Astro 101 *Useful Physics Notes*

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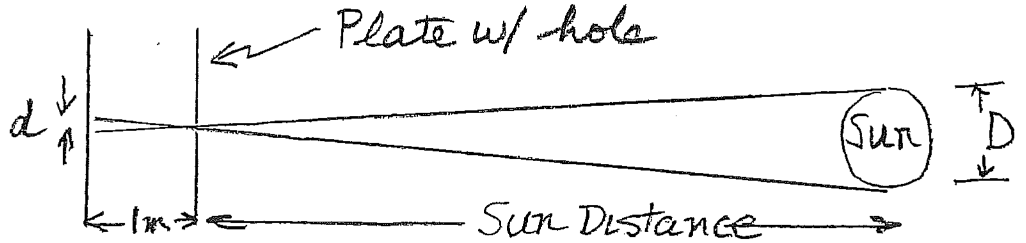
Important **physics** from Chapter 9:
Table 9.1 lists a number of properties of the Sun. You may find it interesting to see how these are measured!

1. **Sun's radius:** You can do this yourself! You need two stiff pieces of paper (*e.g.* paper plates). Make a small hole in one. Then align them with the plate with the hole between the Sun and the other plate as sketched here:



Then adjust the distance between the two plates to be some known distance, *e.g.* 1 m, and measure the diameter, d, of the Sun's image on the "other" plate. If you measure, d, in inches, or millimeters, etc you need to convert it to meters! Curiously you have measured the angular diameter of the Sun:

$$angular\ diameter \equiv \frac{d(m)}{1m} = \frac{D(km)}{Sun\ distance(km)}$$

where D is the diameter of the Sun in km and the Sun's distance is 1 A.U. which is calibrated using the Venus-Earth system, Fig. 1.14 of the text, to be $1.496\ 10^8 km$.

If the two plates are separated by about 1m then the diameter of the Sun's image will be about 1cm; actually $d = 0.0093m$ for 1m separation. Then the formula above gives the Sun's diameter as:

$$D(km) = \frac{0.0093m}{1m} \times 1.496\ 10^8 km = 1.391\ 10^6 km$$

Finally the radius of the Sun is 1/2 of this value or $R_{Sun} = 696,000km = 6.96\ 10^8 m$.

2. **Sun's mass:** Recall from my Chapt 3-4 notes that the mass, M, of an astronomical body is determined by the orbit parameters of a satellite (of that body):

$$M = \frac{rv^2}{G}$$

where r is the radius of the satellite's orbit in meters, v is the velocity (speed) of the satellite in meters/second, and $G = 6.7\ 10^{-11} Nm^2/kg^2$ is the known Newton's gravitational constant, see Figure 1.18 of the text.

Let's use the parameters of the Earth's orbit to *weigh* the Sun! We already know the radius of the Earth's orbit (reviewed above) as $r\ =\ 1.496\ 10^8 km\ =\ 1.496\ 10^{11} m$. The average speed, v, of the Earth in it's orbit is given by:

$$v\ =\ \frac{distance}{time}\ =\ \frac{orbit\ circumference}{orbit\ time}\ =\ \frac{2\pi r}{1\ year\ in\ seconds}$$

which gives $v = 2.99\ 10^4 m/s$. Putting it all together:

$$M_{Sun}\ \equiv\ M\ =\ \frac{1.496\ 10^{11} m \times (2.99\ 10^4 m/s)^2}{6.7\ 10^{-11} Nm^2/kg^2}$$

giving $M_{sun} = 2.0\ 10^{30} kg$.

Once we know the Sun's mass then we can use a generalization of Kepler's 3^{rd} law (P40 of text) to first measure an object's mass in terms of the Sun's mass:

$$M_{object}\ =\ \frac{a^3}{P^2} \cdot M_{Sun}$$

where a is the radius of the satellite's orbit **in astronomical units** and P is the period of the satellite's orbit **in Earth years**. Then for example the mass of the Earth (in Solar masses) is:

$$M_{Earth}\ =\ \frac{((3.85 \times 10^5 km)/(1.50 \times 10^8 km))^3}{((27.3 days)/(365.25 days))^2} \cdot M_{Sun}\ =\ 3.03 \times 10^{-6} M_{Sun}$$

But we just calculated the mass of the Sun, so the mass of the Earth = $6.05\ 10^{24}$kg. which is about right; see page A-3 of text!

3. **Sun's average density:** Recall from Chapt 3-4 notes that

$$density\ =\ M/volume$$

where M is the Sun's mass and the Sun's volume is given by the volume ($\frac{4}{3}\pi r^3$) of a sphere of radius: $r = R_{Sun}$. Thus the density is:

$$density\ =\ \frac{2.0\ 10^{30} kg}{(\frac{4}{3})\ \pi\ (6.96\ 10^8 m)^3}\ =\ 1410 kg/m^3$$

which is curiously close to the density of the Jovian planets (see Table 7.1).

4. **Sun's luminosity:** The energy per second and per (unit) area in sunlight at the Earth's orbit is called the *solar constant*. This is about 1370 watts/m^2 above the Earth's atmosphere. As we expect that the sun radiates energy uniformly in every direction, then every square meter of surface (area $4\pi r^2$) at a distance of $r = 1$ A.U. from the Sun has this amount of energy passing across it every second! Thus the total energy per second from the Sun is given by:

$$Sun\ luminosity \ = \ (\frac{energy/s}{m^2}) \ \times \ (number\ of\ square\ meters\ at\ 1\ A.U.)$$

$$= \ (1370\ watts/m^2) \ \times \ (4\ \pi\ (1.496\ 10^{11} m)^2) \ = \ 3.86\ 10^{26}\ watts$$

5. **Sun's (photosphere) temperature:** The total energy/second radiated by a Black Body is given by Stefan's law (2-2 More Precisely):

$$energy/time \ \equiv \ luminosity \ = \ Surface\ area \times \sigma \times T^4$$

where the $Surface\ area$ of the Sun is the area of a sphere ($4\pi r^2$) with $r = R_{Sun}$ and σ is the known Stefan's constant: $\sigma \ = \ 5.67\ 10^{-8}\ watts/(m^2 K^4)$. As we know the Sun's luminosity and surface area we can solve for surface temperature, T:

$$T \ = \ \sqrt[\frac{1}{4}]{\frac{3.86\ 10^{26} watts}{(4\ \pi\ (6.96\ 10^8 m)^2)\ (5.67\ 10^{-8} watts/(m^2 K^4))}} \ = \ 5780K$$

Pretty fun!!