ASTRO 536 HW #5 - Due 11/22/17 in class

Each problem is worth 15 points, with (any) one problem counted as Xtra Credit

1)

(a) Consider the Lane-Emden equation which describes polytropic star models:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\phi}{d\xi}\right) = -\phi^n$$

with $\phi(0)=1$, $\left.\frac{d\phi}{d\xi}\right|_{\xi=0} = 0$

(b) Find analytic solutions for $n=0$ and $n=1$. For each of these cases compute the mass, mean density, radius, and central pressure as a function of K and λ (see lecture notes for definition of K & λ).

Try to find an analytic solution for $n=5$.

2)

Consider a star described by the polytropic equation of state $P = K\rho^{(n+1)/n}$ and let R and M be the radius & mass of the star, respectively. Prove that:

$$\text{Gravitational P.E.} \equiv \Omega = -\frac{3}{5-n}\cdot\frac{GM^2}{R}$$

Internal Energy $\equiv \mathcal{U} = \dfrac{1}{(\gamma-1)(5-n)} \dfrac{GM^2}{R}$

Total Energy $\equiv E = -\dfrac{3(\gamma - 4/3)}{(\gamma-1)(5-n)} \dfrac{GM^2}{R}$

Mean Temperature $\equiv \bar{T} = \dfrac{\int \rho T \, dV}{\int \rho \, dV} = \dfrac{\mu M_p G}{k(5-n)} \cdot \dfrac{M}{R}$

[Hint: start with Ω]

3)

Consider the standard set of stellar structure equations for a completely convective star for which $\gamma = 5/3$ throughout. Prove that P & ρ are given by the polytropic relation for $n = 1.5$

$P = K \rho^{\frac{n+1}{n}}$ where K is given by formula $E = 4\pi K^{-3/2} G^{3/2} M^{1/2} R^{3/2} = 45.5$

Given M and R, calculate the central Temp, T_c.

It can be shown that the Hayashi protostars for $M \lesssim M_\odot$ correspond to this model during their descent along the Hayashi track after quasi-static equil. is established.

4)

Calculate the central temperature, T_c, and radius of a star of $M = M_\odot$ assuming that it can be described as a polytrope of $n = 1.5$ and its total internal energy is equal to the energy spent in dissociation of H_2 and ionization of H and He. Take $X = 0.70$ and $Y = 0.28$.