ASTRO 536 HW #4 - Due 11/8/17 in class

Problems/exercises for the Virial Theorem, Hydrostatic Equil.:

Problem 1 is for 20 points, half for extra credit ; The rest are worth 10 points each

1) Start with the Equation of Motion:

$$\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} P - \rho \vec{\nabla} \phi + \frac{1}{c} \vec{J} \times \vec{B}$$

for a cloud of gas with density ρ, velocity $\vec{v}$, pressure P, grav. potential ϕ, electric current density $\vec{J}$, and magnetic field $\vec{B}$. Using Ampere's Law <u>without</u> displacement currents, show that:

$$\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} P - \rho \vec{\nabla} \phi - \frac{1}{8\pi} \vec{\nabla}(B^2) + \frac{1}{4\pi} (\vec{B} \cdot \vec{\nabla}) \vec{B}$$

From this equation derive the Virial Theorem by taking the scalar product of both sides with $\vec{r}$ and integrating over a volume V bounded by surface S. You should get:

$$\frac{1}{2} \frac{d^2 I}{dt^2} = 2J + 3U + M + \Omega + \oint_S (\vec{r} \cdot \vec{B}) \vec{B} \cdot d\vec{S} - \oint_S \left(P + \frac{B^2}{8\pi}\right) \vec{r} \cdot d\vec{S}$$

where:

$$I = \int \rho r^2 dV$$

$$U = \int P dV \qquad M = \frac{1}{8\pi} \int B^2 dV$$

$$J = \frac{1}{2} \int \rho v^2 dV \qquad \Omega = -\int \rho \vec{r} \cdot \vec{\nabla} \phi \, dV$$

2) Consider a gaseous sphere of uniform temperature T and density ρ, radius R, in a medium of constant pressure P. From the virial theorem, show that such a sphere can be in equilibrium with the external medium only if the pressure is less than a critical value:

$$P_0 = \frac{3^4 10^3}{2^{13} \pi} \left(\frac{kT}{\mu m_p}\right)^4 \cdot \frac{1}{M^2 G^3}$$

(3)

(1) Consider the equation of hydrostatic equilibrium for a star of radius R, mass M.

(a) Show that the pressure at the center, $P(0)$, is given by the equation:

(5 pts)

$$P(0) = \frac{1}{3}\left(\frac{4\pi}{3}\right)^{1/3} G \int_0^R \left(\overline{\rho(r)}\right)^{\frac{4}{3}} M^{-\frac{1}{3}}(r)\, dM(r)$$

Here, $\overline{\rho(r)}$ is the average density of a sphere of radius $r < R$ inside the star.

(b) Assume that $\overline{\rho(r)}$ is a monotonically decreasing function of r. Prove that:

(5 pts)

$$\frac{3}{8\pi}\left(\frac{G M^2}{R^4}\right) \leq P(0) \leq \frac{1}{2} G \left(\frac{4\pi}{3}\right)^{1/3} \rho^{4/3}(0)\, M^{2/3}$$