Advanced Optics II (PHYS 554 002 & ECE 554 002)
CRN: 54521 (ECE) and 54520 (PHYS)
Spring Semester 2020

Instructor: Dr. Vitaly Gruzdev

Department of Physics and Astronomy

E-mail: vgruzdev@unm.edu

PAIS Building office 2244

Office hours: Tuesdays & Thursdays, 7:00 pm – 8:00 pm

Lectures: January 20 through May 7; Tuesdays & Thursdays from 5:30 pm through 6:45 pm

Mid-term exam: March 12 from 5:30 pm till 8:00 pm

Spring Break: March 15 through March 20

Week of Finals: May 11 through May 16

Final: May 12; makeup final: May 14

Location: Room 1140 PAIS Building

Teaching assistant: TBA

General overview

PHYS/ECE 554 002: Advanced Optics II

CRN: 54520 & 54521

Description of the class

This class is a continuation from Advanced Optics I of the Fall Semester 2019. It covers three major sections: crystal optics; coherence; and Fourier optics.

The first section – crystal optics – is focused on optical response of dielectric and semiconductor crystals and covers the following topics: classical Lorentz oscillator model for ideal dielectric crystals; complex optical response; Kramer-Kronig relations; dispersion if optical response; quantum-mechanical model of optical response; overview of absorption mechanisms; tensor of optical response of anisotropic crystals; strong-field and non-perturbative effects; magneto-optic effects; electro-optic effects; acousto-optic effect; and applications of those effects in devices.

The second section – coherence – covers temporal and spatial coherence; coherence function; interference of partially coherent light; transmission of partially coherent light through optical systems; image formation; and Van-Cittert-Zernike theorem.

The third section – Fourier optics – includes analysis of 2D signals and systems; foundations of scalar diffraction theory; wave-optics and frequency analysis of optical imaging systems; applications of Fourier optics in spatial filtering, holography, and analog processing of optical information.

The fourth section includes a selected special topic. Offered are basics of plasmonics, introduction to fiber optics, optical tweezers, and fundamentals of quantum optics. Also, topics suggested by students will be under consideration for this section.

This class is calculus based. Complex algebra, Fourier transform, and series expansions are routinely utilized in the three major sections. This class also includes an oral presentation.

Sources

Recommended books:

Topic 1: Crystal optics:

- (P3) Frank L. Pedrotti, Leno M. Pedrotti, Leno S. Pedrotti, "Introduction to Optics", 3d edition.
- (FOX) Mark Fox, “Optical Properties of Solids”, 2nd Ed., Oxford University Press, 2010.

Topic 2: Coherence:

- (MF) Miles V. Klein, Thomas E. Furtak, “Optics”, 2nd or 3d Edition.
- (GS) Joseph W. Goodman, “Statistical Optics”, 2nd Ed.

Topic 3: Fourier Optics:

- (GF) Joseph W. Goodman, “Fourier Optics”, 2nd Ed.

Additional resources:

- (HK) Hartmut Haug, Stephan W. Koch, "Quantum Theory of the Optical and Electronic Properties of Semiconductors", 4th or later edition.
- Jacques I. Pankove, “Optical Processes in Semiconductors”.
- Max Born, Emil Wolf, "Principles of Optics", 6th or later edition.
- Lev D. Landau, E. M. Lifshitz and L. P. Pitaevskii, “Electrodynamics of Continuous Media”, 2nd Edition or later (Pergamon Press, 1984).

Workload 1

- **Homework assignments**
- There are planned 14 homework assignments this semester, approximately one assignment per week. Each assignment includes a few problems from the recommended textbooks and other sources. The assignments will be given throughout the semester a week before they are due. Homeworks must be turned in to instructor’s mailbox on the due date by 8:00 pm.
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- **Grading**
- The final grade will be based on the homework assignments, mid-term exam, oral presentation and report, and final exam. The contributions to the final grade are as follows:
- Homework: 26% (2% each homework);
- Report and presentation: 14%
- Mid-term exam: 25%
- Final exam: 35%
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- **Exam dates (subject to change):**
- Mid-term: 03/12 (no makeup date).
- Final exam: 05/12; makeup date: 05/14.

Workload 2

- **Formal Report:**

A formal report is focused on one of suggested topics related to fundamental concepts and/or applications. It can be your lab report or a part of your lab report if the report topic is acceptable for this class. The objective of this task is to master students' writing skills and improve their style of technical/scientific writing. The report is prepared using LaTeX, which is a standard tool used in the scientific community in various areas including physics and engineering. The style should follow the format of a scientific paper from Physical Review, Optics Letters, or Applied Physics Letters. Length range is from 2 journal pages (minimum) to 4 journal pages (maximum). The report should be submitted to the instructor as a PDF file via e-mail (subject "Formal Report") by noon of Thursday, 05/07. The file name should include your last name in the style "Name_Report". Formal reports and presentations are individual. Samples of journal styles and LaTeX/TeX templates will be provided by instructor via e-mail.

- **Presentation:**

The objective of this task is to improve the skills of public presentation on scientific and technical topics. You will deliver a presentation at the end of the semester on 05/07/20 on the topic of your Formal Report. Duration: 15 minutes of presentation + 5 minutes for questions/answers. It should cover fundamentals, relevant theoretical background, development/state-of-the-art in the field, and applications in science and/or technology. Slides for the presentation can be run from either instructor's laptop or your own laptop. Preferable formats, tips for slide preparation, and suggestions on presentation style will be shared later.

Syllabus Topic 1: Crystal optics and nonlinear effects

- 1) Classical model of optical response of ideal dielectrics; dispersion; Kramer-Kronig relations.
- 2) Classical model of absorption by free carriers in solids; optical response of semiconductors.
- 3) Tensor of optical response; birefringence; polarization rotation (chirality); anisotropic reflectivity.
- 4) Basics of quantum theory of crystals; overview of absorption mechanisms in crystals.
- 5) Quantum theory of linear absorption in non-metal crystals; semiconductor Bloch equations.
- 6) Strong-field electron dynamics in crystals; laser-driven electron oscillations; non-perturbative approaches; the Keldysh photoionization formula.
- 7) Crystals in magnetic fields: Faraday effect; Cotton-Mouton effect; Zeeman effect; Landau levels; magneto-absorption.
- 8) Crystals in electric fields: Pockels effect; Kerr effect; Stark effect; Bloch oscillations; Franz-Keldysh effect.
- 9) Acousto-optic effect; crystal-optic devices.

Syllabus Topic 2: Coherence

1) Temporal coherence; quasimonochromatic light; spectral flux density; interference spectroscopy; coherence function; contrast of interference pattern; properties of coherence function; examples for typical models of light sources.

2) Statistical optics; autocorrelation function; second and third-order coherence functions; examples for quasimonochromatic and thermal light; influence of spectrum broadening; coherence time and spectral bandwidth;

3) Spatial coherence; Young's experiment; two-point and continuous sources; Van Cittert-Zernike theorem; proof of the theorem; assumptions of the theorem; examples; influence of frequency spread; transverse coherence; longitudinal coherence; coherence length.

4) Fluctuations; light as a stochastic process; basics of statistical description; correlation; correlation momenta; their relation to coherence functions of various orders; correlation interferometry; quantum analysis; quantum coincidence.

5) Transmission of partially coherent light through optical systems; image formation from incoherent objects; point spread function; extended sources of light; optical transfer function; correlation form of the optical transfer function (OTF); examples of OTFs for typical optical elements; OTFs of real optical systems.

Syllabus Topic 3: Fourier Optics

1) Analysis of 2D signals and systems: 2D Fourier analysis; local spatial frequency; linear systems; basics of 2D sampling theory.

2) Foundations of scalar diffraction theory: transition from vector to scalar model; Helmholtz equation; Fresnel-Kirchhoff diffraction; Rayleigh-Sommerfeld diffraction; Huygens-Fresnel principle; generalization to non-monochromatic waves; diffraction by boundaries; angular spectrum of plane waves.

3) Fresnel and Fraunhofer diffraction: the Fresnel approximation; the Fraunhofer approximation; examples of diffraction patterns.

4) Wave-optics analysis of coherent optical systems: phase transformation by thin lens; Fourier transforming properties of lenses; image formation under monochromatic illumination; analysis of complex coherent optical systems.

5) Frequency analysis of opticl imaging systems: general treatment of imaging systems; frequency response of diffraction limited coherent and incoherent imaging; effect of aberrations on frequency response; coherent vs incoherent imaging; resolution beyond the classical limit.

6) Applications in spatial filtering: wave-front modulation, spatial light modulators; diffractive optical elements.

7) Applications in analog optical information processing; incoherent and coherent image processing systems; the VanderLugt filter; the joint transform correlator; character recognition; image restoration; discrete analog optical processors.

8) Applications in holography: wave-front reconstruction problem; the hologram; the Leith-Upatnieks hologram; image locations and magnification; some specific types of holograms; thick holograms; computer-generated holograms; hologram degradation; holography with spatially incoherent light; some applications of holography.

Topic 4: Selected Special Topics

5-6 lectures; 2 homework

OPTIONS:

- 1) Basics of plasmonics.
- 2) Optical tweezers.
- 3) Introduction to guided wave optics (fiber optics).
- 4) Radiation pressure and ponderomotive force.
- 5) Other topics if proposed by students.

Topic suggestions are welcome!

Tentative schedule

Topic	Date	Subject	Reading	Homework	HW Due	Solutions
Crystal optics	01/21	Lorentz model of optical response	P3; Ch. 25			
	01/23	Absorption by free carriers	P3; Ch. 25	HW1	01/30	
	01/28	Tensor of optical response				
	01/30	Absorption mechanisms	Pn; Ch. 1	HW2	02/06	
	02/04	Quantum theory of absorption	Pn; h. 3			
	02/06	Strong-field electron dynamics	Lecture	HW3	02/13	
	02/11	Crystals in magnetic fields	HK			
	02/13	Crystals in electric fields	HK	HW4	02/20	
	02/18	Acousto-optic effect				
Coherence	02/20	Temporal coherence	KF; Ch. 8.1	HW5	02/27	
	02/25	Statistical optics	KF; Ch. 8.2			
	02/27	Spatial coherence;	KF; Ch. 8.3	HW6	03/05	
	03/03	Light as a stochastic process	GSO; Ch.			
	03/05	Fluctuations of light	KF; Ch. 8.4	HW7	03/12	
	03/10	Optical transfer function	KF; Ch. 8.5			
	03/12	Midterm test	Equation list			
	03/17	SPING BREAK				
	03/19	SPING BREAK				
Fourier optics	03/24	2D signals and systems	GFO; Ch. 2	HW 8	03/31	
	03/26	Basics of scalar diffraction theory	GFO; Ch. 3			
	03/31	Fresnel & Fraunhofer diffraction	GFO; Ch. 4	HW 9	04/07	
	04/02	Wave-optics analysis	GFO; Ch. 5			
	04/07	Frequency analysis	GFO; Ch. 6	HW 10	04/14	
	04/09	Applications: spatial filtering	GFO; Ch. 7			
	04/14	Applications: analog processing	GFO; Ch. 8	HW 11	04/21	
	04/16	Applications: holography	GFO; Ch. 9			
Selected topic	04/21	Selected topic lecture 1		HW 12	04/28	
	04/23	Selected topic lecture 2				
	04/28	Selected topic lecture 3		HW 13	05/05	
	04/30	Selected topic lecture 4				
	05/05	Selected topic lecture 5				
	05/07	Student presentations				
	05/12	Final exam	Equation list			
	05/14	Makeup final exam	Equation list			

Topic 1: Crystal optics and nonlinear effects

LECTURE 1

Topics:

Classical model of optical response of ideal dielectrics: linear Lorentz model of an oscillator

Refractive index

Absorption

Dispersion

Kramer-Kronig relations

Multi-frequency oscillators; oscillator force

General structure of absorption spectrum of crystals.

Math tools:

Differential equations; Fourier transform; complex algebra; integration; functions of complex variables (poles; Cauchy formula for integrals)

Introduction

Optics of crystals – optical response of crystals to electromagnetic waves.

<u>Measurable optical response:</u>

refractive index n: $\boldsymbol{n = c_0/v_{phase}}$; $\boldsymbol{c_0}$ - speed of light in vacuum

absorption coefficient α: $\boldsymbol{F(x) = F_0 \exp(-\alpha x)}$

Objective: express refractive index and absorption coefficient (**<u>optical constants</u>**) via microscopic parameters of the crystals related to the particles (atoms, molecules, electrons) constituting the crystals.

Traditional approach:

n, α ⇐ complex dielectric function ε ⇐ optical susceptibility χ ⇐ model of particle dynamics driven by light

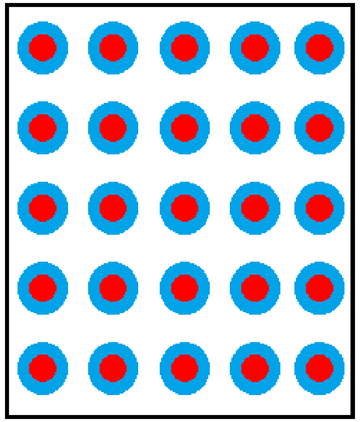
Magnetic field is neglected since magnetic force scales as v_{part}/c_0. Typical values of v_{part} are close to Fermi speed in crystals at room temperature (about 10^5 - 10^6 m/s) and is 2-3 orders of magnitude smaller than the force form electric field of light.

Models to calculate susceptibility from microscopic parameters:

- Classical (energy of the particles varies continuously) – Lorentz, Drude;
- Quantum (energy of the particles is quantized).

Classical model of optical response of ideal dielectrics

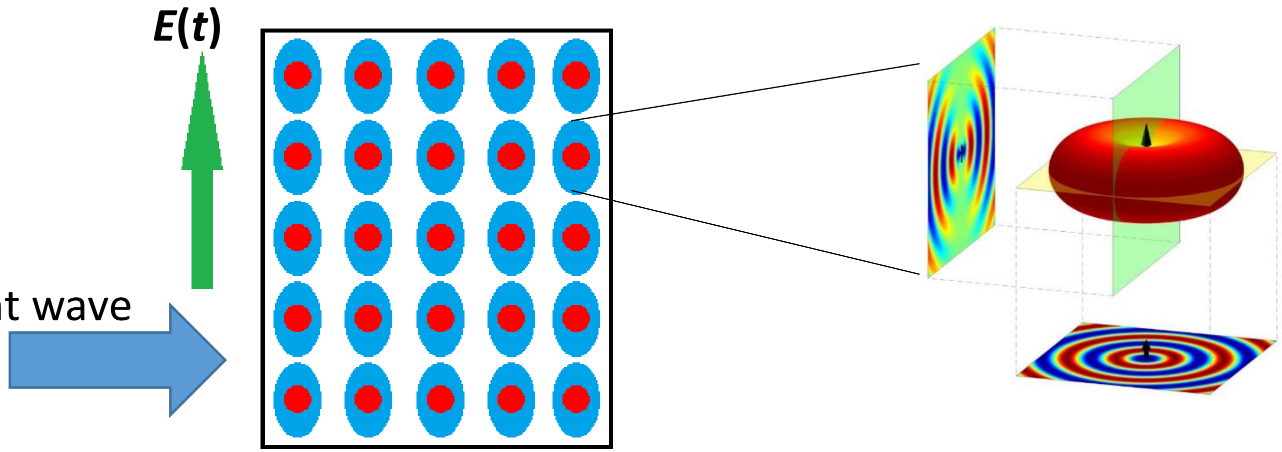
Non-polar dielectrics: spherical symmetry of electron clouds prior to light action.



NOTE: in polar dielectrics, the dipoles are randomly oriented so that they produce total time-averaged zero polarization.

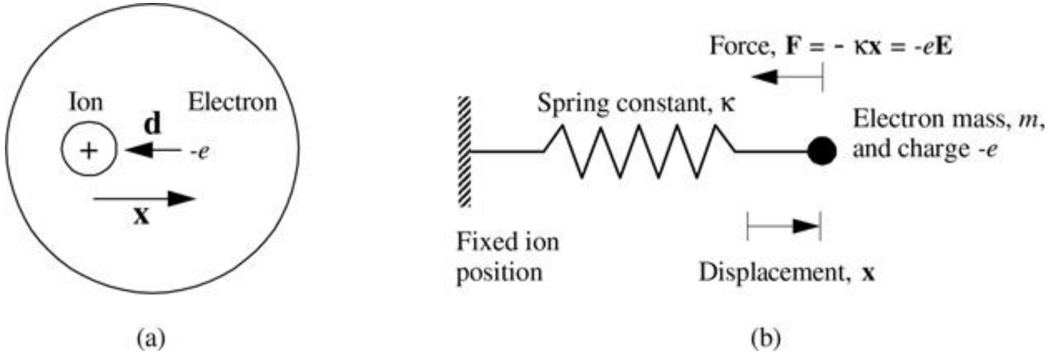
Electric field of light stretches the clouds along the field direction and produces dipoles.

Incident light wave



The classical model replaces the clouds and the atoms with a damped oscillator.

Origin of the damping: electron interactions with other electrons and atoms that result in energy transfer from the oscillating electron.

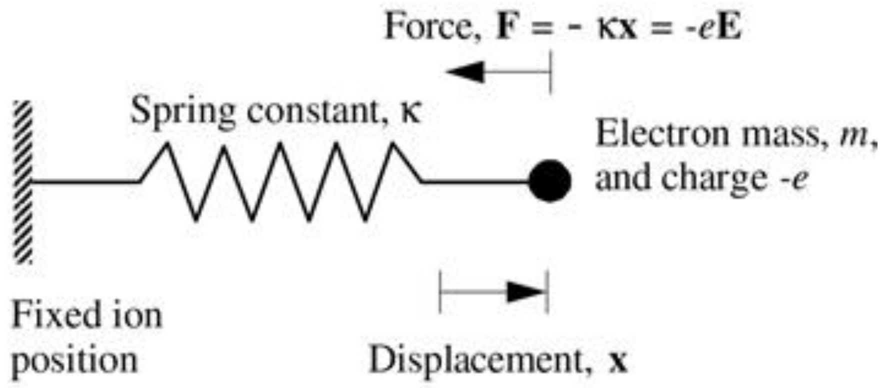


Linear Lorentz model

Dipole momentum of a single oscillator with charge *q*:

$$\vec{\boldsymbol{p}} = -q\vec{\boldsymbol{r}}$$

Polarization of volume *V* with volumetric density *N* of the oscillators:



$$\vec{\boldsymbol{P}} = \frac{1}{V}\sum_i \overrightarrow{\boldsymbol{p}_i} = N\langle\vec{\boldsymbol{p}}\rangle$$

For a system of identical oscillators:

$$\vec{\boldsymbol{P}} = -Nq\vec{\boldsymbol{r}}$$

Below we consider electrons as the oscillating particles: *q* = *e* – electron charge (1.6 x 10^{-19} Q).

Lorentz oscillator equation:

$$m_0\frac{d^2\vec{\boldsymbol{r}}}{dt^2} = -e\vec{\boldsymbol{E}}(t) - m_0\omega_0^2\vec{\boldsymbol{r}} - m_0\gamma\frac{d\vec{\boldsymbol{r}}}{dt}$$

Acceleration; **m_0 – electron mass**

Electric force

Restoring force at **natural frequency ω_0**

Frictional (damping) force at **damping rate γ** [1/s]

In [Pedrotti] an elastic constant is introduced:

$$K_S = m_0\omega_0^2$$

Solution of the oscillator equation

$$m_0 \frac{d^2\vec{\boldsymbol{r}}(t)}{dt^2} = -e\vec{\boldsymbol{E}}(t) - m_0\omega_0^2\vec{\boldsymbol{r}}(t) - m_0\gamma\frac{d\vec{\boldsymbol{r}}(t)}{dt}$$

Fourier transform:

$$\vec{\boldsymbol{r}}(t)\xrightarrow{F}\vec{\boldsymbol{r}}(\omega)$$
$$\vec{\boldsymbol{E}}(t)\xrightarrow{F}\vec{\boldsymbol{E}}(\omega)$$

$$m_0(-j\omega)^2\vec{\boldsymbol{r}}(\omega) = -e\vec{\boldsymbol{E}}(\omega) - m_0\omega_0^2\vec{\boldsymbol{r}}(\omega) - m_0\gamma(-j\omega)\vec{\boldsymbol{r}}(\omega)$$

After simplification – **average travel of an oscillating electron**:

$$\vec{\boldsymbol{r}}(\omega) = -\frac{e}{m_0}\frac{\vec{\boldsymbol{E}}(\omega)}{\omega_0^2 - \omega^2 - j\omega\gamma}$$

From definition of the dipole momentum: $\vec{\boldsymbol{p}} = -e\vec{\boldsymbol{r}}$

$$\vec{\boldsymbol{p}}(\omega) = \frac{e^2}{m_0}\frac{\vec{\boldsymbol{E}}(\omega)}{\omega_0^2 - \omega^2 - j\omega\gamma} = \alpha(\omega)\vec{\boldsymbol{E}}(\omega)$$

Average polarizability of dipoles:

$$\alpha(\omega) = \frac{e^2}{m_0}\frac{1}{\omega_0^2 - \omega^2 - j\omega\gamma}$$

Towards the optical response

Material polarization is a part of **constitutive relations**:

$$\vec{\boldsymbol{D}} = \varepsilon_0 \vec{\boldsymbol{E}} + \vec{\boldsymbol{P}}$$

Constitutive relation in terms of relative permittivity ε and susceptibility χ:

$$\vec{\boldsymbol{D}} = \varepsilon_0 \varepsilon \vec{\boldsymbol{E}} = \varepsilon_0 \vec{\boldsymbol{E}} + \varepsilon_0 \chi \vec{\boldsymbol{E}}$$

Polarization to field relation: $\vec{\boldsymbol{P}} = \varepsilon_0 \chi \vec{\boldsymbol{E}}$

But in terms of average dipole momentum: $\vec{\boldsymbol{P}}(\omega) = N\langle \vec{\boldsymbol{p}}(\omega) \rangle = N\alpha(\omega)\vec{\boldsymbol{E}}(\omega) = \varepsilon_0 \chi \vec{\boldsymbol{E}}$

That delivers susceptibility in terms of the microscopic model:

$$\boldsymbol{\chi(\omega) = \frac{N\alpha(\omega)}{\varepsilon_0} = \frac{Ne^2}{\varepsilon_0 m_0} \frac{1}{\omega_0^2 - \omega^2 - j\omega\gamma}}$$

By introduction of **plasma frequency**: $\boldsymbol{\omega_P^2 = \frac{Ne^2}{\varepsilon_0 m_0}}$

the susceptibility receives its standard notation:

$$\boldsymbol{\chi(\omega) = \frac{\omega_P^2}{\omega_0^2 - \omega^2 - j\omega\gamma}}$$

Almost here: dielectric function (permittivity)

Now the oscillator model can deliver the dielectric function due to the permittivity to susceptibility relation:

$$\varepsilon(\omega) = 1 + \chi(\omega) = 1 + \frac{\omega_P^2}{\omega_0^2 - \omega^2 - j\omega\gamma}$$

It can be split into real and imaginary parts:

$$\boldsymbol{\varepsilon(\omega) = \varepsilon'(\omega) + j\varepsilon''(\omega)}$$

Real part of permittivity:

$$\boldsymbol{\varepsilon'(\omega) = 1 + \omega_P^2 \frac{\omega_0^2 - \omega^2}{\left(\omega_0^2 - \omega^2\right)^2 + \omega^2\gamma^2}}$$

Imaginary part of permittivity:

$$\boldsymbol{\varepsilon''(\omega) = \omega_P^2 \frac{\omega\gamma}{\left(\omega_0^2 - \omega^2\right)^2 + \omega^2\gamma^2}}$$

Towards the optical constants: Maxwell equations

Assuming: ideal dielectric that contains no free charges; no polarization (time-averaged polarization is zero):

$$\nabla \vec{\boldsymbol{P}} = 0$$

Displacement current: $\vec{\boldsymbol{j}} = \frac{\partial \vec{\boldsymbol{P}}}{\partial t}$

Wave equation for electric field:

$$c_0^2 \nabla^2 \vec{\boldsymbol{E}}(\vec{\boldsymbol{r}}, t) = \frac{\partial^2 \vec{\boldsymbol{E}}(\vec{\boldsymbol{r}}, t)}{\partial t^2} + \frac{1}{\varepsilon_0} \frac{\partial^2 \vec{\boldsymbol{P}}(\vec{\boldsymbol{r}}, t)}{\partial t^2}$$

Time-to-frequency and coordinate-to-wave vector Fourier transforms:

$$\vec{\boldsymbol{E}}(\vec{\boldsymbol{r}}, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-j\omega t} dt \iiint_{-\infty}^{\infty} \vec{\boldsymbol{E}}(\vec{\boldsymbol{k}}, \omega) e^{j\vec{\boldsymbol{k}}\vec{\boldsymbol{r}}} d^3k$$

delivers a dispersion relation:

$$k^2 = \frac{\omega^2}{c_0^2}[1 + \chi(\omega)] = \frac{\omega^2}{c_0^2}\left[1 + \frac{\omega_P^2}{\omega_0^2 - \omega^2 - j\omega\gamma}\right] = \frac{\omega^2}{c_0^2}\varepsilon(\omega) = \frac{\omega^2}{c_0^2}[\varepsilon'(\omega) + j\varepsilon''(\omega)]$$

Wave vector must be complex:

$$k(\omega) = k_{Re}(\omega) + jk_{Im}(\omega)$$

Connection to complex refractive index

Complex refractive index is introduced via the complex wave vector:

$$k(\omega) = k_{Re}(\omega) + jk_{Im}(\omega)$$

$$k(\omega) = k_{Re}(\omega) + jk_{Im}(\omega) = \frac{\omega}{c_0}\tilde{n}(\omega) = \frac{\omega}{c_0}\big(n(\omega) + j\kappa(\omega)\big)$$

Here n_{Re} is the usual refractive index, and κ is called **extinction coefficient** and is related to absorption coefficient.

For a plane monochromatic wave with the complex wave vector:

$$\vec{\boldsymbol{E}}(\vec{\boldsymbol{r}}, t) = \overrightarrow{\boldsymbol{E}_0} e^{j(\vec{\boldsymbol{k}}\vec{\boldsymbol{r}} - \omega t)}$$

Irradiance of the damped plane wave:

$$I(x) = I_0 e^{-\alpha x}$$

$$\boldsymbol{\alpha(\omega) = 2k_{Im}(\omega) = 2\frac{\omega}{c_0}\kappa(\omega)}$$

Complex permittivity vs complex refractive index

General relation:

$$\tilde{n}^2 = \left(n(\omega) + j\kappa(\omega)\right)^2 = \varepsilon(\omega) = \varepsilon'(\omega) + j\varepsilon''(\omega)$$

Complex permittivity to optical constants (*n* and *κ*) :

$$\boldsymbol{\varepsilon'(\omega) = n^2(\omega) - \kappa^2(\omega) = 1 + \omega_P^2 \frac{\omega_0^2 - \omega^2}{\left(\omega_0^2 - \omega^2\right)^2 + \omega^2\gamma^2}}$$

$$\boldsymbol{\varepsilon''(\omega) = 2\, n(\omega)\, \kappa(\omega) = \omega_P^2 \frac{\omega\gamma}{\left(\omega_0^2 - \omega^2\right)^2 + \omega^2\gamma^2}}$$

Optical constants to real and imaginary parts of permittivity:

$$\boldsymbol{n = \sqrt{\frac{1}{2}\left[\varepsilon' + \sqrt{\varepsilon'^2 + \varepsilon''^2}\right]}}$$

$$\boldsymbol{\kappa = \sqrt{\frac{1}{2}\left[-\varepsilon' + \sqrt{\varepsilon'^2 + \varepsilon''^2}\right]}}$$

where: $\boldsymbol{\varepsilon'(\omega) = 1 + \omega_P^2 \frac{\omega_0^2-\omega^2}{\left(\omega_0^2-\omega^2\right)^2+\omega^2\gamma^2}}$ and $\boldsymbol{\varepsilon''(\omega) = \omega_P^2 \frac{\omega\gamma}{\left(\omega_0^2-\omega^2\right)^2+\omega^2\gamma^2}}$

Final results

Reflectivity

- For simplicity consider only normal incidence.

Amplitude reflection coefficient:

$$r(\omega) = \frac{1 - n(\omega) - j\kappa(\omega)}{1 + n(\omega) + j\kappa(\omega)}$$

Power reflection coefficient:

$$R(\omega) = r(\omega)r^*(\omega) = \frac{[1 - n(\omega)]^2 + \kappa^2(\omega)}{[1 + n(\omega)]^2 + \kappa^2(\omega)}$$

Brief summary

Calculation of either complex susceptibility or complex permittivity delivers the optical constants and their scaling with frequency.

Also, reflectivity and transmissivity can be calculated from those values.

Next steps:

1) nonlinear oscillator – nonlinear optics;

2) Multiple oscillators – dispersion over extended range of wavelength;

3) Quantum oscillator – quantum-mechanical model.

Comment: local electric fields

Polarization to field relation:

$$\vec{\boldsymbol{P}} = \varepsilon_0 \chi \vec{\boldsymbol{E}}$$

Includes local electric field acting on the dipole rather than the electric field of incident light wave.

From electrodynamics: $\overrightarrow{\boldsymbol{E}_L} = \frac{\vec{\boldsymbol{P}}}{3\varepsilon_0} + \vec{\boldsymbol{E}}$

Polarization: $\vec{\boldsymbol{P}}(\omega) = \varepsilon_0 \chi \overrightarrow{\boldsymbol{E}_L} = \varepsilon_0 \chi \left[\frac{\vec{\boldsymbol{P}}}{3\varepsilon_0} + \vec{\boldsymbol{E}}\right]$

$$\vec{\boldsymbol{P}}(\omega) = \frac{\varepsilon_0 \chi \vec{\boldsymbol{E}}}{1 - \frac{\chi}{3}} = \frac{\varepsilon_0 \omega_P^2 \vec{\boldsymbol{E}}}{\omega_0^2 - \omega^2 - j\omega\gamma} \frac{1}{1 - \frac{1}{3}\frac{\omega_P^2}{\omega_0^2 - \omega^2 - j\omega\gamma}} = \frac{\varepsilon_0 \omega_P^2 \vec{\boldsymbol{E}}}{\omega'^2_0 - \omega^2 - j\omega\gamma}$$

where

$$\omega'^2_0 = \omega_0^2 - \frac{1}{3}\omega_P^2$$

<u>The local-field correction results in a shift of the resonance frequency of the Lorentz oscillators.</u> This effect is frequently missed in many textbooks (but not in Pedrotti!).

The local-field corrections do not affect the functional form of permittivity and susceptibility.

Dispersion of refractive index and absorption

$$\boldsymbol{\kappa} = \sqrt{\frac{1}{2}\left[-\boldsymbol{\varepsilon}' + \sqrt{\boldsymbol{\varepsilon}'^2 + \boldsymbol{\varepsilon}''^2}\right]}$$ where $$\boldsymbol{\varepsilon}'(\boldsymbol{\omega}) = \mathbf{1} + \boldsymbol{\omega}_P^2 \frac{\boldsymbol{\omega}_0^2 - \boldsymbol{\omega}^2}{\left(\boldsymbol{\omega}_0^2 - \boldsymbol{\omega}^2\right)^2 + \boldsymbol{\omega}^2\boldsymbol{\gamma}^2}$$ and $$\boldsymbol{\varepsilon}''(\boldsymbol{\omega}) = \boldsymbol{\omega}_P^2 \frac{\boldsymbol{\omega}\boldsymbol{\gamma}}{\left(\boldsymbol{\omega}_0^2 - \boldsymbol{\omega}^2\right)^2 + \boldsymbol{\omega}^2\boldsymbol{\gamma}^2}$$

For transparent materials: $\omega_0^2 - \omega^2 \gg \omega\gamma$

Therefore: $\varepsilon'' \ll \varepsilon'$ and $\kappa \ll n$

$$n^2(\omega) = \varepsilon'(\omega) = 1 + \omega_P^2 \frac{1}{\omega_0^2 - \omega^2}$$

Assuming $\omega^2 \ll \omega_0^2$:

$$\frac{1}{\omega_0^2 - \omega^2} \approx \frac{1}{\omega_0^2}\left[1 + \frac{\omega^2}{\omega_0^2} + \frac{\omega^4}{\omega_0^4}\right]$$

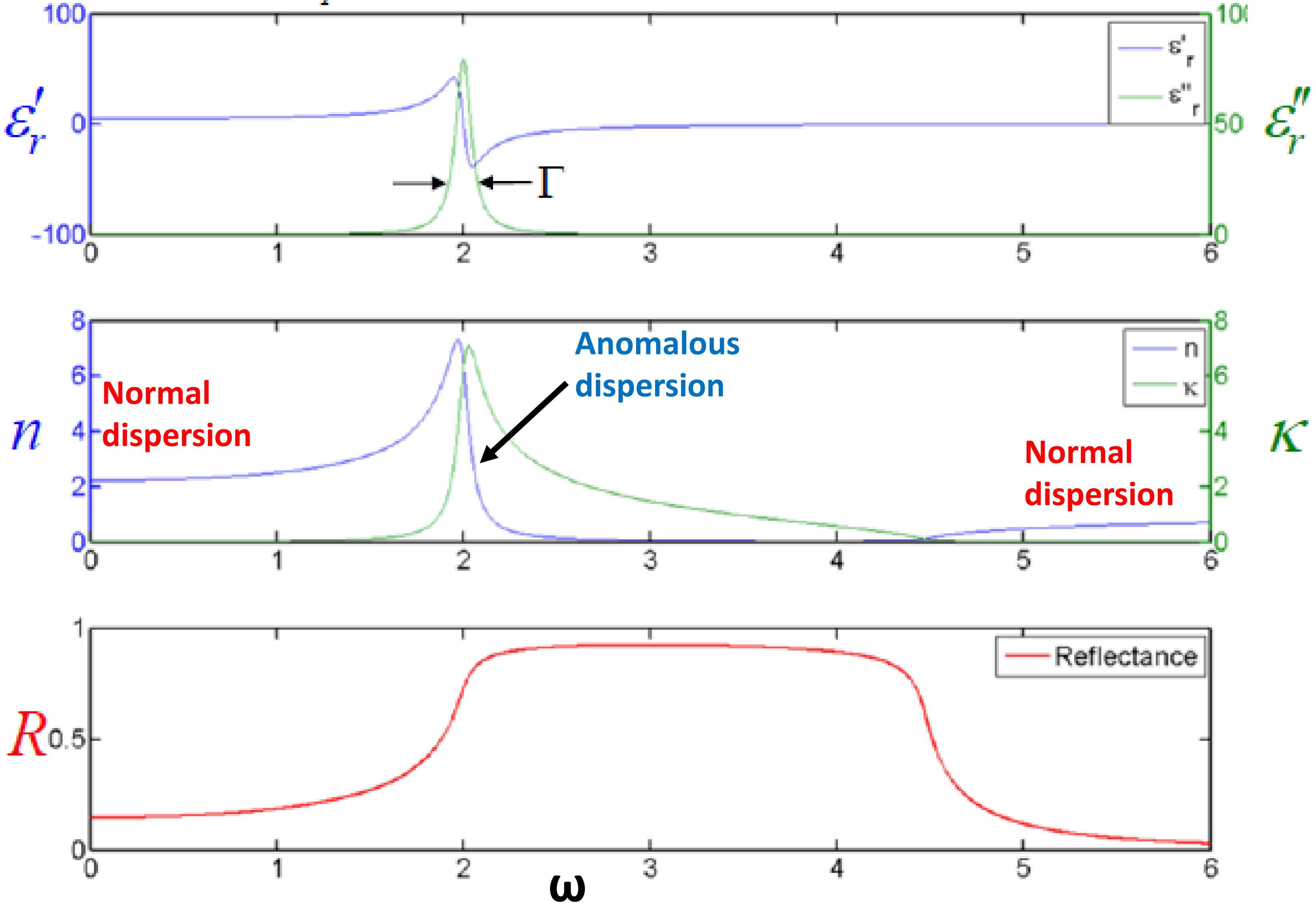
$$n^2 = A' + \frac{B'}{\lambda^2} + \frac{C'}{\lambda^4} + \cdots$$

Cauchy dispersion relation:

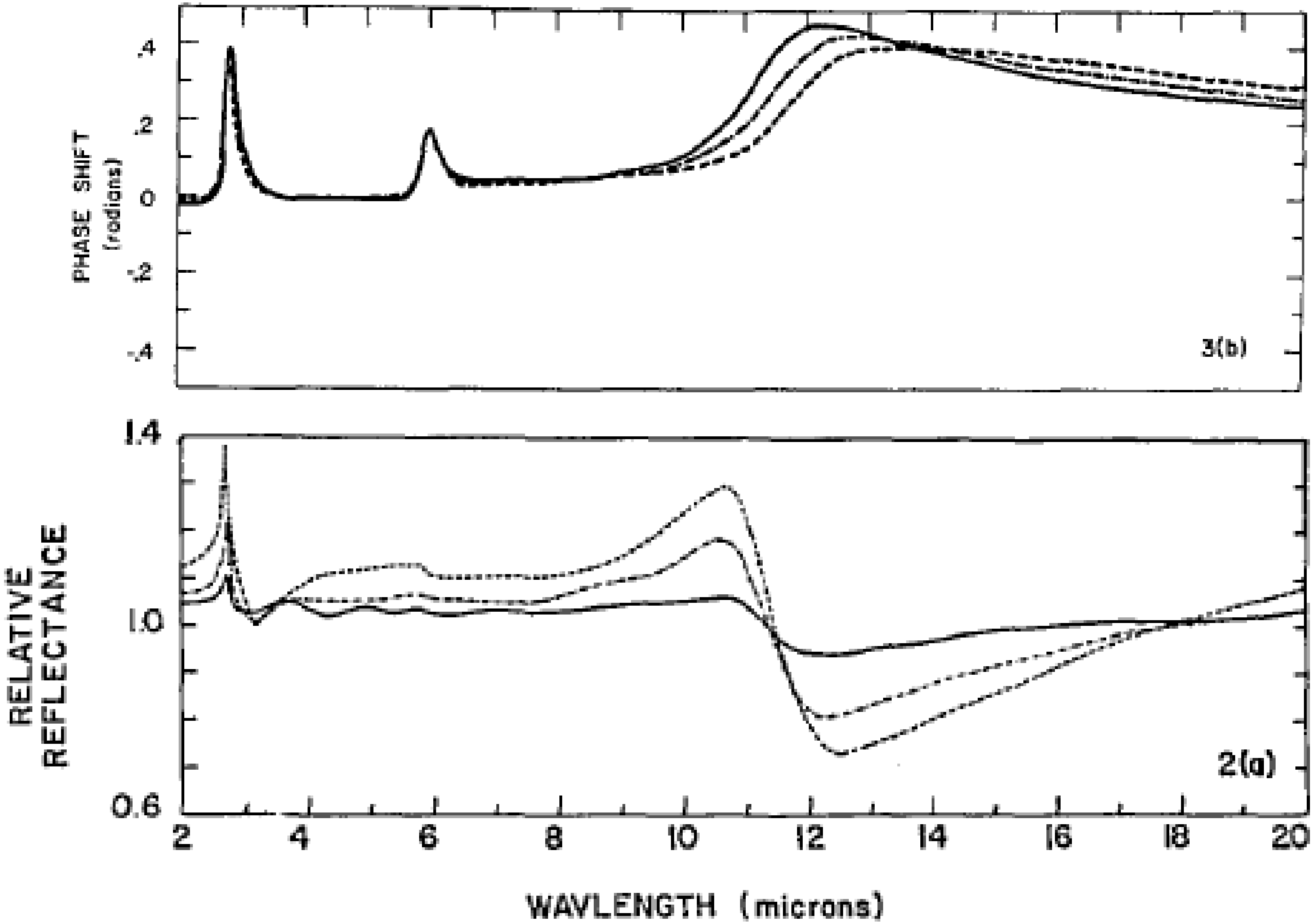
$$n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots$$

Typical dispersion curve

$\omega_p = 4 \qquad \omega_0 = 2 \qquad \Gamma = 0.1$



Experimental data: water solution n of NaCl



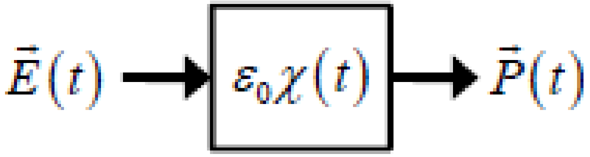
M. R. Querry, R. C. Waring, et al, J. Opt. Soc. Am. **62** (7) 849-855 (1972)

Towards Kramers-Kronig relations: analytical properties of permittivity/susceptibility

Standard form of susceptibility:

$$\chi(\omega) = \frac{\omega_P^2}{\omega_0^2 - \omega^2 - j\omega\gamma} = \omega_P^2\left[\frac{1}{\omega'_0 + \omega + 1/2j\gamma} - \frac{1}{\omega'_0 - \omega - 1/2j\gamma}\right]$$

where $\omega'^2_0 = \omega_0^2 - \frac{\gamma^2}{4}$



$$\vec{P}(t) = \varepsilon_0 \int_{-\infty}^{\infty} \vec{E}(\tau)\chi_e(t-\tau)\,d\tau$$

$$\vec{P}(\omega) = \varepsilon_0 \chi(\omega)\vec{E}(\omega)$$

- Two poles in the complex frequency plane.

$$\chi(\omega) = \chi'(\omega) + j\chi''(\omega)$$

From Fourier theory, if $\chi(t)$ is purely real then

$\chi'(\omega)$ is an even function

$\chi''(\omega)$ is an odd function