PHYC 581/480, HW 1

Due: 21-Sep-16

1)

(a) (5 points) We demonstrated in class that if a space appears isotropic from 2 or more points then it must be homogeneous. Does a homogeneous space also imply isotropy? Support your answer with examples.

(b) ~~Cons~~ (10 points) Consider a test for homogeneity and isotropy of the local universe (i.e., $z \ll 1$, curvature neglected). Some class of objects all have roughly the same absolute or intrinsic luminosity, $\underline{L}$, and have approximately a constant number density, $\underline{n}$. Derive a relationship between the number of such objects observed brighter than a flux $\underline{F}$ (or if you like, magnitude $\underline{M}$) and n and $\underline{L}$. That is, the number of objects brighter than $\underline{F}$, $\underline{N(F)}$, is proportional to what power of $\underline{F}$? Locally, observations indeed agree with this prediction but at larger distances the relationship ~~becomes~~ ~~ste~~ changes such that $\underline{N(F)}$ falls more steeply with $\underline{F}$ than predicted. Discuss possible reasons for this deviation. This test was used in support of Big Bang Cosmologies over the Steady State Cosmology. Explain why and how.

② (15 points) Using the Robertson-Walker metric, in any form you like, derive the expression for the area and volume enclosed between the origin out to some proper radial distance (at a given epoch t!) Do this for each of the 3 geometries: positively curved, flat, and negatively curved. Based on your results discuss whether the respective spaces are: bounded or unbounded; finite or infinite. To explicitly show the dependence on the curvature of the space, expand your results in the limit of small curvature (i.e., $\underline{K} = \frac{1}{R_c^2}$, where $\underline{R}_c$ is the radius of curvature).

3)
(5 points) For a uniformly expanding isotropic and homogeneous universe show that the relative velocity of fundamental observers, $\underline{v}$, is proportional to their distance, $\underline{r}$: $v \propto r$.

4)
(5 points) If some object which expands with the universe has a size $\underline{l_0}$ today, what would its angular size, $\underline{\Delta\theta}$, have been at redshift $\underline{z}$? Expand your result in the limit of low redshifts and negligible curvature and express your result in terms of $\underline{z}$, $\underline{H_0}$, and q_0 using appropriate relations derived in class for $\underline{r(z)}$, where $\underline{r}$ is the comoving radial distance.

(5) (10 points) Derive a relationship for the observed surface brightness, SB, of an object at redshift z with absolute luminosity $\mathcal{L}$ and proper size ℓ:

$$SB \equiv \frac{\text{Flux } (\text{ergs} \cdot s^{-1} \cdot m^{-2})}{\text{angular size } \Delta\Omega \text{ } (\text{arcsec}^2)}$$

$$= \text{function of } z, \mathcal{L}, \ell$$

What is interesting about the result when compared with other relationships between observed and intrinsic properties of objects which were derived in class? What can this test be used for?