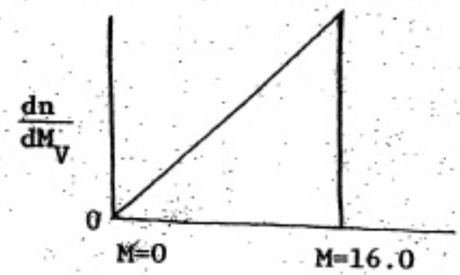
ASTRO 536 HW #2 - Due 10/2/17 in class

(1) Consider the luminosity function of the nearby stars (those 60 stars within a radius of 5.2 pc):

10 pts

$L/L_\odot$	Number	M_V
7.6 - 23	2	0–2.5
.7 - 7	3	2.5–5
.07 - .7	6	5–7.5
.007 - .07	6	7.5–10
7×10^{-4} - .007	19	10–12.5
7×10^{-5} - 7×10^{-4}	18	12.5–15
$2\cdot10^{-5}$ - $7\cdot10^{-5}$	6	15–17.5

We decide to approximate this by a triangular distribution in M_V:



i.e., we express the number of stars per unit M_V per pc^3 by:

$$\frac{dn}{dM_V} = kM_V \ , \quad 0 \le M_V \le 16.0 \quad (k=\text{const.})$$

$$\frac{dn}{dM_V} = 0 \ , \qquad M_V \ge 16.0 \ .$$

a) Calculate k.

b) Suppose that the more distant stars have the same luminosity function as the nearest neighbors. Also assume that the unaided human eye can detect stars as faint as apparent magnitude $m_o = +6.0$. Calculate the number of stars visible to the human eye, neglecting interstellar absorption and reddening. At what distance would a star with $M_V = 0$ ($L \sim 70\ L_\odot$) be barely detectable to the human eye? The same for $M_V = 4.7$ ($L \sim L_\odot$), and for $M_V = 10$ ($L \sim 0.007\ L_\odot$).

(2) (20 pts) Consider the following reaction which happens in a volume V in equilibrium at a temperature T :

$$H \rightleftarrows H^+ + e^- \qquad (1)$$

Derive the Saha Eqn for this,

$$\frac{N_{H^+} N_{e^-}}{N_H} = \frac{(2\pi m_e k T)^{3/2} e^{-\chi/kT}}{h^3} \qquad (2)$$

where the N_i are # density, k is Boltzmann's const, $\chi = 13.6$ eV is the ionization energy of H, and h is Plank's const. The derivation I'd like should start with the Thermodynamic definition of the Chemical Potential μ, which can be written in many equivalent forms, e.g. :

$$\mu_i = \left(\frac{\partial E}{\partial N_i}\right)_{\underbrace{S,V,N_j}_{\text{constant}}} \quad (j \neq i) \equiv \text{dimension of energy}$$

$$\text{or} \quad = -T\left(\frac{\partial S}{\partial N_i}\right)_{E,V,N_j} \quad (j \neq i)$$

$$\text{or} \quad = \left(\frac{\partial F}{\partial N_i}\right)_{T,V,N_j} \quad (j \neq i)$$

$$\text{or} \quad = \left(\frac{\partial G}{\partial N_i}\right)_{T,P,N_j} \quad (j \neq i) \qquad \text{etc}$$

Therefore, μ is pertinent to systems where the # of particles can change.

In equilibrium show that the reaction (1) leads to:

$$\mu_H = \mu_{H^+} + \mu_{e^-} \qquad (3)$$

Equilibrium, for example, could be stated as a thermodynamic condition like $dS = 0$.

Next show that the chemical potential is related to the partition function of internal energies of the particles of type i:

$$\mu_i = -kT \ln \frac{Z_{\text{internal}, i}}{n_i} \qquad (4)$$

n_i — # of particle of type i (NOT # density)

From this and (3) you should show that you get the law of mass action:

$$\cancel{\frac{N_{H^+} N_{e^-}}{N_H}} \quad \frac{n_{H^+}\, n_{e^-}}{n_H} = \frac{Z_{int, H^+}\, Z_{int, e^-}}{Z_{int, H}} \qquad (5)$$

n — these n are again # of particles.

Finally, with the approximation $m_p \approx m_{H^+} \sim m_H$, and approx. the $Z_{int, H}$ only by its 1st term, derive (2).

(3)

15 pts

Consider a large volume V containing equations
10^{12} Hydrogen atoms per cm^3, <u>initially</u> at
$T \sim 0\,K$. Calculate (i.e., derive an equation)
the energy which must be supplied to raise
the gas to a temperature T and plot E vs. T,
in the range $0 < T < 10^4\,K$. At what temp.
T has <u>half</u> the energy gone into ionization?
[Hint: the equations you'll need to solve for
this temperature will require numerical methods;
make tables of values, use a calculator, etc,
to <u>estimate</u> the answer!]

(4) As discussed in class, the "metals" in the Solar atmosphere are important for understanding the spectra. Specifically, metals with low ionization potentials supply an abundance of electrons which can be captured by hydrogen to form H^- ions (a famous "problem" solved by Chandrasekhar!). The H^- ion has ONLY one bound state (I.P. = 0.75 eV); it can be easily photo-ionized by light from Sun. Since 0.75 eV is equivalent to a kT for $T = 8600\,K$ and the $T_{effective} \approx 6000\,K$ for the Solar surface, it is clear that H^- could be very important for the opacity of the Solar atmosphere. The other possible absorber of visible continuum radiation from the Sun is an H atom itself. However, the H atom must be in some excited state, because the ground state H atoms can only absorb in the far U.V.

15 pts

Assume thermal equilibrium and calculate the number ratio of H^- ions to H atoms in the appropriate states (a single principle quantum number $n > 1$ determined by you!) Show that

$$\frac{N(H^-)}{N(H_n)} \approx 10^2 \quad \text{for } T = 6000\,K \text{ and } n_e = \frac{10^{14}}{cm^3}$$

The conclusion is that the H^- do indeed play the important role in scattering radiation from the solar surface.

(5) 15 points

For a pure hydrogen gas with a gas pressure of $P_g = 10^3$ dyn cm^{-2} and a temperature $T = 10\,080$ K, calculate the ratio H^+/H and the electron pressure $P_e = n_e kT$. Remember that $n_e = H^+$ and $P_g = nkT$, with $n = e + H^+ + H$. $\chi_{ion} = 13.6$ eV.

(6) Extra credit for 15 points:

For a pure helium gas with a gas pressure $P_g = 10^3$ dyn cm^{-2} and $T = 15\,000$ K, calculate He^{2+}/He^+, He^+/He, and P_e. Remember $n = n_e + He^{2+} + He^+ + He$. $\chi_{ion}(He) = 24.58$ eV; $\chi_{ion}(He^+) = 54.4$ eV. Here $\chi_{ion}(He^+)$ is the energy needed in order to remove the additional electron from the He^+ ion to make He^{2+}.