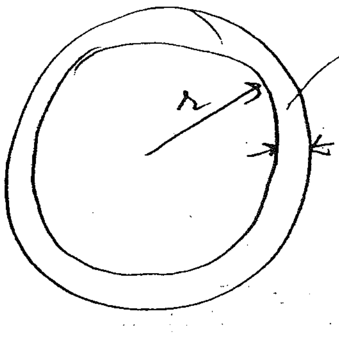
Solutions to HW #2 →

(1)



\# of stars in shell:

$dN = 4\pi r^2 dr\, n$

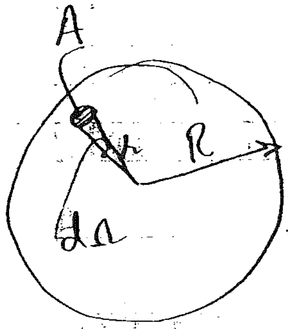
$\Phi = \frac{L}{4\pi r^2}$ from 1 star

For dN stars:

$d\Phi = \Phi\, dN = nL\,dr$

(a) $\int_0^\infty d\Phi = nLr\Big|_0^\infty = \infty$!

(b)



$r^2 d\Omega = A \rightarrow d\Omega = \frac{A}{r^2}$

$d\Omega \equiv$ angular area of a star at distance r

The angular area covered by stars in a shell (e.g. (a)) is $(n4\pi r^2 dr)\,d\Omega = n4\pi r^2 dr \frac{A}{r^2}$

Integrating this from $r = 0 \rightarrow R$ should give 4π

$$\int_0^R 4\pi n A\, dr = 4\pi$$

$$\rightarrow nAR = 1$$

$$\rightarrow \boxed{R = \frac{1}{nA}}$$

Flux from stars out to R:

$$\Phi = \int_0^R nL\,dr = nLR = \frac{L}{A}$$ the absolute lumin. of sun

1 (c) $R = \frac{1}{nA}$

$$n = \rho/M_\odot = \rho_c/M_\odot = \frac{2.8 \times 10^{11}}{4} / \text{Mpc}^3$$

$$\approx \frac{7 \times 10^{10}}{\text{Mpc}^3} \approx \frac{2 \times 10^{11}}{\text{Mly}}$$; use $1 \text{ pc} \approx 3.3 \text{ lyr}$

so $1 \text{Mpc}^3 = (10^6 \text{ pc})^3 = 10^{18} \text{ pc}^3 = (3.3 \times 10^6 \text{ lyr})^3$

$$= 3.5 \times 10^{19} \text{ lyr}^3$$

$$\rightarrow n = 2 \times 10^{-9} / \text{lyr}^3$$

$A = \pi R_\odot^2 = \pi \times (7 \times 10^8 \text{m})^2$; $3.3 \text{ lyr} = 3.1 \times 10^{16} \text{ m}$

so $7 \times 10^8 \text{m} \times \frac{3.3 \text{ lyr}}{3.1 \times 10^{16} \text{m}} \sim 7 \times 10^{-8} \text{ lyr}$

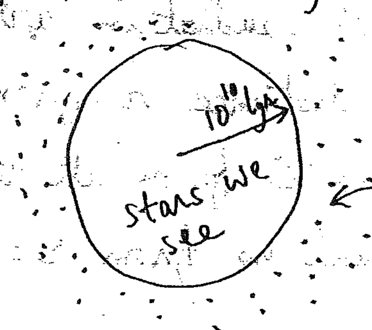
$$= \pi (7 \times 10^{-8} \text{ lyr})^2$$

$$\rightarrow A = 49\pi \times 10^{-16} \text{ lyr}^2$$

so $R = \frac{1}{nA} = \frac{\text{lyr}}{2 \times 10^{-9} \times 49\pi \times 10^{-16}} = \frac{10^{25}}{98\pi} \text{ lyr} \sim \frac{10^{23}}{\pi} \text{ lyr}$

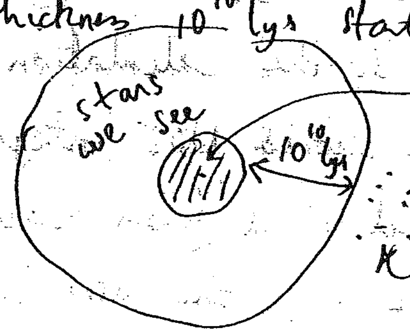
$$\boxed{R \approx 3 \times 10^{22} \text{ lyr}}$$

(d) τ of stars $\sim 10^{10}$ lyr or $\sim 10^{-12} R$



light from these hasn't reached us yet

$> 10^{10}$ lys later stars nearest us start dying out so ~~this~~ the "shell" of thickness 10^{10} lys starts to move outward:



dead stars

light from these hasn't reached us yet.

So the night sky will be 10^{-12} times as bright as the surface of sun!

Or the flux of light at night:

$$F \approx 10^{-12} \frac{L}{A} = \frac{(10^{-12})(3.8 \times 10^{26} J/s)}{\pi R_\odot^2}$$

$$F \approx 10^{-12} (2.5 \times 10^{8} J/s)$$

$$F \approx 2.5 \times 10^{-4} J/s$$

For a discussion of these results and the resolution of the paradox, please read the handout from Harrison's "Cosmology. The Science of the Universe."