Spring 2006

Physics 467: METHODS OF THEORETICAL PHYSICS II

Tu Th 2:00 pm - 3:15 pm, P&A Rm 184

Instructor: S. PRASAD, Rm 1119 P&A (North Wing); 277-5876; sprasad@unm.edu

The present course will continue in the spirit of Physics 466 with the development of further mathematical concepts and tools that define the essential mathematical foundation of our graduate physics courses. The course will stress physical applications drawn from classical and quantum mechanics, relativity, optics, solid-state physics, electromagnetism, and statistical mechanics.

We shall cover five areas of mathematical physics in this course, namely linear algebra, tensors, group theory, nonlinear dynamics, and probability/statistics. The concepts of linear spaces and linear operations on them will motivate our discussions of linear algebra and of tensors. The subject of matrices and determinants will be covered in detail, and their relevance to modern optimization methods and analysis will be explored. The most important aspects of both continuous and discrete groups, particularly in the context of symmetries, invariances, and conservation laws of physics, will be discussed. Our coverage of nonlinear dynamics will be brief, touching upon the notions of linear and nonlinear instabilies of physical systems described by one or more coupled nonlinear ODE's. Finally, a couple of lectures will be devoted to probability and statistics, with some applications drawn from quantum and statistical mechanics.

Texts:

I know of no text that includes a sufficiently comprehensive coverage of all of the topics I intend to cover in this course. However, since G. Arfken and H. Weber's *Mathematical Methods for Physicists* (Academic Press, 5th/6th edition) does contain a majority of the topics, it will nominally serve as the main text for the course.

Our treatment of linear algebra will be steeped in the discussion of linear spaces and transformations. Two excellent sources for such a treatment are: Gelfand's *Lectures on Linear Algebra* (Wiley Interscience, 1961) and Kolmogoroff and Fomin's *Introductory Real Analysis* (Dover, 1970). A third useful text is Curtis's *Linear Algebra* (Springer Verlag, 4th edition, 1984). *Numerical Recipes* by Press, *et al.* is often an excellent source for discussion of methods of linear algebra, particularly those methods that are well suited for numerical computation.

Topics in nonlinear dynamics, an area of increasing interest in a variety of physical settings, will be drawn from Nicolis's *Introduction to Nonlinear Science* (Cambridge, 1995).

For the material on probability and statistics, good sources are the introductory chapter in F. Reif's *Fundamentals of Statistical and Thermal Physics* (McGraw Hill, 1965) and G. Roussas's *A Course in Mathematical Statistics* (Academic Press, 2nd edition, 1997).

Grading:

The grading in the course will be based on your performance in homework (HW) assignments (25%), two mid-term (MT) exams (25% each), and a final exam (25%). There will be about 9 HW assignments in all with 4-5 problems each. MT I is tentatively scheduled for February 21; MT II for April 4; and the Finals on May 9 at 12:30 pm.

Grader: Pablo Reyes

Office Hrs: *Instructor's*: M F 1-2 pm or by appointment
Grader's: To be determined

PHYSICS 467

I. LINEAR ALGEBRA (∼4 weeks)

*Linear vector and function spaces, including Hilbert space
*Dual vector spaces, inner and outer products, Gram-Schmidt orthogonalization
*Eigenvectors and eigenvalues of operators, diagonalization
*Matrix representation of operators and expansion in basis functions
*Hermitian and unitary matrices
*Dirac notation
*Important concepts of matrix algebra

II. TENSOR ANALYSIS (∼2 weeks)

*Definition, outer product, contractions
*Pseudotensors, dual tensors
*Tensor differential operations

III. GROUP THEORY (∼4 weeks)

*Definition
*Representation theory
*Generators of continuous groups
*Symmetries, invariants, and conservation laws in physics
*Special groups – SU(2), SU(3), O(3), and Lorentz groups
*Discrete groups – classes and character table

IV. NONLINEAR DYNAMICS (∼2.5 weeks)

*Dynamical systems, phase spaces, invariant manifolds
*Linear stability analysis of fixed points
*Nonlinear behavior around fixed points: bifurcations

V. ELEMENTARY STATISTICS (∼1.5 weeks)

*Probability, statistical ensembles
*Conditional and joint probabilities, probability densities, Bayes theorem
*Law of large numbers, central-limit theorem, Gaussian prob. density
*Binomial and Poisson distributions