PHYC 581/480, HW 4

Due: 29-Sep-16

(1) BIG BANG NUCLEOSYNTHESIS. The following questions involve arguments and calculations done in class.

(a) 5 pts) Since BBN occurs during the first few minutes after the Big Bang, this clearly means that one is dealing with the radiation dominated epoch from the Friedman Equations. The Friedman Eqn of interest here is:

$$\frac{\dot{R}(t)^2}{R(t)^2} = \frac{8\pi G}{3}\left(\rho_m(t) + \rho_r(t)\right) - \frac{c^2}{\mathcal{R}^2 R(t)^2} \quad (1)$$

Since we're dealing with very early times <u>show</u> that the matter and curvature terms are unimportant relative to the radiation term, resulting in:

$$\frac{\dot{R}(t)^2}{R(t)^2} \approx \frac{8\pi G}{3}\rho_r(t), \quad (2)$$

and rewrite the right hand side in terms of the present radiation inertial mass density $\rho_{r,0}$.

(b) 5 pts) Integrate the above final equation to get a relation for $R(t)$. Make sure you include all of the constants! ~~And make sure the equation~~

(1c) Using your result in (1b) derive a relationship
(5 pts) for $H(t)$.

(1d) Eliminate the variable t from $R(t)$ and $H(t)$
(10 pts) to get a relationship $H(R)$. Knowing how the temperature T scales with the scale factor $R(t)$, convert your relationship for $H(R)$ to $H(T)$. This result tells you what the <u>expansion rate</u> of the universe was at early times <u>as a function of temperature T</u>.

(1e) Your result in (1d) for $H(T)$ will have some constants in it one of which will be $\rho_{r,0}$ and another
(10 pts) will be T_0, the present radiation temperature. The relation between these is found using:

$$\varepsilon_{r,0} = a T_0^4 = \rho_{r,0} c^2$$

This is the case when the energy density $\varepsilon_{r,0}$ is due to only radiation. As discussed in class, however, there are other sources of relativistic particles that contribute to ε_r at early times besides photons: $e^+, e^-, \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$. So our equation for $\varepsilon_{r,0}$ becomes $\varepsilon_{r,0} = \chi a T_0^4$. Rewrite $H(T)$

by elimination of $\rho_{s,0}$ and T_0 to get a relationship that has $H(T)$ in terms of $\chi, G, a,$ and c as the only constants (besides a numerical one).

(1f) Take you final $H(T)$ from above and call the inverse of this the expansion timescale at temperature

(10 pts) T: $$\tau_{expansion} \equiv \frac{1}{H(T)}$$

Compare this with the weak interaction timescales at high temperatures given by:

$$\tau_{weak} = \left(\frac{1.7 \times 10^{10}}{T}\right)^5 \text{ seconds} \quad ; \ T \text{ is in Kelvin}$$

What is the condition for the "freeze-out" of the weak interactions? <u>At what temperature, T_f, does this happen</u>? To calculate this <u>you will need to calculate χ</u> as we did in class taking into account what the degrees of freedom are for all the particles listed at the bottom of the previous page.

(1g) Using the non-relativistic Maxwell-Boltzmann distribution for number densities:

(10 pts) $$N = g\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{-mc^2/kT}$$

derive the neutron to proton ratio, $\frac{n}{p}$, at the freeze-out temperature for weak-interactions, T_f. Use this to calculate the mass fraction of ^4He assuming all neutrons go into making ^4He:

$$Y \equiv \frac{\text{Mass}(^4He)}{\text{Mass(protons)} + \text{Mass}(^4He)}$$

(1h) (10 pts) Your value for Y should be quite a bit larger than the "real" value of around 25%. Why is that? Give detailed explainations and calculate the correct value based on your arguments.

(2) $\boxed{\text{EXTRA-CREDIT}}$

(20 pts) Using diagrams for $\tau_{expansion}$ and τ_{weak} versus T describe in detail how the $\frac{n}{p}$ ratio will vary when there are more or less neutrino families than the standard value of 3. How does this effect the value for Y above? Explain why the theoretical abundances of D, ^3He, and ^7Li vary with the density of baryonic matter in the manner they do.