture: the quantity is equal to $n(\omega)\,\mathrm{d}\omega\,\hbar$, where $\hbar$ is a new universal dimensional constant signalling transition to quantum effects and $n(\omega)\,\mathrm{d}\omega$ is a number of quantum 'particles' (photons) determined (and denoted) by the wave vector $\mathbf{k}$ where the values of $\mathbf{k}$ belong to the interval $[k, k + \mathrm{d}k]$ with $k = k^0 = |\mathbf{k}| = \omega/c$. Then,

$$P^\mu = \int_0^\infty n(\omega)\,\mathrm{d}\omega\,\hbar k^\mu \tag{8}$$

and the total energy and momentum of the field is interpreted as a sum over all quantum particles, each carrying 4-momentum equal $\hbar k^\mu$. In this sense, apart from and independently of the quantization formalism, there is a direct and physically clear relationship between classical and quantum electromagnetic fields.

In linearized general relativity, the relationship between the classical and quantum fields may be derived in an analogous way though there are some troubles. There are two approaches to the problem. The first approach is based on the use of the field equations alone. A metric perturbation $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ around Minkowski space gives rise to the linearized Riemann tensor which is gauge invariant and applying the harmonic gauge condition ${h^{\mu\nu}}_{,\nu} = 0$ and $h \equiv \eta^{\mu\nu}h_{\mu\nu} = 0$ the field equations are reduced to $\Box h_{\mu\nu} \equiv \partial^\alpha\partial_\alpha h_{\mu\nu} = 0$. Analogously to the electromagnetic case, a general solution in this gauge is

$$h_{\mu\nu} = \frac{1}{2}\frac{1}{(2\pi)^{3/2}}\int_{-\infty}^{\infty}\frac{\mathrm{d}^3k}{k^0}\left[a_{\mu\nu}(\mathbf{k})\,\mathrm{e}^{-\mathrm{i}k_\alpha x^\alpha} + \overline{a}_{\mu\nu}(\mathbf{k})\,\mathrm{e}^{\mathrm{i}k_\alpha x^\alpha}\right] \tag{9}$$

with $k^\mu k_\mu = 0$ and the wave amplitude (the polarization pseudotensor) is restricted by $a_{\mu\nu}k^\nu = 0$ and $\eta^{\mu\nu}a_{\mu\nu} = 0$. One then introduces a Lorentz covariant energy–momentum pseudotensor $t_{\mu\nu}$ which is the second-order part in $h_{\mu\nu}$ of the Ricci tensor, see e.g. [6]. For a single monochromatic wave, the pseudotensor averaged over a spacetime region of size much larger than $|\mathbf{k}|^{-1}$ is

$$\langle t_{\mu\nu}\rangle = \frac{c^4}{16\pi G}k_\mu k_\nu a_{\alpha\beta}\overline{a}^{\alpha\beta}. \tag{10}$$

The pseudoscalar $a_{\alpha\beta}\overline{a}^{\alpha\beta}$ is nonnegative; in fact, by a Lorentz transformation one may get $k^\mu = k^0(1, -1, 0, 0)$ and the harmonic gauge implies $a_{\mu 1} = a_{\mu 0}$ and $a_{33} = -a_{22}$. The polarization pseudotensor has then five independent components and under the remaining gauge transformations, $a_{\mu\nu} \to a_{\mu\nu} + \epsilon_\mu k_\nu + \epsilon_\nu k_\mu$, where $\epsilon_\mu(\mathbf{k})$ is a Lorentz covariant complex vector subject to $\epsilon_\mu k^\mu = 0$, three of them are changed and may be set equal to zero and only the other two, a_{22} and a_{23}, remain gauge invariant. The helicity ± 2 plane wave is described by a_{22} and a_{23}. One gets

$$a_{\alpha\beta}\overline{a}^{\alpha\beta} = 2|a_{22}|^2 + 2|a_{23}|^2 > 0$$

[3] It is worth noting that one cannot obtain an analogous Lorentz covariant formula using the spectral decomposition of the field strength instead of the potential.

and this fact allows one to assign the number n of gravitons, each having the 4-momentum $\hbar k^{\mu}$, to unit volume of the plane wave [6],

$$n = \frac{c^2}{8\pi G\hbar}\omega(|a_{22}|^2 + |a_{23}|^2). \tag{11}$$

The other approach seems more reliable since one formulates the linearized general relativity as a Lagrangian theory for a massless spin-2 field in flat spacetime. To this end, one generates a Lagrangian for the field by taking the second variation of Einstein–Hilbert Lagrangian with respect to the metric perturbation $h_{\mu\nu}$ around the flat background [7]. The resulting Lagrangian appeared first in the textbook [8] and will be referred to as Wentzel Lagrangian,

$$L_W = \tfrac{1}{4}(-h^{\mu\nu;\alpha}h_{\mu\nu;\alpha} + 2h^{\mu\nu;\alpha}h_{\alpha\mu;\nu} - 2h^{\mu\nu}{}_{;\nu}h_{;\mu} + h^{;\mu}h_{;\mu}), \tag{12}$$

with $h \equiv g^{\mu\nu}h_{\mu\nu}$ and $g_{\mu\nu}$ is the flat spacetime metric in arbitrary coordinates. The Lagrangian and the field equations are gauge invariant and again the harmonic gauge $h^{\mu\nu}{}_{;\nu} = 0 = h$ is most convenient (the covariant derivatives are introduced for later use). As is well known, the theory is defective for there is no gauge invariant energy–momentum tensor for $h_{\mu\nu}$: both the canonical [9] and the variational (metric) energy–momentum tensors [7, 10] are gauge dependent and cannot be improved. This is a particular case of a generic situation where the variational energy–momentum tensor (hereafter denoted as the stress tensor) does not inherit the symmetries of the underlying Lagrangian [11]. As a consequence, it is impossible to attach a physical meaning to the local distribution of energy already for a linear spin-2 field. Only in special cases, e.g. for plane waves, one can use a particular gauge and then the canonical energy–momentum tensor may be invariant under the remaining gauge transformations [9]. This notion of local energy has, however, a limited meaning.

In general relativity, the (matter) stress tensor acts as the source of gravity and one expects that it plays a distinguished role also for the linearized gravity itself, though it is not gauge invariant. Wentzel Lagrangian is expressed in terms of covariant derivatives, and formally assuming that the metric $g_{\mu\nu}$ in (12) is arbitrary one may use the standard definition to calculate the tensor. It is very complicated [11] and we apply the harmonic gauge condition to simplify it, then it reads

$$\begin{aligned} T^{W}_{\mu\nu}(h,\eta) = {} & -h_{\mu\nu;\alpha\beta}h^{\alpha\beta} - 2h_{\alpha\beta;(\mu}h_{\nu)}{}^{\alpha;\beta} + \tfrac{1}{2}h_{\alpha\beta;\mu}h^{\alpha\beta}{}_{;\nu} + 2h_{\mu}{}^{\alpha;\beta}h_{\nu(\alpha;\beta)} \\ & + \tfrac{1}{4}g_{\mu\nu}(-h^{\alpha\beta;\sigma}h_{\alpha\beta;\sigma} + 2h^{\alpha\beta;\sigma}h_{\sigma\alpha;\beta}). \end{aligned} \tag{13}$$

This expression holds only in flat spacetime since in deriving it one assumes that the covariant derivatives commute. In Cartesian coordinates, the general solution for $h_{\mu\nu}$ is given by equation (9) and the total 4-momentum for the field is

$$P^{\mu} = \frac{1}{c}\int_{\mathbf{R}^3} T^{W0\mu}\,\mathrm{d}^3x = \frac{1}{4c}\int_{-\infty}^{\infty}\frac{\mathrm{d}^3k}{k^0}a_{\alpha\beta}(\mathbf{k})\,\overline{a}^{\alpha\beta}(\mathbf{k})k^{\mu}. \tag{14}$$

Clearly, this formula agrees with (10) and (11) for a suitably chosen normalization factor in $T^{W}_{\mu\nu}$. These expressions are invariant under the remaining gauge transformations $a'_{\mu\nu} = a_{\mu\nu} + \epsilon_{\mu}k_{\nu} + \epsilon_{\nu}k_{\mu}$. Furthermore, Deser and McCarthy [10] have proved a 'folk theorem' to the effect that under an arbitrary gauge transformation of a gauge invariant quadratic Lagrangian (i.e., L_W or any other equivalent to it in Minkowski space) the stress tensor computed 'on shell' (the field equations hold) is varied only by superpotential terms. A superpotential term, by its construction, gives no contribution to the Poincaré generators. In other terms, the Poincaré generators are gauge invariant. This means that the total P^{μ} evaluated in the harmonic gauge using equation (14) is equal to the 4-momentum evaluated in any other gauge or without any gauge (beyond the harmonic gauge equations (9)–(11) and (13)–(14) are not valid). The gauge invariance of the total P^{μ} together with the gauge dependence of the stress tensor implies that while P^μ is numerically the same, when computed in any gauge, the integrand in each case is different from that in the second integral in (14). When P^μ is computed in classical electrodynamics in various gauges, in each case the integrand has a form different from that in equation (7) but its value for a given **k** is always the same. Yet in the linearized gravity the integrand has both different forms and different values for various gauges. This is why the standard interpretation [6] of (11) as the number of gravitons with the momentum $\hbar k^\mu$ raises doubts.

The exact nonlinear general relativity is different in this aspect from any linear field theory. Due to the principle of equivalence the notion of local distribution of energy is meaningless (this is a stronger cause than the gauge non-invariance in the linearized gravity) and one should search for the relationship between the quantum and the classical theory in a different way. It is commonly accepted that exact purely radiative fields are described by the plane-fronted gravitational waves with parallel rays (pp waves) [12]. Comparing the Weyl tensor for pp waves with the Maxwell field strength for a plane electromagnetic wave, one finds similarities permitting analogous physical interpretation [12, 13]. A subclass of pp waves, the plane wave manifolds, is geodesically complete and for this reason is regarded as classical analogues of gravitons [12, 14]. However, energy considerations indicate that this interpretation is not well grounded. Although the pp waves are exact solutions of the nonlinear vacuum Einstein field equations, they also constitute their own linear approximation (sometimes this feature is used as an argument supporting that these spacetimes correspond to gravitons). As a consequence, the total energy assigned to these solutions is always zero. In fact, in the ADM approach to gravitational energy of a solution one introduces a pseudotensor defined as the quadratic and higher order terms part in the decomposition of Ricci tensor in the metric perturbations $h_{\mu\nu}$. A physical meaning has only the total ADM energy being a surface integral of the pseudotensor over a sphere at spatial infinity (assuming that the spacetime is asymptotically flat there). For the pp waves the full Ricci tensor is equal to its linear approximation, hence the pseudotensor and the total ADM energy are zero (regardless of that the plane waves are not asymptotically flat). One cannot view plane gravitational waves as a classical low energy approximation to a swarm of quantum particles carrying energy.

In ordinary quantum mechanics one associates with any microscopic object having rest mass and energy E (elementary particle, nucleus, atom) a quantum wave of frequency ω and assumes $E = \hbar\omega$. Contrary to the case of electromagnetic field, equations (7) and (8), one cannot recognize this frequency on classical grounds. In fact, if a microscopic system has a classically defined length scale λ it may be used to formally define a frequency $\omega_{\rm cl} = 2\pi c/\lambda$; this frequency, however, is unrelated to the true quantum frequency $E/\hbar$. For instance, the classical electron radius $\frac{e^2}{mc^2}$ is some hundred times smaller than the quantum electron Compton wavelength $\hbar/mc$. One expects that the same holds for these gravitational fields which are analogous in a sense to classical particles. A black hole is essentially an elementary object which cannot be decomposed into its constituent parts and is as fundamental as the electron. The static black hole of mass M defines its length scale, the horizon radius $r_g = \frac{2GM}{c^2}$, and the classical frequency $\omega_{\rm cl} = \frac{\pi c^3}{GM}$ is assigned to it. Since the length scale is not independent from the black hole energy E, a bizarre relation arises:

$$E = \frac{\pi c^5}{G\omega_{\rm cl}},$$

clearly $\omega_{\rm cl}$ has nothing to do with a hypothetical quantum frequency which might be associated with the black hole in quantum gravity.

In standard quantum field theory, the particle–wave duality establishes a universal correspondence between energy (mass) and frequency. This correspondence is inconsistent with classical general relativity. In Einstein's theory (and in other metric theories of gravity), a good notion of energy is provided only by the total ADM energy. This notion of energy has a unique status in general relativity. In no other theories of physics is energy effectively a charge and the same holds for momentum. In classical electrodynamics, where all the relevant calculations can explicitly be done, the electromagnetic field of a system of charges can be decomposed at the spatial infinity into the radiative field which gives no contribution to the total charge as being transversal and the 'Coulomb field' which falls off as r^{-2}. The latter can be spectrally decomposed into longitudinal waves with zero frequency [4]. Thus, the electric charge is universally associated with classical zero frequency waves. We emphasize that in general relativity the Poincaré generators being charges are calculated as integrals over a 2-sphere at the spatial infinity. This means that the energy and momentum of an asymptotically flat spacetime are zero frequency characteristics of this spacetime. In consequence, this implies that any general relationship between energy and frequency is *a priori* impossible and in particular the Einstein formula $E = \hbar\omega$ cannot hold.

5. Conclusions

The standard notion of the graviton as a quantum of the radiation gravitational field, which is akin to the photon, may be justified (with some reservation) only in the framework of the linearized gravity (which itself is a defective theory). In this theory, it gives rise to an amusing large number coincidence between the number of gravitons radiated away by solar planets in a time interval equal to the wave period and the number of nucleons in these planets. In full (nonlinear) general relativity, the (ADM) energy is, for fundamental reasons, unrelated to wave phenomena, in particular it is disconnected from the wave frequency. This statement does not mean that gravitons do not exist. If gravitons as quanta of a quantum gravitational field do exist, their properties are different from those of photons in QED. The nature of gravitons may be determined only in the framework of full quantum theory of gravity and without knowing it one can say nothing about them. The case of electromagnetism is misleading in this aspect: either classical general relativity bears no traces of quantum effects at all or the traces of gravitons are different from those of photons and at present are unrecognizable in the theory. In particular, one cannot expect that in the low energy classical approximation the gravitons may be interpreted as the pp waves.

Here, it has been assumed that the gravitational field carries a fundamental interaction and as such is of quantum nature. If gravity is an 'emergent' phenomenon, i.e. if it arises as a kind of averaging of other elementary interactions [15], then its quantization makes no sense.

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