Contemporary Physics Laboratory

Spring 2008

Table of Contents

PHYS 493L-001
Tuesday and Thursday 1400-1650 hr
Room P&A 116, Call #28340
Instructor:
  Paul R. Schwoebel
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  Phone: 277-2616
  E-mail: kas.unm.edu
  Office hours: By appointment

Text (optional): *Building Scientific Apparatus*, 3rd edition, by John Moore, Christopher Davis and Michael Coplan (Perseus Books, Cambridge MA, 2003). Some reading in this book will be required. Other reading material will be recommended as it pertains to a particular lab element.

Purpose: Senior Lab will expose students to the diverse experimental techniques required of a graduate student or professional scientist. These techniques will be developed with the hands-on experience gained through a series of experiments that each require multiple class sessions. These experiments will involve holography, elementary particle decay, Doppler shifts, interference phenomena, and gas discharges. The student will be exposed to a variety of important techniques including interferometry, precision timing and coincidence techniques, particle detection, and sensitive velocity measurements. The laboratory will also include an element devoted to teaching the methodology behind the construction of experimental apparatus. Here the student will be exposed to elementary aspects of machine shop practice.

Grading: Grades will be based on your performance in the laboratory, such as thorough, complete laboratory notebooks, experimental analysis, and laboratory reports. In addition, each student will be given a one-half hour oral final exam. Laboratory performance and the oral exam will be, respectively, 85% and 15% of your final grade. Attendance is required.

You will need to complete four laboratory elements that must include laboratories 1 and 2. The time required to complete a lab element will vary depending on the particular lab; however, plan to spend roughly four weeks of class sessions on each. Lab reports will be turned in for grading within 2 weeks of their completion. If experiments are performed with a partner, each student should keep their own laboratory note book and write their own laboratory reports.

Laboratory Reports: Use the standard format for laboratory reports:

1. Abstract
2. Introduction
3. Theory
4. Experimental Apparatus
5. Results
6. Discussion & Conclusions
7. References

Reports should appear as they would in a scientific journal, with graphics embedded in the text.

LABORATORY SAFETY

Elements of this laboratory class pose certain hazards. No instructions can substitute for common sense. If you are unclear about something ask the instructor or the TA. Safety topics include: Machine Shop Safety, Laser Safety, High Voltage Safety, and Radiation Safety. Read the supplied and recommended information. Then print and sign the form on page iv, and turn this in to the instructor. This signed form will be required before you can use the laboratory.

Safety rules and guidelines are designed to reduce the possibility of accidents in routine situations. In a research laboratory where much equipment is custom made there is no routine, and there is no safety rule or policy that can substitute for an intelligent and careful handling of the equipment. The following notes are intended to make you aware of the risks of working in the Senior and Optics Laboratories.

Machine Shop Safety.

Some basic directions are given in the lab write up. Duplicate and additional items are listed below.
- Never work alone in the shop.
- Safety glasses must be worn in the shop at all times.
- Do not enter the shop with bare feet, sandals, slippers, or open toed shoes.
- Tie back long hair.
- Do not wear loose clothing or jewelry.
- Maintain a clean work area.
- Do not touch chips while the machine is operating.
- Do not leave chuck keys in chucks.
- Secure all work (clamp in vice, chuck, etc.).

Laser Safety

For a complete introduction to laser safety see:

http://www.osha.gov/SLTC/laserhazards/

The American National Standard Institute (ANSI) has classified lasers according to what they perceive as hazard level. Class IIIb and class IV lasers are considered as hazardous. The HeNe lasers used in the experiments you will be doing typically will not cause permanent eye damage, however, it is good practice to: 1) Never look directly at the beam of the laser, 2. Keep the laser beam in one horizontal plane close to the table, 3) Never bend down to the table level. We have purposely chosen to have the optical table surfaces as low as possible which reduces the chances of having your eyes at the beam height. Low chairs are not allowed in the lab. Sitting accommodations are limited to high stools.

High Voltage (&Line Voltage) Safety

You can supplement the following general electrical safety description with reading at:

http://www.osha.gov/SLTC/electrical/index.html

The following guidelines are to protect you from potentially deadly electrical shock hazards as well as the equipment from accidental damage. Note that the danger to you is not only in your body providing a conducting path, particularly through your heart. Any

involuntary muscle contractions caused by a shock, while perhaps harmless in themselves, may cause collateral damage due to contact with sharp edges and points inside various things like stamped sheet metal shields and the cut ends of component leads. In addition, the reflex may result in contact with other electrically live parts.

- Don't work alone because in the event of an emergency another person's presence may be essential.

- Always keep one hand in your pocket when near a line-powered or high voltage system.

Radiation Safety

For a complete introduction to radiation safety see:

http://www.osha.gov/SLTC/radiation/

It is possible that several types of radioactive source will be encountered in the laboratory. Most of the sources you will use in this laboratory are sealed. Low activity sources and therefore present no health issues.

NEVER eat, drink, or smoke in the laboratory. Wash your hands at the conclusion of each laboratory session. Sealed gamma ray sources having activities of ~ 1 μCi can be handled with your fingers. For sealed and unsealed source of roughly 10 μCi or greater use tongs or other devices. Do not handle directly.

Laboratory Safety Form

Spring 2008

My signature below acknowledges that I have read and understood the safety information supplied for the Optics and Senior Laboratories in the Department of Physics and Astronomy.

Name (please print)	email	Date	Signature

LAB 1: Mechanical Practices in Experimental Science

Paul R. Schwoebel

Purpose

Introduce the student to mechanical practices used in the design and construction of scientific apparatus through exposure to mechanical drafting and the fundamental operations performed in a machine shop

Reading Assignment

Chapter 1: *Building Scientific Apparatus*, 3rd edition, by John Moore, Christopher Davis and Michael Coplan (Perseus Books, Cambridge MA, 2003). Familiarity with the material in Chapter 2 is useful for future reference.

Background

The experimental scientist must routinely design and construct scientific apparatus in order to conduct research. Typically, advanced undergraduate and graduate students in the sciences have had an introductory electronics class. Often, however, students are not introduced to the mechanical aspects of the designing and constructing scientific apparatus until after they begin their graduate research. The student's research career is greatly facilitated if they acquire the proper foundations in these mechanical practices as an undergraduate.

The mechanical aspects of building scientific apparatus involve conceptualizing the requirement, producing a mechanical drawing that defines the apparatus to fulfill that requirement, and fabricating the apparatus to the necessary specifications. As an advanced undergraduate or graduate student you will often be required to accomplish all of these tasks. As a practicing scientist, most often you will perform tasks one and two and submit task three to a professional machine shop. In either case, understanding the basic principles behind metal working, glass blowing, and materials joining will aid you in making mechanical drawings to communicate your needs and designing apparatus to fulfill these needs.

Mechanical Drawing

The book, *Building Scientific Apparatus*, describes the basics of mechanical drawing. More complete treatments[1] can be consulted as your skill levels and needs grow. Typically, modern mechanical drawing is done with the aid of computer programs, referred to a Computer-Aided Drafting (CAD) programs, developed specifically for this task. The limited drawing required for this module will only require pencil, graph paper, a scale, a right triangle, and a compass.

Make a drawing of a 1/2-13 hex head bolt 2.5 inches long and a 1/2-13 slotted nut. Include a hole in the bolt, 0.40" from the beginning of the threads for a 5/32 cotter pin to lock the nut in place. Refer to the *Machinery's Handbook*[2] on reserve in the library for the necessary dimensions of the bolt, nut, and cotter pin. Complete this drawing and have your instructor check it by no later than the beginning of the 3rd class period.

Shop Safety

The 2rd class session on will be spent will be spent in the shop actually applying some of the fabrication techniques about which you have been reading. Of utmost importance is you safety in the shop. While you are working in the shop it is <u>REQUIRED</u> that you:

1) Wear safety glasses at all times.
2) Wear long pants.
3) Do not wear sandals or jewelry.
4) Roll up long sleeves past the elbows and tie back long hair.
5) Keep you work area neat and organized.

If you have a question it is much better to ask the shop personnel for help than proceed with an operation with which you are unfamiliar. Lastly remember that <u>SAFETY GLASSES MUST BE WORN AT ALL TIMES</u> when you are in the shop.

Machine Shop Practices

After having some exposure to the theory of machine tool practices, as presented in your text, it is best to continue learning by experience. For the second class period you will go to the machine shop and be introduced to the most basic equipment by one of the machinists. Under the machinist's supervision, you will then use the band saw, hand file, and drill press to make the part shown in the attached drawing.
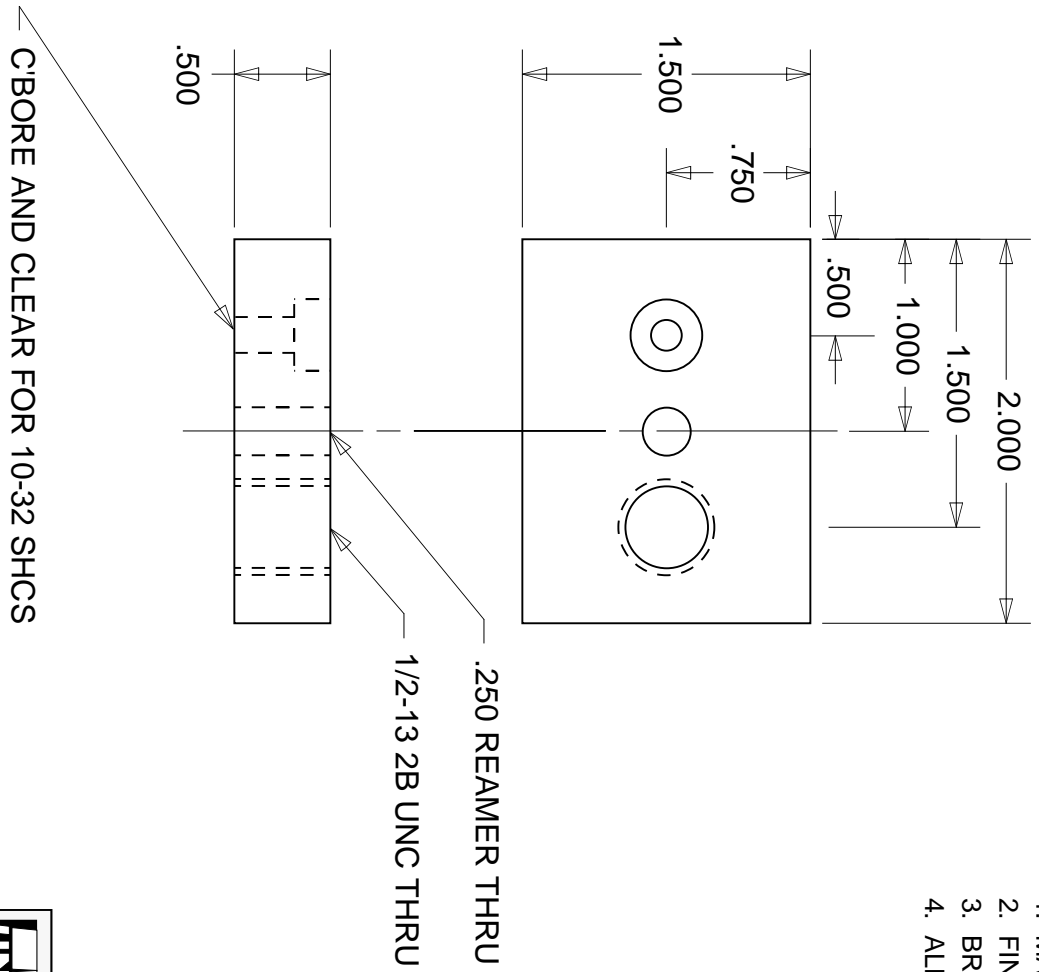
Your next machining exercise will begin in the 3rd lab period in which, again under the supervision of a machinist, you will begin to make the 1/2" slotted nut and bolt drawn for in your mechanical drafting exercise. Making these parts will require you will carry out many of the most important operations done in the machine shop using the lathe and milling machine; the machines on which the majority of the work in a machine shop is performed. Completion of the nut and bolt will require most of the remainder of this laboratory module. During this time basic joining processes such as soldering and welding will be demonstrated so that you are also familiar with these techniques.

Submit your completed nut and bolt to the course instructor for examination. If the parts are within tolerance you will have just completed your introduction to mechanical practices in experimental science.

REFERENCES

1) *Technical Drawing*, by F. E. Giesecke et al., 12th edition (Prentice Hall, 2002) ISBN: 0130081833. {This is an updated version of a book that has been a classic in the area for 60 years}
2) *Machinery's Handbook* by Erik Oberg et al, 26th edition (Industrial Press, 2000) ISBN: 0831126663 (CD ROM and Cloth). {This book has a wealth of information and is a standard reference in the metal working industry}

NOTES:
1. MATERIAL: Al 6061-T6.
2. FINISH: #32 OR BETTER, ALL OVER (NO SCRATCHES).
3. BREAK EDGES AND CORNERS .005 MIN. R (SMOOTH).
4. ALL DIMENSIONS ±.025 UNLESS NOTED.

2.000
1.500
1.500
1.000
.500
.750
.500

C'BORE AND CLEAR FOR 10-32 SHCS

1/2-13 2B UNC THRU

.250 REAMER THRU

LAB 2: Experiments in Nuclear Physics

John A. J. Matthews and Paul R. Schwoebel

Purpose

Introduce the student to some of the basic techniques and approaches used in nuclear physics.

Reading Assignment

Reading required as per references in text of experiment.

Preface

This laboratory is divided into two sections. The first section is an introduction to gamma-ray spectroscopy. γ-ray spectroscopy is of both fundamental and applied interest. The techniques introduced in γ-ray spectroscopy will be expanded upon and used in the second section to measure the mean-life of the muon. The mean life of the muon is directly related to the fundamental strength of the Weak Nuclear Force; one of the four fundamental forces in nature.

Part 1: Gamma-Ray Spectroscopy

Introduction

 The decay of many radionuclides involves the emission of γ-rays. Processes that leave the daughter in an excited state can lead to gamma emission. Alpha emission and beta emission precede gamma decay in the natural radionuclides. For example, there can be a large difference between the nuclear spin of the ground states of the parent and the daughter. Then the beta transition directly to the ground state of the daughter is forbidden and therefore most of the transitions leave the daughter in an excited state. Decay schemes for some radionuclides are shown below.
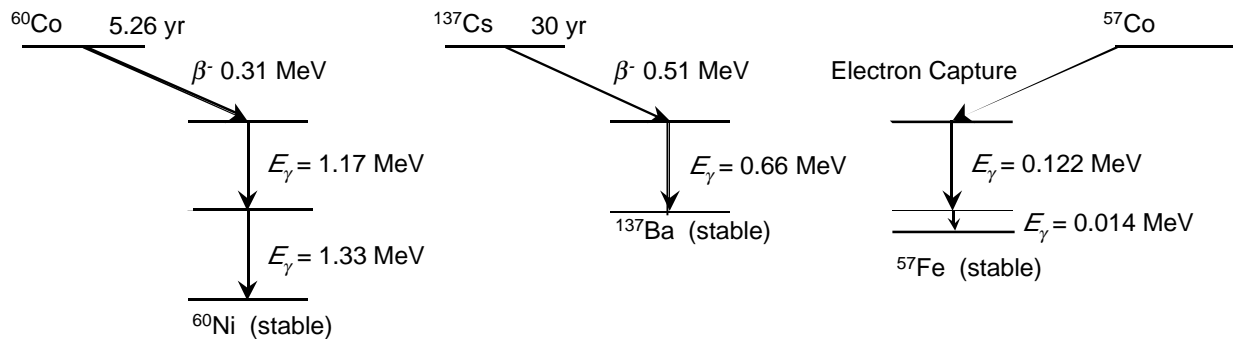
Often the half-life of the parent is very long relative to the half-life of the daughter. In this case gamma decay is in transient equilibrium with the decay of the parent and the γ-ray intensity falls off with the half-life of the parent. This is the reason it is customary to name the parent as the γ-ray source.

γ-ray spectroscopy has a number of important uses in the applied sciences. For example, it can be used to identifying much of the elemental composition of an unknown sample. To do this the

unknown sample is irradiated with neutrons which makes the sample radio active. This is so-called 'neutron-activation". One can then measure the γ-rays (and β-rays) and sample half-life to determine the constituents and their relative concentrations. This technique is used in the petroleum industry and areas of geology, medicine, and criminology, to name a few.

To learn about γ-ray spectroscopy and standard instrumentation used in nuclear physics you will:

1. Observe γ-ray energy spectra,[1]
2. Identify the processes taking place,[2-8]
3. Complete an energy calibration of the apparatus. [9, 10]
4. Determine the identity of an unknown isotope.
5. Determine the attenuation coefficients of γ-rays as a function of γ-ray energy.[2, 10, 11]

Procedure:

A radioactive γ-ray source is placed near a NaI(Tl) scintillation detector. The NaI(Tl) absorbs the γ-ray and gives a light burst proportional to the amount of energy absorbed. The light is converted into electrons by a photocathode mounted on the input of a photomultiplier tube (PMT). The PMT is interfaced by the PMT base to a high voltage power supply and amplifier (or preamplifier plus amplifier). The PMT outputs a current pulse which is proportional to, and much greater than, the initial photoelectron current. Finally a multichannel analyzer (MCA) digitizes the pulses and stores a histogram of the number of pulses versus pulse amplitude. This is shown schematically in Fig. 1.
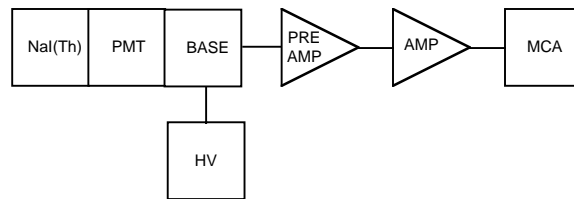

Fig. 1: Schematic drawing of the electronics for γ-ray spectroscopy. The NaI(Th) scintillator crystal, PMT, and PMT Base are a single unit. The high voltage applied to the base is negative and less than 1500 V. The preamp and amplifier are typically Nuclear Instrumentation Modules (NIM) powered by a NIM crate. The MCA is a special purpose electronics plug-in board in a PC.

1. Observe γ-ray energy spectra, identify the processes taking place, energy calibration the apparatus, and determine the identity of an unknown isotope.

Using the ^{137}Cs source, observe the output from the anode output of the PMT base on an oscilloscope using 50Ω termination. The pulse will be negative. Set the high voltage on the PMT so that the pulses are > 50-100 mV. Typically ~ *positive* 1000 V is adequate high voltage; **do not exceed +1500V**. If you use a preamplifier (before the amplifier) then connect the anode output to the input of the preamp. If you do not use a preamplifier, then change from the *anode* to the *dynode* output of the PMT base. The pulse will now be positive and about one-half the size of the anode signal. This pulse can go directly into the amplifier (which typically want positive polarity signals). Before continuing it is instructive to see the γ-ray line(s) directly on the oscilloscope. Each line will appear as a brighter *band*. To see this you will need to get the correct trigger polarity: negative if you take the signal from the anode output of the PMT base or

positive if you take the signal from the dynode output of the PMT base. You will also need a
sweep rate that is matched to the time response of the NaI(Tl) detector. Once you find the signal,
vary the high voltage (modestly) to see how the signal magnitude varies with applied high
voltage. At this point you may want to determine what voltage change will cause a doubling of
the signal amplitude!

A preamplifier-amplifier combination, or simply an amplifier, is used to shape the pulse. Pulse
shaping is important for high rate applications and/or for minimizing electronic noise. These
units also provide additional amplification if needed. Generally these modules perform best at
or below the midrange of their settings. To see how the amplifier changes the pulse, pass the
signal (from the PMT base) through channel 1 of the oscilloscope and into the input of the
amplifier (check that you have the polarity correct). Look at the output of the amplifier in
channel 2 of the oscilloscope to check that it is not clipped, i.e. limited in amplitude, and that it
meets the input specifications of the MCA. For example for the PCA-3 card the input signal
specifications are:
  - 0 to +10 volts,
  - unipolar or positive edge leading bipolar,
  - pulse rise times from 0.5 to 30 μs.

Finally look at the signal on the MCA. For simple applications you should set the MCA to
accumulate/display the maximum number of channels. The final choice of high voltage and
amplifier gain settings should place the highest energy γ-ray line near the upper end of the MCA
range. As different combinations of high voltage and amplifier gain will result in the same
amplitude signal you may want to investigate which combination gives you the sharpest signal:
i.e. the narrowest line for a fixed signal amplitude. At this point you should also accumulate γ-
ray spectra from the ^{60}Co source to learn the range of γ-ray energies and the number of distinct
lines. Then you should consider building a cave, from lead bricks, to shield the NaI(Tl) detector
from extraneous (i.e. background) γ-rays. Where does this background come from? You should
also experiment with the distance between the source and the front of the NaI(Tl) detector. Does
this make any discernable difference other than count rate? A good rule of thumb is to place the
source at least 2 detector diameters from the detector. Why? The effective solid angle of the
detector is then $(\pi r^2)/4\pi d^2$, where r is the detector radius and d is the source-to-detector distance.
With your optimal setup you should accumulate a spectrum from each of the γ-ray sources. Do
the spectra look different from your first spectra? How and why? Do the spectra look like the
text book spectra? Identify as many of the features and lines as you can. Now take individual
spectra for a couple sources such as the ^{137}Cs and ^{60}Co. You may also want to try the ^{57}Co source
if it is not too old. Do this in as short a time period as is possible. Repeat to be sure that your
peaks have not drifted! Fit the peaks with the PCA3 program on the PC to determine the channel
numbers for the center of each γ-ray line. If the DAQ electronics and the MCA are linear there
should be a linear relation between peak channel number and γ-ray energy. To check this make
a plot of channel number versus energy. Are the points in a line? Does the curve go through
(0,0) or is there an offset? To what energy does channel 1 correspond? Now get an unknown γ
source from the instructor. Using references[2, 10] identify the unknown source.

Other issues you should consider include:

Do you have a circuit diagram including all equipment device types/numbers, SETTINGS, etc so
that you could easily rebuild your setup. Have you sketched pulse shapes at different places in

the circuit? What should you check for when looking at pulse shapes? What change in phototube high voltage results in a 100% increase in the observed γ-ray pulse heights (i.e. channel number in MCA)?

To a first approximation the gamma ray line width, that is its Full-Width at Half-Maximum (FWHM) is 2.35σ, and related to the statistical fluctuations in the number of photo-electrons, N_e, that are collected from the photocathode of the phototube. In turn $N_e \propto E_\gamma$, thus:

$$\sigma / E_\gamma = 1/\sqrt{N_e} \propto 1/\sqrt{E_\gamma}$$

Thus this ratio measures N_e. Check this by plotting $(E_\gamma/\sigma)^2$ versus E_γ. If the plot is linear then our approximation was valid; that is there should be an essentially constant γ-ray energy required per observed photo-electron. What is the average γ-ray energy/photo-electron in your experiment? The inverse question is how many photoelectrons result from a 1 MeV γ-ray?[12]

γ-ray (i.e. photon) cross sections for interacting with the NaI(Tl) are rather small in the energy range of a few hundred keV to ~MeV. The photon interaction processes include the photoelectric effect, Compton scattering and pair production. What photon cross section is most directly related to the total conversion of the γ-ray to visible light in the energy range of this experiment? Is this the dominant cross section at these energies? What is the dominant photon interaction? Does this dominant process result in events in the observed γ-peaks? If not how do events get to be in the peak?

2. Measure γ-ray attenuation coefficients:

Just as γ-rays interact with the NaI(Tl) to be detected or with lead shielding to reduce background counts, γ-rays interact with all matter. The physical processes include the photoelectric effect, Compton scattering and pair production as noted above. These cross sections are combined (in a variety of ways depending on the precise definition) into an absorption or attenuation coefficient, μ. Thus following a distance, X, an initial number of γ-rays, $N(0)$, is attenuated to a final number, $N(X)$:

$$N(X) = N(0) \, e^{-\mu X}$$

Because the photon cross sections change rapidly with energy and depend on the absorber material's nuclear charge, it is interesting to measure μ at different energies and for more than one absorber material.

To measure the energy dependence of μ, start with the ^{137}Cs source and the MCA. To know $N(0)$ for each γ-ray line, you need to take (and fit) a MCA spectra with no absorber and for a known "live-time" interval. Then take additional spectra with different thickness of absorber and for different types of absorber. Aluminum and lead are available. Plot $N(X, E_\gamma)$ versus absorber thickness, X, as you accumulate the data. Remember to include the statistical uncertainty in each measurement, δN, in your plot:[13]

$$\delta N = \sqrt{N}$$

Are your statistics sufficient such that $\delta N << N$? If not, accumulate spectra for longer periods of time. If spectra are accumulated for different time intervals how do you record them in one plot? Have you taken spectra for enough absorber thicknesses to measure the X-dependence of $N(X)$ at small-X where $N(X) \sim N(0)$, and also at large X where $N(X) << N(0)$. Why is this important? Now try the ^{60}Co source. Should your steps in absorber thickness be the same at different γ-ray energies and/or for different absorbing materials? Why or why not?

Do you need to correct for the NaI(Tl) efficiency, ε? Why or why not? Because you expect an exponential decrease with absorber thickness you should plot your data on semi-log paper (or use a log scale on the y-axis in your computer graphing program). Do your results agree with the exponential dependence on absorber thickness? Do your results agree with smaller values for μ at larger γ-ray energies? If the answer to either of the last two questions is no, then you may want to reconsider the *geometry* of your experimental setup. Can the absorber provide a scattering path for γ-rays not initially directed at the NaI(Tl) detector to scatter into the detector? How can you minimize this experimental problem? Once you have a reliable experimental geometry and analysis procedures, take sufficient spectra to measure μ at several energies and for at least two absorber materials. How do your results compare with tabulated values for μ?

Part 2: Measurement of the Mean Life of the Muon

Introduction

The muon[14] is an elementary particle indistinguishable from the electron except that its mass is ~ 200 times greater. Muons are produced primarily from the decays of charged pions, $\pi^{\pm}$, which are themselves produced (copiously) in extensive air showers caused by cosmic rays. Primary cosmic rays cover the spectrum from protons to intermediate mass nuclei (< iron). The primary cosmic rays interact with nuclei in the atmosphere creating large numbers of charged and neutral π mesons. These subsequently interact or decay. Depending on the energy of the initial cosmic ray, millions or billions of secondary particles can be produced. This is called an extensive air shower.

Generally the neutral mesons, π^o, decay before interacting. Depending on their energy, the charged pions may interact with nuclei in the atmosphere or may decay, $\pi^{\pm} \rightarrow \mu^{\pm} + \nu$, to charged muons and neutrinos. To understand this behavior look up the lifetimes, τ_{π}, and masses, m_{π}, of charged and neutral pions. The average distance they travel (before decaying) depends on their energy, E_{π}, and is given by:

$$Distance \sim (E_{\pi}/m_{\pi}c^2)\ c\ t$$

where c is the speed of light. What are typical distances if $E_{\pi} = 10^9$ eV or if $E_{\pi} = 10^9$ eV? If this distance is large then it is likely the π interacts before it decays.

Unlike pions, muons do not interact strongly. Thus to first order they will decay before they interact. The distance a *typical* $E_{\mu} = 10^9 - 10^{10}$ eV muon travels is thus:

$$Distance = (10^9 \sim 10^{10}\ [eV])/(105.7\ [MeV])(3 \times 10^8\ [m/s])(2.197 \times 10^{-6}\ [s]) =$$

$$6.24 \sim 62.4\ [km].$$

Where the muon mass $m_{\mu} = 105.7$ MeV. This distance is sufficiently great that many muons reach the earth surface. In fact at the earth's surface muons are the dominant component of secondary particles from cosmic ray showers. Most of the muons are of modest energy by the time they reach ground level. Thus some will range out, i.e. stop, in a tank of liquid scintillator. The study of the decay of these stopped muons is the basis of this experiment.

Muons decay via the weak interaction similar β-decay of free neutrons and nucleons in nuclei:

$$\mu^{\pm} \rightarrow \nu_{\mu} + e^{\pm} + \nu_e$$

Because neutrinos only interact via the weak nuclear force, muon decay is one of very few natural processes that only involves the weak interaction. The decay rate is actually a measure of the <u>strength</u> of the weak interaction, much like the electronic charge is a measure of the strength of the electromagnetic interaction.

As with nuclear β-decay the energy (E_e) spectrum of the resultant $e^\pm$ is that for a typical three body weak decay:[1]

$$d\Gamma(E_e)/dE_e = (G_F^2/12\pi^3) \, m_\mu^2 \, E_e^2 \, (3 - 4 \, E_e /m_\mu).$$

where $d\Gamma$ is the muon decay rate. If this is integrated over possible electron energies:

$$\Gamma = 1/\tau_\mu = G_F^2 \, m_\mu^5/192\pi^3$$

where τ_μ is the muon lifetime and G_F is the Fermi coupling constant. The Fermi coupling constant is the fundamental coupling constant of the charge changing weak interaction. Thus a measurement of the muon lifetime provides a measurement of G_F once the muon mass is known!

A fraction of the muons that reach the earth's surface have just the correct energy to stop in a block or tank of scintillator. As the muons stop they deposit $\sim$ 2 MeV/(gm/cm^2) in the scintillator. Because the density of scintillator is $\sim$1 (gm/cm^3), muons deposit $\sim$ 2 MeV/cm of path length. This is much greater than the $\sim$ 1 MeV/cm of typical γ-rays in Part 1 of this lab. Thus these stopping muons result in a pulse of light (in the scintillator) which is easily detected.

Roughly 5% of the μ^- will be captured into low Bohr orbits and then interact with the nucleus of the scintillator atoms before decaying. Thus the majority of μ^- and virtually all the stopped μ^+ decay before interacting with electrons or nuclei in the scintillator. Each muon decay results in an electron with a energy up to $m_\mu/2 \sim$ 53 MeV (i.e. neutrinos are essentially massless). These electrons also can result in a pulse of light (in the scintillator) which is also easily detected.

If one starts a clock each time a muon stops, i.e. this defines $t = 0$, then for a total of N_{stop} stopped muons the number of muons remaining at a time t later is:

$$N(t) = N_{stop} \, exp \, (-t /\tau_\mu).$$

Note: clearly this assumes that muons are not lost due to interactions with the scintillator (see comments above). Process other than weak decays that remove muons will result in a low value for τ_μ. Random accidentals will be flat in time and will result in a high value for τ_μ, unless you analyze your data properly.

The number of muon decays in the time interval from t_1 and t_2 is:

$$\Delta N(<t>) = N(t_1) - N(t_2) = N_{stop} \{exp \, (-t_1/\tau_\mu) - exp \, (-t_2/\tau_\mu)\} \sim N_{stop} \, (\Delta t/\tau_\mu) \, exp \, (-<t>/\tau_\mu).$$

where $\Delta t = t_2 - t_1$ and $<t> = (t_2 + t_1)/2$, and the approximate relation is valid when $\Delta t << \tau_\mu$. Thus a histogram of the number of the observed decays, $\Delta N(<t>)$, binned in time bins of width Δt, is predicted to be a simple exponential in $<t>/\tau_\mu$. A semi-log plot of $\Delta N(<t>)$ versus $<t>$ will have a slope $-1/\tau_\mu$.

Procedure

The muon decay experiment starts with a large tank of liquid scintillator viewed by two phototubes (PMTs). If one PMT is sufficient to trigger on cosmic ray muons and on the electrons from muon decay, why use two PMTs? The basic setup is shown schematically in Fig.

1. As depicted in Fig. 1, the difference between a through going muon and a stopped muon followed by a β-decay, is one pulse versus two pulses.
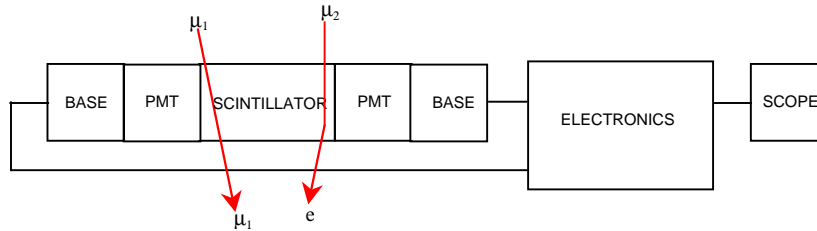

Fig. 1: Schematic setup for muon lifetime experiment. μ_1 passes through the scintillator losing some energy: A single voltage pulse appears on the scope. μ_2 stops in the scintillator and decays after time t to an electron: Two voltage pulses appear on the scope.

A sketch of a realistic experimental setup is shown in Fig. 2. The PMTs require *negative* HV and ~ -1500 V or less should provide adequate output signals. In practice you need to adjust the HV for each PMT to get approximately the same output signals. Typically the PMT output signals are discriminated with $V_{threshold}$ = 30 mV. Set the discriminator output pulse length to ~ 20 ns. Are the pulse lengths sufficient to allow for the variation in pulse timing between the 2 PMTs and still give a coincidence? Have you adjusted the relative time delay between the two scintillator signals so the signals are in time on average? Set the coincidence unit to require a 2-fold coincidence. The coincidence requires that both PMTs are above threshold to avoid noise triggers or triggers from cosmic ray muons that are clippers.

To monitor and set up your experiment pass signals of interest through the scope; i.e. put the oscilloscope between outputs of interest and the next device in the logic/signal chain.
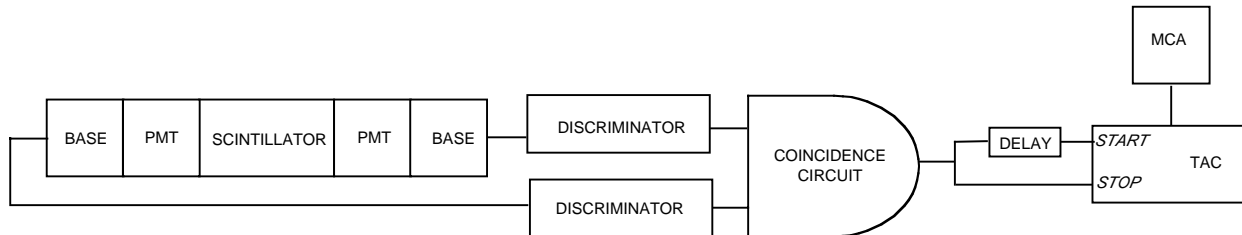

Fig. 2: A practical experimental arrangement for the muon lifetime experiment.

Two outputs are taken from the coincidence unit. One is delayed and used to START the TAC. The other is used to *STOP* the TAC. For details on how a TAC works see Appendix 2. At first this order appears to be counter intuitive. This is explained by Fig. 3 and by the fact that only when the TAC receives a good *START-STOP* combination will it produce an output pulse. To delay the signal to the TAC *START*, use the delay cable supplied with the experiment. How much delay does this cable introduce and why was this length chosen?

Thus all the events that have a *START* but no *STOP* within the TAC time window will result in no TAC output. For events with a good START-STOP combination the TAC output pulse has an amplitude proportional to the difference in time between the *START* and *STOP*. The TAC output signal is analyzed in the MCA. This is the good news. The bad news is that if a second

muon passes through the scintillator close in time to first, then the second muon is indistinguishable from a decay electron. This results in a random accidental signal that should be uniform in time and thus produce a flat background. Start to think how you will analyze the data to accommodate this background!
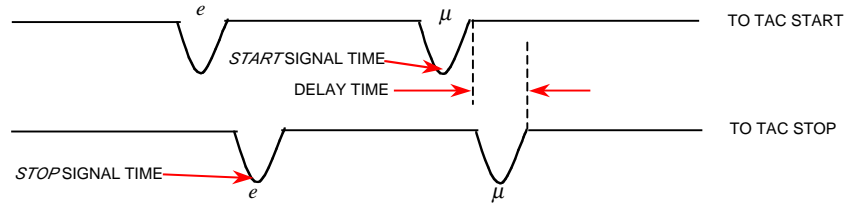

FIG. 3: Sketch of the signals entering the TAC (not to scale in time). The effect of the delay is to cut off the first part of the histogram stored in the MCA. It does not change the exponential nature of the histogram.

Set the time window on the TAC to ~ 5-10 muon lifetimes. Thus the data at large times will be essentially all accidentals. But don't set the time window too long or you will only be studying accidentals. If you have time you should accumulate and analyze data taken with different TAC time windows.

To obtain adequate statistics you will need to run for at least 24 hours. Remember to leave a big **DANGER HIGH VOLTAGE** sign on you apparatus.

The raw data from the MCA is a histogram of counts versus channel number. You need to calibrate the system.. That is, you supply a well defined time signal into the TAC/MCA combination to obtain the conversion from channel number to time. This is shown schematically in Fig. 4. Use a pulse generator followed by a discriminator (or simple splitter) to create two *in-time* signals. Put one through a precision delay, e.g. BNC model 7020 digital delay NIM module. Run with various values of delay to calibrate the full scale of the TAC/MCA. If you take data sets with different TAC time windows you will need to calibrate for each TAC setting. Remember to calibrate the TAC/MCA immediately before or after your data run, i.e. before you inadvertently change something.
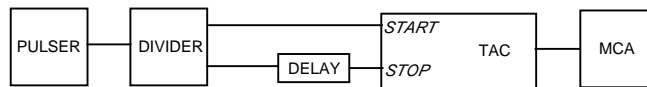

FIG. 4: Arrangement for time calibration of the TAC-MCA.

The recommended technique to analyze the data is to extract an *ASCII* file from the MCA and transfer this to a PC or workstation to manipulate and fit the data. You will need to correct for background counts. Remember if your time bins become too wide then the simple 1-exponential form is no longer correct. How does your measurement compare with the world average of $\tau_\mu = 2.197 \ \mu s$?[15] If you agree within 5-10 % you are measuring the Fermi coupling constant to that same precision!

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3. W.R. Leo, Ch. 7, *Scintillation Detectors.*

4. W.R. Leo, Ch. 8, *PMTs.*

5. W.R. Leo, Ch. 9 - Sect 9.7, *Scintillation Detector Operation.*

6. W.R. Leo, Ch. 11, *Pulse Signals.*

7. W.R. Leo, Ch. 12, *NIM Electronics.*

8. W.R. Leo, Ch. 14, *Pulse Signal Shaping and MCAs*.

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13. W.R. Leo, Ch. 4, *Statistics and Error Analysis.*

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15. Particle Data Group, Phys Rev. D **54**, 1 (1996)

APPENDIX 1: Time-to-Amplitude Converters

Bill Miller, edited by Paul Schwoebel

A Time-to-Amplitude Converter (TAC) is a device that accepts a start pulse and waits for a stop pulse. Circuitry inside the TAC determines how much time has elapsed between the two pulses. The TAC produces a voltage pulse with an amplitude that is proportional to the elapsed time.

To use the TAC:

1. Check if there are any special power requirements like ±6 V. This can usually be found on the front panel. Some units have a rear panel switch that allows for either ±12 V of ±6 V. Make sure that your NIM bin has correct voltages available.

2) Check to see what logic family the unit uses, NIM logic (a "V" looking character) or TTL (a representation of a positive going pulse). Some units can be switched between the logics.

3) Set up the TAC for the proper time scale. If you set it up for a 1 μs scale and give it a signal with 2 μs between start and stop you will not get an output from the TAC. Similarly, If you select 1 μs and deliver only 1 ns the TAC will provide no output.

4) There can be a number of extra functions on the front panel. For *COINC* and *ANTICOINC* select *ANTICOINC*. *GATE* should be *OPEN*. The SCA is not important for this experiment. Leave the ULD at 10 and the LLD at 0. Buttons or switches associated with the SCA should be set to *OFF* or *OUT*. Anything that says *DELAY* is not important. This adjusts the time between the accepted *STOP* signal and the TAC output pulse. For slow count rates, 10 μs is nothing to worry about.

APPENDIX 2: Gamma-Ray Spectroscopy
EG&G ORTEC

Gamma-Ray Spectroscopy Using NaI(Tl)

EQUIPMENT NEEDED FROM EG&G ORTEC FOR EXPERIMENTS 3.1 THROUGH 3.7, 3.9, and 3.10

Bin and Power Supply
905-3 NaI(Tl) Crystal and Phototube Assembly
266 Photomultiplier Tube Base
556 High Voltage Power Supply
113 Scintillation Preamplifier
575A Amplifier
^{137}Cs gamma source, 5 μCi $\pm$5%
SK-1G Source Kit (see Appendix)
Absorber Kit Model 3-Z2
Absorber Kit PbAl-23
M-NaI-3 Stand for Sodium Iodide Detector

ACE-2K MCA System including suitable IBM PC (other EG&G ORTEC MCAs may be used)
Oscilloscope

ADDITIONAL EQUIPMENT NEEDED FROM EG&G ORTEC FOR EXPERIMENT 3.8
427A Delay Amplifier
551 Timing Single-Channel Analyzer
426 Linear Gate
875 Counter

Purpose

The purpose of this experiment is to acquaint the student with some of the basic techniques used for measuring gamma rays. It is based on the use of a sodium iodide (NaI) detector that is thallium (Tl) activated.

Gamma Emission

Most isotopes that are used for gamma measurements also have betas in their decay schemes. The typical decay scheme for the isotope will include a beta decay to a particular level followed by gamma emission to the ground state of the final isotope. The beta particles will usually be absorbed in the surrounding material and not enter the scintillator at all. This absorption is normally assured with aluminum absorbers (ref. 10). For this experiment the betas offer no real problem, and so absorbers are not specified. There will be some beta absorption by the light shield over the phototube. The gammas, however, are quite penetrating and will pass easily through the aluminum light shield.

Generally there are two unknowns that we would like to investigate about a gamma source. One is the energies of the gammas from the source; the other is the number of gammas that leave the source per unit of time. In this experiment the student will become familiar with some of the basic NaI(Tl) measurements associated with gamma-emitting unknowns.

A total time of ~6 h is required to complete all the parts of Experiment 3 (3.1 through 3.10). The complete series can be done in two 3-h lab periods, since each is written to be fairly independent of the others.

EXPERIMENT 3.1

Energy Calibration

Setup of Equipment

Set up the electronics in the arrangement shown in Fig. 3.1. There are two parameters that ultimately determine the overall gain of the system: the high voltage that is furnished to the phototube and the gain of the linear amplifier. The gain of the photomultiplier tube is quite dependent upon its high voltage. A rule of thumb for most phototubes is that a 10% change of the high voltage will change the gain by a factor of 2. The high-voltage value depends on the phototube being
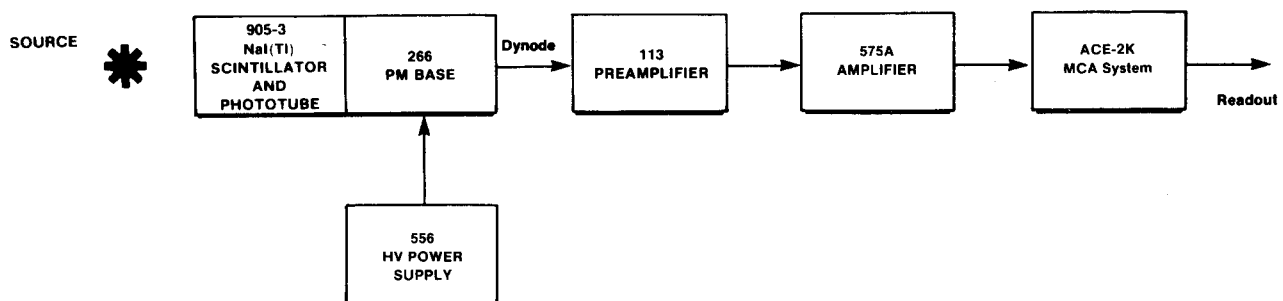

Fig. 3.1. Electronic Block Diagram for Gamma-Ray Spectroscopy System with NaI(Tl) Detector.

15

used; consult your instruction manual for the phototube and select a value in the middle of its normal operating range. (The instructor may wish to recommend a value.)

Set the indicated modules as follows:

556 High Voltage: See phototube instructions and set the level at about the middle of the acceptable operating range (normally about +1000 V).

113 Scintillation Preamplifier: Set the Input Capacity switch at 200 pF. The output pulses will be positive.

575A Amplifier: Positive input and Bipolar output. Shaping time set to 0.5 μsec. The gain will be adjusted during the experiment.

Multichannel Analyzer: PHA Analysis mode; 1000 channels are adequate for this experiment.

Procedure

1. Place the ^{137}Cs source from SK-1G ($E_\gamma = 0.662$ MeV) ~2 cm in front of the NaI(Tl) crystal.

2. Adjust the coarse and fine gain controls of the linear amplifier so that the 0.662-MeV photopeak for ^{137}Cs falls at approximately channel 280. For the illustrations shown in Figs. 3.2 and 3.3, the gain of the system has been set so that 1 MeV falls at about channel 420 to 425. Since the system is linear, 2 MeV would therefore fall at approximately channel 840 to 850.

3. Accumulate the ^{137}Cs spectrum for a time period long enough to determine the peak position. Figure 3.2 shows a typical ^{137}Cs spectrum that has been plotted. Although these spectra are usually plotted on semilog graph paper, the figures shown in this experiment are plotted on linear paper to point out some of the features of the spectra.

4. After the ^{137}Cs spectrum has been read out of the MCA, erase it and replace the ^{137}Cs source with a ^{60}Co source from SK-1G.

5. Accumulate the spectrum for a period of time long enough for the spectrum to be similar to that in Fig. 3.3.

6. Read out the MCA.

EXERCISES

a. Plot both the ^{137}Cs and ^{60}Co spectra and fill in items 1, 2, and 3 in Table 3.1.

b. From items 1, 2, and 3 in Table 3.1 make a plot of energy of the photopeaks vs channel number. Figure 3.4 shows this calibration for the data taken from Figs. 3.2 and 3.3. If other calibration sources are available, additional data points can be added to Fig. 3.4. The other entries in Table 3.1 will be filled out in Experiment 3.3.

c. Use the energy calibration feature of the MCA and compare the results with those found in Exercise **b**.

Event	Energy (MeV)	Channel No.
Photopeak	0.662	280
Compton edge	0.478	180
Backscatter	0.184	100

$$\text{Resolution} = \frac{\delta E}{E} \times 100\%$$

$$= \frac{32}{280} \times 100\%$$

$$= 11.5\%$$
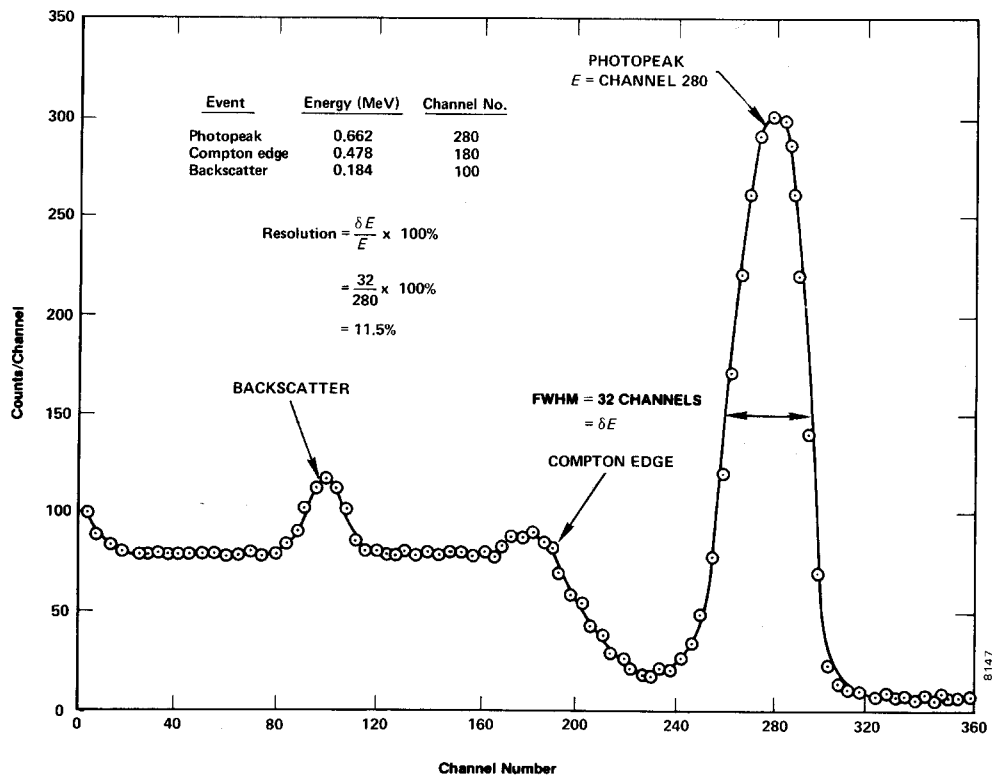
Fig. 3.2. NaI(Tl) Spectrum for ^{137}Cs.

16

Event	Energy (MeV)	Channel No.
Photopeak	1.33	560
Photopeak	1.17	496
Backscatter	~0.210	80
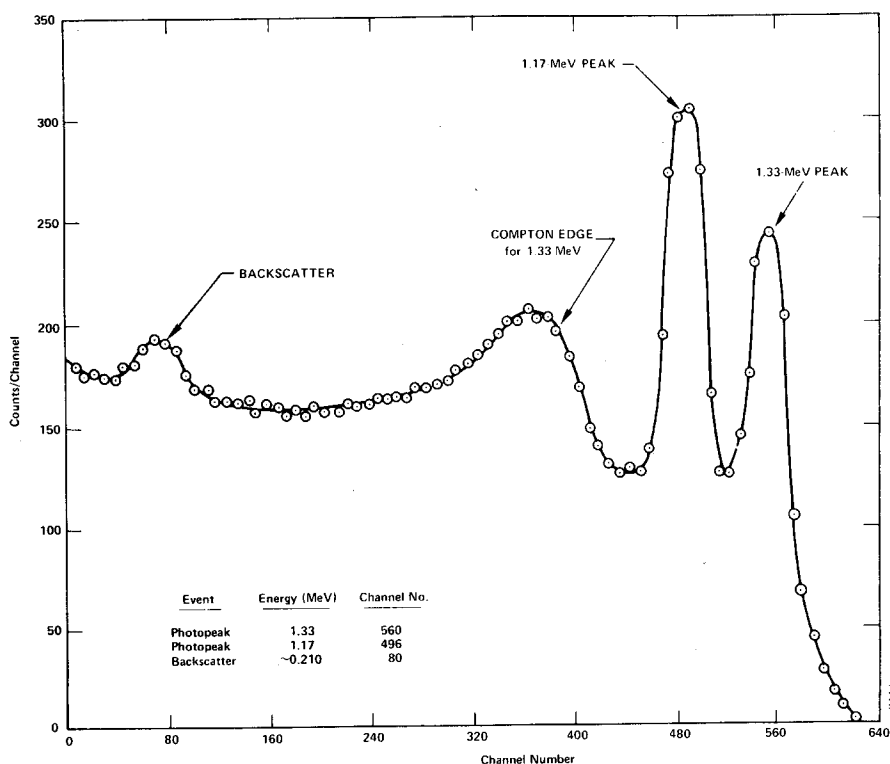
Fig. 3.3. NaI(Tl) Spectrum for ⁶⁰Co.

Table 3.1

	Event	Energy (MeV)	Channel Number
1.	0.662-MeV photopeak	0.662	
2.	1.17-MeV photopeak	1.17	
3.	1.33-MeV photopeak	1.33	
4.	Compton edge ^{137}Cs		
5.	Backscatter ^{137}Cs		
6.	Backscatter ^{60}Co		

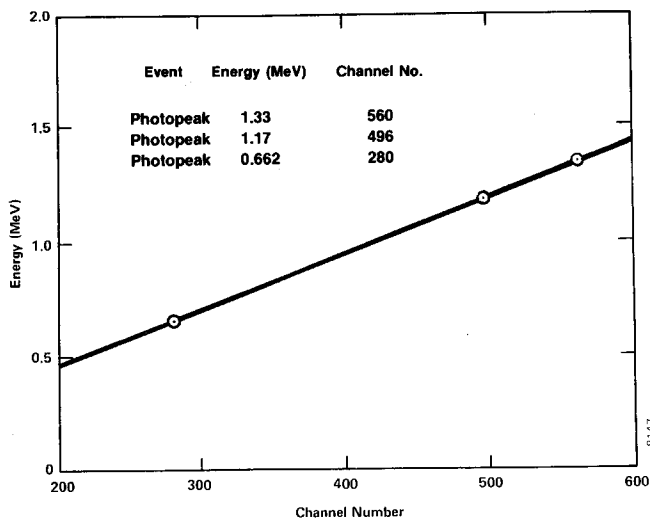| Event	Energy (MeV)	Channel No.
Photopeak	1.33	560
Photopeak	1.17	496
Photopeak	0.662	280

Fig. 3.4. Energy Calibration Curve for NaI(Tl) Detector.

EXPERIMENT 3.2

Energy Analysis of an Unknown Gamma Source

Purpose

The purpose here is to use the calibrated system of Experiment 3.1 to measure the photopeak energies of an unknown gamma emitter and to identify the unknown isotope.

Procedure

1. Erase the ⁶⁰Co spectrum from the MCA, but do not change any of the gain calibration settings of the system.

2. Obtain an unknown gamma source from the instructor. Accumulate a spectrum for the unknown source for a period of time long enough to clearly identify the photopeak(s) of the source. From the calibration curve, determine the energy for each photopeak.

———————— **EXERCISE** ————————

Use refs. 7 and 8 to identify the unknown isotope.

17

EXPERIMENT 3.3
Spectrum Analysis of ^{60}Co and ^{137}Cs

Purpose

The purpose of this experiment is to explain some of the features, other than the photopeaks, that are usually present in a pulse-height spectrum. These are the Compton edge and the backscatter peak.

The Compton interaction is a pure kinematic collision between a gamma photon and what might be termed a free electron in the NaI(Tl) crystal. By this process the incident gamma gives up only part of its energy to the electron. The amount given to the recoil electron (and the intensity of the light flash) depends on whether the collision is head-on or glancing. For a head-on collision the gamma imparts the maximum allowable energy for the Compton interaction. The energy of the scattered gamma can be determined by solving the energy and momentum equations for this billiard ball collision. The solution for these equations in terms of the scattered gamma can be written approximately as

$$E_{\gamma'} \cong \frac{E_\gamma}{1 + 2E_\gamma \, (1 - \cos\theta)} , \quad (1)$$

where
$E_{\gamma'}$ = energy of the scattered gamma in MeV,
θ = the scattering angle for γ',
E_γ = the incident gamma-ray energy in MeV.

If $\theta = 180°$ due to a head-on collision in which γ' is scattered directly back, Eq. (1) becomes

$$E_{\gamma'} \cong \frac{E_\gamma}{1 + 4E_\gamma} . \quad (2)$$

As an example, we will calculate $E_{\gamma'}$ for an incident gamma energy of 1 MeV:

$$E_{\gamma'} = \frac{1 \text{ MeV}}{1 + 4} = 0.20 \text{ MeV}. \quad (3)$$

The energy of the recoil electron, E_e, for this collision would be 0.80 MeV. This is true since

$$E_e = E_\gamma - E_{\gamma'} . \quad (4)$$

Then the position of the Compton edge, which is the maximum energy that can be imparted to an electron by the Compton interaction, can be calculated by Eq. (4).

─────────── **EXERCISES** ───────────

a. Calculate the energy of the Compton edge for the 0.662-MeV gammas from ^{137}Cs. Enter this value in Table 3.1. From your plot and calibration curve, does this calculation agree with your measured value?

b. Backscatter occurs when gammas make Compton inter-

actions in the material that surrounds the detector. Figure 3.5 was taken from ref. 10 and is a good illustration of the various events that can take place in a typical source–NaI(Tl) detector–lead shield arrangement. Backscattered gammas from these interactions $(E_{\gamma'})$ make photoelectric interactions in the NaI(Tl) when they enter the crystal. The energy of the backscattered peak can be found by solving Eq. (2).

Solve Eq. (2) for the background gammas from ^{137}Cs and for the 1.33-MeV gammas from ^{60}Co. Fill in the rest of Table 3.1. How do your measured energies compare with the theoretical energies from Eq. (2)? If the backscatter peak is not very pronounced in your spectrum, it can be improved by accumulating a spectrum with a sheet of lead absorber placed slightly to the left of the source in Fig. 3.1.

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EXPERIMENT 3.4
Energy Resolution

Purpose

The resolution of a spectrometer is a measure of its ability to resolve two peaks that are fairly close together in energy. Figure 3.2 shows the gamma spectrum that was plotted for the ^{137}Cs source. The resolution of the photopeak is found by solving the following equation:

$$R = \frac{\delta E}{E} \times 100, \quad (5)$$

where
    R = the resolution in percent,
    δE = the full width of the peak at half of the maximum count level (FWHM) measured in number of channels,
    E = the channel number at the centroid of the photopeak.

In Fig. 3.2 the photopeak is in channel 280 and its FWHM = 32 channels. From Eq. (5) the resolution is calculated to be 11.5%.

─────────── **EXERCISE** ───────────

Calculate the resolution of the system from your ^{137}Cs spectrum. Record this value for later reference.

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EXPERIMENT 3.5
Activity of a Gamma Emitter (Relative Method)

Purpose

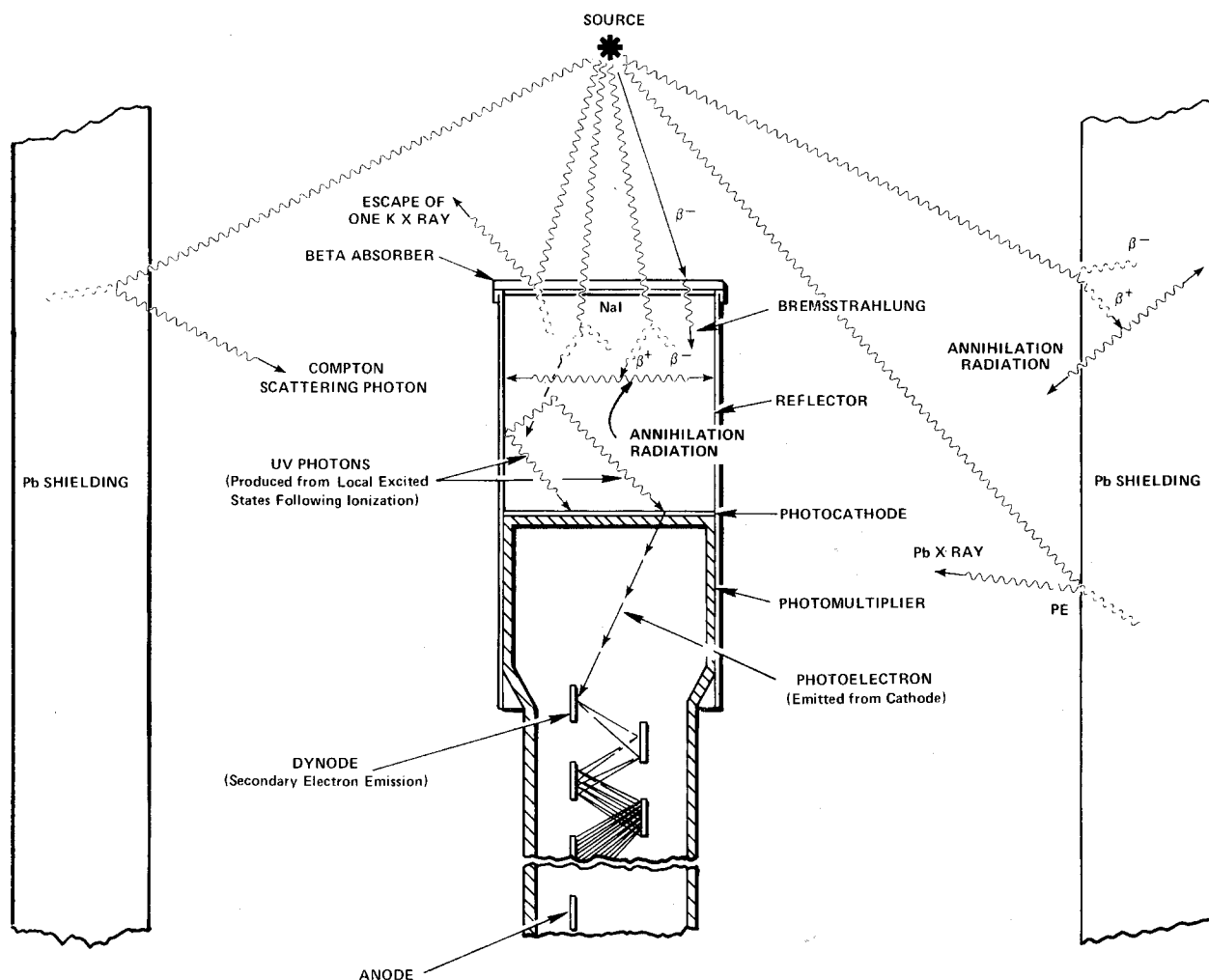In Experiments 3.1 and 3.3, procedures were given for determining the energy of an unknown gamma source. Another

ESCAPE OF
ONE K X RAY

BETA ABSORBER

NaI

BREMSSTRAHLUNG

COMPTON
SCATTERING PHOTON

ANNIHILATION
RADIATION

REFLECTOR

ANNIHILATION
RADIATION

UV PHOTONS
(Produced from Local Excited
States Following Ionization)

PHOTOCATHODE

Pb SHIELDING

Pb X RAY

PHOTOMULTIPLIER

Pb SHIELDING

PHOTOELECTRON
(Emitted from Cathode)

DYNODE
(Secondary Electron Emission)

ANODE

**Fig. 3.5. Various Events in the Vicinity of a Typical Source–Crystal
Detector–Shield Configuration.**

unknown associated with the gamma source is the activity of the source, which is usually measured in curies (Ci); 1 Ci = 3.7 x 10^{10} disintegrations/s. Most of the sources that are used in nuclear laboratory experiments have activities of the order of microcuries (μCi). The purpose of this experiment is to outline one procedure by which the activity of a source can be determined, called the relative method.

In using the relative method, it is assumed that the unknown source has already been identified from its gamma energies. For this example, assume that the source has been found to be ^{137}Cs. Then all that is necessary is to compare the activity of the unknown source to the activity of a standard ^{137}Cs source that will be supplied by the laboratory instructor. For convenience, call the standard source S1 and the unknown source U1.

Procedure

1. Place the S1 source about 4 cm from the face of the detector (or closer if necessary to get reasonable statistics)

and accumulate a spectrum for a period of live time, selectable on the analyzer, long enough to produce a spectrum similar to Fig. 3.2.

2. Use the cursor to determine the sum under the photopeak. In the example shown in Fig. 3.2, this would correspond to adding up all counts in channels 240 through 320. Define this sum to be Σ_{s1}.

3. Erase the MCA spectrum. Remove source S1 and replace it with source U1, positioned **exactly** the same distance from the crystal as the S1 source was. Accumulate a spectrum for the same period of live time that was used in step **1**. Sum the peak as in step **2**.

4. Erase the spectrum from the MCA. Remove the U1 source and accumulate background counts for the same period of live time that was used in steps **1** and **3** above.

5. Sum the background counts in the same channels that were used for the photopeaks in steps **2** and **3** above. Call this sum Σ_b.

19

—————————— EXERCISE ——————————

Solve for the activity of the U1 by using the following ratio:

$$\frac{\text{activity of U1}}{\text{activity of S1}} = \frac{\Sigma_{U1} - \Sigma_b}{\Sigma_{S1} - \Sigma_b} \qquad (6)$$

Since the efficiency of the detector is only energy dependent, the standard and unknown sources do not have to be the same isotope. It is only necessary that their gamma energies be approximately the same ($\pm10\%$) in order to get a fairly good estimate of the absolute gamma activity of the unknown.

EXPERIMENT 3.6
Activity of a Gamma Emitter (Absolute Method)

Purpose
The activity of the standard used in Experiment 3.5 can be determined by the absolute method. The purpose of this experiment is to outline the procedure for this method. Here the source that is to be measured will be called U1.

Procedure
1. Place the U1 source 9.3 cm away from the face of the detector.

2. Accumulate a spectrum and note the live time that is used.

3. Use the cursor to determine the sum under the photopeak, Σ_{U1}. Then erase the spectrum, remove the source, and accumulate background for the same live time and calculate Σ_b.

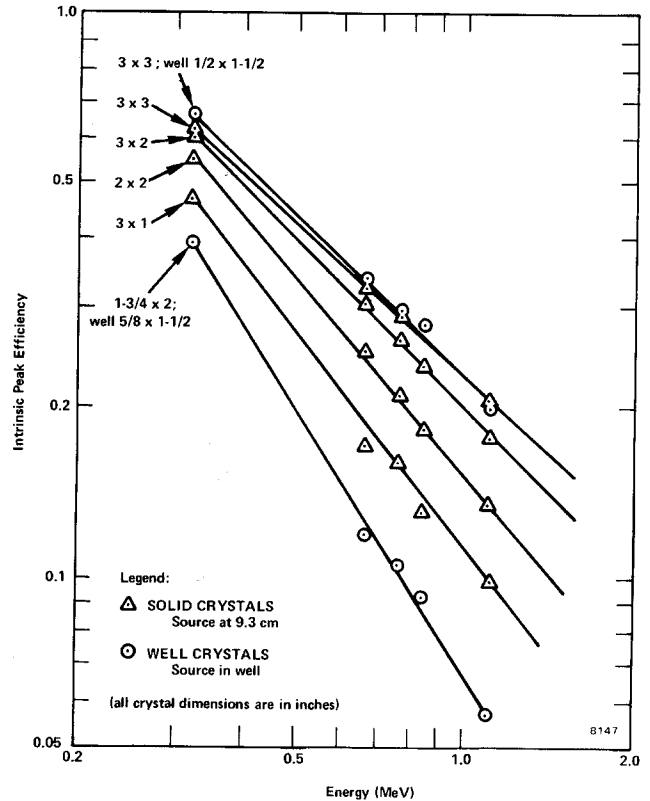**4.** Use the following formula to calculate the activity of U1:

$$\text{activity of U1} = \left(\frac{\Sigma_{U1} - \Sigma_b}{t}\right) \frac{1}{G\epsilon_p f}, \qquad (7)$$

where

t = live time in seconds,

ϵ_p = intrinsic peak efficiency for the gamma energy and detector size used (Fig. 3.6 and ref. 10),

f = the decay fraction of the unknown activity which is the fraction of the total disintegrations in which the measured gamma is emitted (refs. 7 and 8 and Table 3.2),

G = area of detector $(cm^2)/4\pi s^2$,

s = source-to-detector distance in cm.

Fig. 3.6. Intrinsic Peak Efficiency of Various NaI(Tl) Crystals vs Gamma Energy.

Table 3.2. Gamma Decay Fraction, (f), for Some Common Isotopes.

Isotope	Gamma Energy (MeV)	f
^{137}Cs	0.662	0.92
^{51}Cr	0.323	0.09
^{60}Co	1.17	0.99
^{60}Co	1.33	0.99
^{22}Na	1.276	0.99
^{22}Na	0.511	0.99
^{54}Mn	0.842	1.00
^{65}Zn	1.14	0.44

EXPERIMENT 3.7
Mass Absorption Coefficient

Purpose
The purpose of the experiment is to measure experimentally the mass absorption coefficient in lead for 662-keV gamma rays.

References 2, 3, and 5 point out that gammas interact in matter primarily by photoelectric, Compton, or pair-production interactions. The total-mass absorption coefficient can be measured easily with a gamma-ray spectrometer. In this experiment we will measure the number of gammas that are removed from the photopeak by photoelectric or Compton

interactions that occur in a lead absorber placed between the source and the phototube.

From Lambert's law (ref. 1) the decrease of intensity of radiation as it passes through an absorber is given by

$$I = I_0 e^{-\mu x}, \qquad (8)$$

where

I = intensity after the absorber,

I_0 = intensity before the absorber,

μ = total-mass absorption coefficient in cm^2/g,

x = density thickness in g/cm^2.

The density thickness is the product of the density in g/cm^3 times the thickness in cm.

The half-value layer (HVL) is defined as the density thickness of the absorbing material that will reduce the original intensity by one-half. From Eq. (8):

$$\ln I/I_0 = -\mu x. \qquad (9)$$

If $I/I_0 = 0.5$ and $x =$ HVL, ln $0.5 = -\mu$(HVL) and hence

$$HVL = \frac{0.693}{\mu} \qquad (10)$$

In this experiment we will measure μ in lead for the 0.662-MeV gammas from ^{137}Cs. The accepted value is 0.105 cm^2/g. Values for other materials can be found in ref. 8.

Procedure

1. Place the ^{137}Cs source about 5.0 cm from the NaI(Tl) detector and accumulate the spectrum long enough for the sum under the 0.662-MeV peak ($\Sigma_{Cs} - \Sigma_b$) to be at least 6000 counts. Determine ($\Sigma_{Cs} - \Sigma_b$).

2. Erase the MCA and insert a piece of lead from the absorber kit between the source and the detector. Accumulate the spectrum for the same period of live time as in step **1** above. Determine ($\Sigma_{Cs} - \Sigma_b$).

3. Erase the MCA and insert another piece of lead. Determine ($\Sigma_{Cs} - \Sigma_b$). Repeat with additional thicknesses of lead until the count-sum is >1000. Fill in the data in Table 3.3.

Table 3.3. Data for Mass Absorption Coefficient.

Absorber	Absorber Thickness (mg/cm^2)	$\Sigma_{Cs} - \Sigma_b$
1	0	
2		
3		
4		
5		
6		
7		

a. Using semilog graph paper, plot I vs absorber thickness in mg/cm^2, where I = ($\Sigma_{Cs} - \Sigma_b$)/live time. Determine the HVL from this curve and calculate μ from Eq. (10). How does your value compare with the accepted value of 0.105 cm^2/g?

b. Repeat the above experiment for the aluminum absorbers in the Absorber Kit. The μ for aluminum is 0.074 cm^2/g.

EXPERIMENT 3.8

The Linear Gate in Gamma-Ray Spectroscopy

Purpose

The purpose of this experiment is to show how a linear gate can be used with an MCA in gamma-ray spectroscopy. The linear gate will limit the analysis of input pulse amplitudes to those that will be included within the photopeak.

The measurement of the mass absorption coefficient in Experiment 3.7 required the accumulation of several complete spectra, although the data of interest were included within only a fraction of the total number of channels that were used. The normal time for completing Experiment 3.7 is approximately 45 min. By using a linear gate, the same information can be obtained in about 1/3 of the time. An equivalent saving of time can also be made in Experiments 3.5 and 3.6 (Source Activity Determinations). Since the procedures are about the same as for Experiment 3.7, the student should repeat these experiments with the linear gate to see how much time will be saved.

See equipment list at beginning of Experiment 3 for additional equipment required for Experiment 3.8.

Connect the system components as shown in Fig. 3.7. Connect the bipolar output of the 575A Amplifier to both the 427A Delay and the 551 Timing Single-Channel Analyzer. Connect the Delay output to the linear Input of the 426 Linear Gate and connect the gate output to the analyzer input. Connect the SCA output to both the 875 Counter input and the Enable input of the Linear Gate.

The Linear Gate is a module that permits linear pulses to be passed only during the time interval that follows each Enable input. In normal operation the adjusted time interval will allow only one linear pulse to be furnished into the MCA.

The Timing Single-Channel Analyzer determines whether each input pulse amplitude is within the window and generates a logic output pulse for each input pulse that satisfies the criteria. By adjusting the lower and upper levels of its window, the 551 then can determine what portion of the spectrum is gated through for analysis in the MCA. This is true since it delivers the enable logic pulse to open the linear gate.

From the standpoint of timing, one would like to have the logic pulse arrive at the enable input of the linear gate just prior to the arrival of the corresponding linear pulse that is to be gated. Since the amplifier provides a bipolar output to the SCA, and since the SCA generates an output at 50% of full amplitude on the trailing edge of the positive lobe, the SCA output will occur at about 2 μs after the onset of the pulse. Thus, if the 427A Delay is set for 3 μs and the 426 Linear Gate width is adjusted to maximum, 4 μs, the gate passes the input pulse for a period from 1 μs before the delayed pulse reaches the 426 until 3 μs of elapsed pulse time. This passes the positive portion of the bipolar pulse, which is all that affects the MCA measurement; the negative portion of the bipolar pulse is not used.

The inclusion of a counter in Fig. 3.7 permits a direct total of the counts to be observed, and the adjustment of the window width will limit these to the peak area. This simplifies the summing of counts for peak area integrations.

Figure 3.8 shows how Fig. 3.2 might look if the window of the SCA were set properly to just span the ^{137}Cs photopeak. Since the MCA has a live display while it is accumulating, it is quite simple to adjust the window of the SCA properly.

The single-channel principle and the control of the linear gate are examined here with individual modules. Both functions are also included in the MCA, so the separate modules are not required for other experimental applications.

Module Settings:
Use the same settings for the high-voltage power supply, preamplifier, and amplifier that were used for Experiment 3.1. Set the 426 Linear Gate for Normal with its Gate Width control fully clockwise for 4 μs. Set the 875 Counter for count and use the Positive input from one of the Pos Out connectors on the 551 Timing SCA; reset the 875 Counter to zero. Set the 551 Timing SCA for Normal operation, the Lower-Level control at 030, the Upper-Level control fully clockwise
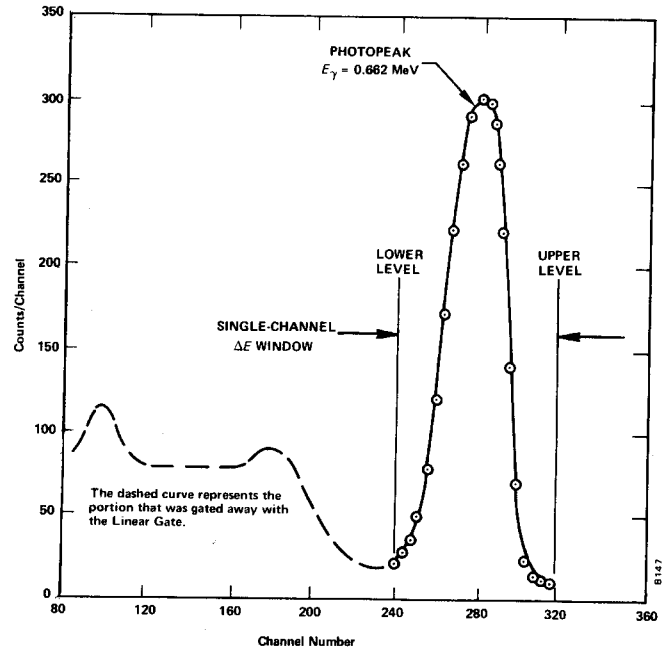

Fig. 3.8. ^{137}Cs **Spectrum with the Linear Gate.**

at 1000 divisions, and Delay at minimum for 0.1 μs. Set the 427A Delay Amplifier for a 3-μs delay.

Procedure
1. Place the ^{137}Cs source from SK-1G about 4 or 5 cm from the crystal face. Accumulate a spectrum in the MCA while adjusting the E and the ΔE window on the 551 Timing SCA. Set the window so that it just brackets the photopeak as in Fig. 3.8. You are now ready to make the first measurements.

2. Clear the MCA and reset the counter to zero. Start both at the same time and accumulate for a period of time long enough to obtain about 6000 counts in the counter. Record
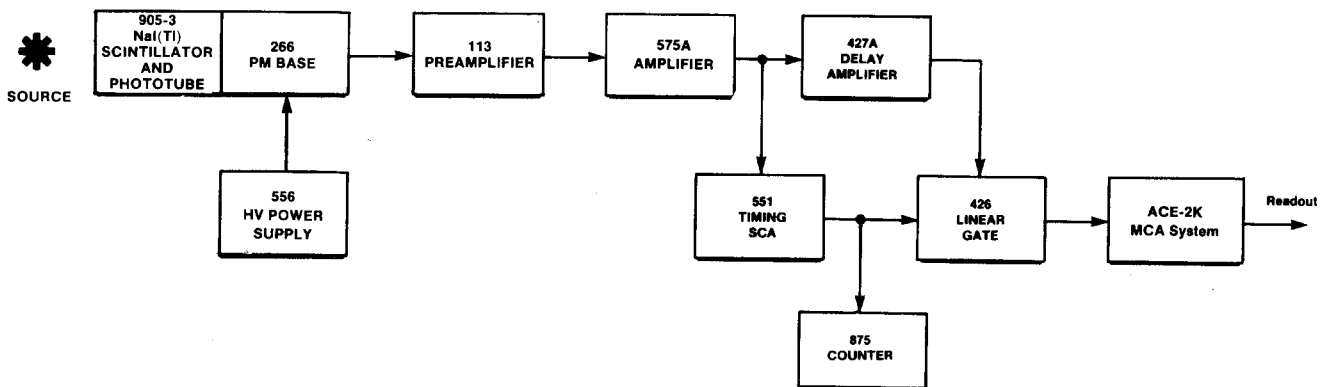

Fig. 3.7. Block Diagram of Electronics for Gamma-Ray Spectrometry System with a Linear Gate.

the total elapsed time for the measurement, the average dead time from the MCA, and the count in the counter. Read out the analyzer and then clear both the MCA and the counter.

3. Place the first lead absorber between the source and the detector as in Experiment 3.7 and accumulate for the same period of time that was used in step **2** above. Record only the counter counts and the total elapsed time. It is not necessary to read out the MCA for each spectrum since the counter is summing the counts under the photopeak. You should observe the MCA for each spectrum to make sure that the proper spectrum is being stored.

4. Repeat step **3** for each added absorber thickness that was used in Experiment 3.7. Make a background run with the source removed, and fill in Table 3.4 as in Experiment 3.7.

Table 3.4

Absorber	Absorber Thickness (mg/cm^2)	$\Sigma_{Cs} - \Sigma_b$
1	0	
2		
3		
4		
5		
6		
7		

5. Calculate the same data as in Experiment 3.7, Exercises **a** and **b**.

6. In step **2** the output of the MCA was read. Sum this output spectrum and compare it with the counter sum that was taken from the same run. The counter sum should be slightly larger since it does not suffer from dead-time corrections at these counting rates. The MCA does suffer, because it requires some amount of time to measure and store each pulse and thus does not actually analyze as many pulses as have been furnished to it. The MCA sum should be equal to the counter count times the percent of live time, which is equal to the live time of the MCA divided by the clock time for the spectrum accumulation.

EXPERIMENT 3.9
Sum Peak Analysis

Figure 3.3 shows the two pronounced peaks in ^{60}Co. Figure 3.9 shows the decay scheme of ^{60}Co.

Most of the time the decay occurs by β emission to the 2.507-MeV excited state of ^{60}Ni. Subsequent decay to the ground state always occurs by gamma emission to the 1.3325-MeV level (a 1.174-MeV gamma) followed almost simultaneously by the 1.3325-MeV gamma to the ground state. In Experi-

ment 19 we will show that these two events are in coincidence and have an angular correlation that deviates from an isotropic distribution by only 16%. For the purposes of this experiment we can assume that each of these gammas are isotropically distributed. In other words, if γ_1 goes in a particular direction, γ_2 can go in any of the 4π steradians that it wishes. There is a certain probability that it will go in the same direction as γ_1. If this occurs within the resolving time of the detector, γ_1 and γ_2 will be summed and hence a sum peak will show up in the spectrum. From the definitions in Experiment 3.6, the number of counts, Σ_1, under the γ_1 peak is given by:

$$\Sigma_1 = \epsilon_1 G f_1 t A, \tag{11}$$

where A is the activity of the sample and t is the time. In a similar calculation, the sum Σ_2 for γ_2 is given by:

$$\Sigma_2 = \epsilon_1 G f_2 t A. \tag{12}$$

From Eqs. (11) and (12) the number of counts in the sum peak, Σ_s is given by:

$$\Sigma_s = \epsilon_1 \epsilon_2 f_1 f_2 G^2 A t \,[W(0°)], \tag{13}$$

where $W(0°)$ is a term that accounts for the angular correlation function. For the case of ^{60}Co, Eq. (13) is quite simple. Σ_s becomes:

$$(\Sigma_s)_{^{60}Co} \cong \epsilon_1 \epsilon_2 G^2 A t, \tag{14}$$

since $W(0°) \cong 1.0$.

In this experiment we will show that the sum peak for ^{60}Co has an energy of 2.507 MeV and that its sum is given by Eq. (14).
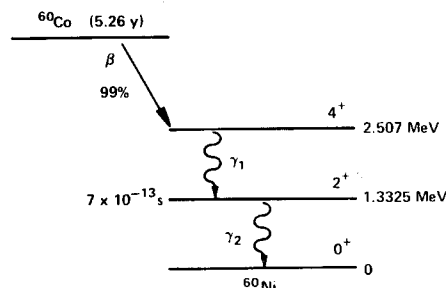

Fig. 3.9. The Decay Scheme of ^{60}Co.

Procedure
1. Set up the electronics as shown in Fig. 3.1.

2. Use the gammas from the source kit to calibrate the MCA so that full scale is ~3.0 MeV. For 1024 channels this would put the ^{137}Cs (0.662 MeV) peak at approximately channel 225.

3. Construct a calibration curve as in Experiment 3.1.

4. Place the ^{60}Co source from the source kit at exactly 9.3 cm from the face of the detector. Count for a period of time long enough so that the area under the sum peak is ~1000 counts. This procedure was outlined in Experiment 3.6.

a. Verify that the energy of the sum peak is 2.507 MeV. Subtract the background from the sum peak and verify its sum from Eq. (14).

b. Repeat this sum peak analysis for the ^{22}Na source. Figure 3.10 shows the decay scheme for ^{22}Na and a typical spectrum with the sum peak.
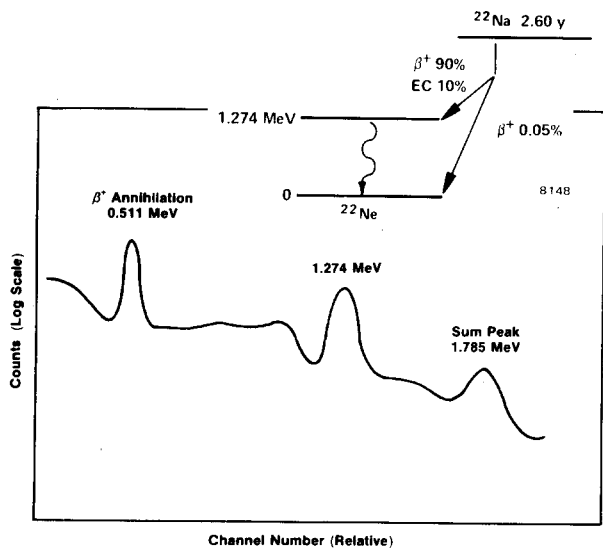

Fig. 3.10. Sum Peak for the ^{22}Na Source from Source Kit SK-1G.

EXPERIMENT 3.10
Photoelectric Absorption

Purpose
The purpose of this experiment is to study the photoelectric absorption of photons and verify the strong dependence of this process on the atomic number of the absorbing material.

When a gamma of energy <150 keV interacts with matter, the interaction has a high probability of being photoelectric. In the photoelectric interaction, the photon interacts with one of the tightly bound electrons in the material. The electron, in general, is knocked out of the atom with an energy given by:

$$E_e = hf - E_b,$$

where f is the frequency of the photon and E_b is the binding energy of the electron that is involved in the interaction. The probability of photoelectric interaction is dependent on the atomic number of the absorbing material and the energy of the gamma or x-ray photon. Although it is difficult to write out an exact analytic expression for this probability, it can be shown that for low energy photons

$$\mu = \frac{K \cdot Z^n}{E_\gamma^3}, \tag{15}$$

where K is a constant, Z is the atomic number, and n is usually between 4 and 5.

Procedure
The set up for this experiment is the same as for Experiment 3.7.

1. Place the ^{57}CO source ~3.8 cm from the NaI detector. Accumulate for a time period long enough to get reasonable statistics in the 122-keV line. As in Experiment 3.7, $\Sigma - \Sigma_b$ should be at least 6000 counts.

2. Clear the MCA and place the thinnest aluminum absorber between the source and the detector. Count for the same period of time as in step **1**. Repeat for the other two aluminum absorbers.

3. Repeat steps **1** and **2** for the other thin absorbers, Fe, Cu, Mo, Sn, Ta, and Pb, in the Model 3-Z2 source kit. Note: The counting time might have to be increased as the atomic number of the absorber is increased.

a. For the three measurements made with the thin aluminum foils, calculate and average μ, Eq. (9). Repeat for the other absorbers.

b. Make a plot of μ vs $Z^{4.5}/E_\gamma^3$ from your experimental data. How do your results compare to the theory?

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LAB 3: Speed Measurement by Optical Techniques

J.-C. Diels and W. Rudolph

Purpose:

Familiarize the student with the Doppler effect as used for speed measurement, spatial filtering, spectrum analyzers, and interferometric techniques.

Reading Assignment

As referenced in text, and Chapter 4 of *Building Scientific Apparatus*, 3rd edition, by John Moore et al. (Perseus Books, Cambridge MA, 2003).

Introduction

The most obvious optical method for measuring the speed of a moving object is to use the Doppler effect: light reflected from the moving object will be shifted in frequency in proportion to the line of sight velocity of the object. While we are familiar with examples in the radio domain (such as police traffic radar and Doppler weather radar) and with acoustic waves (the apparent change in pitch of a passing siren), demonstrating the Doppler effect optically is not a trivial matter. The primary reasons for this are:

1. Optical frequencies are much too high (~ 10^{14} Hz) to be measured directly, and

2. The shift corresponding to typical speeds is on the order of 0.1 - 10 MHz, much less than the line width of a typical He-Ne laser.

It is nonetheless possible to demonstrate the Doppler effect interferometrically using simple equipment. In this experiment, you will use a small He-Ne laser, an electronic spectrum analyzer, a photodiode detector, and some simple optical elements to measure the shift in frequency of light scattered from the back of a small moving train, a rotating disk, and from particles suspended in flowing water. From this information you will be able to determine the velocity profile of the water flowing through a tube, the speed of a train, the speed of a disk, and (after directly measuring the disk speed) the wavelength of the laser.

One disadvantage of the Doppler effect to measure speed is that you can only determine the velocity component along the line of sight. It is possible to measure the transverse speed optically in spite of this handicap, again interferometrically. The trick is to project first an interference pattern onto the object, then measure the frequency of the light fluctuations as it passes through the fringes. The fringe spacing is determined from the geometry of the experimental setup, and the speed is then given by the product of this spacing and the characteristic frequency of the scattered light fluctuations. For this part of the experiment a rotating disk will again be used, although a much faster rotation rate is required for the effect to be readily observable.

Theory and Concepts

At this point the question arises "How do we get around the problems mentioned in the introduction?" The answer is a technique known as *heterodyne interferometry*. This method involves combining the frequency-shifted or modulated optical (or radio, microwave, etc.) signal with an unshifted reference signal. The so-called "square-law" detector (which includes all common optical detectors) will respond only to the intensity (and intensity variation) of the combined beam. If the frequencies of the two signals differ by less than the bandwidth of the detector and are not identical, the electrical output of the detector will be modulated at a frequency equal to the difference between the frequencies of the two beams. For simplicity let us assume two electromagnetic waves of equal amplitude A and frequencies ω_1 and ω_2. If the two waves are incident on a detector we measure a signal

$$S = \langle (A cos\omega_1 t + A cos\omega_2 t)^2 \rangle \qquad (2.1)$$

where $\langle \rangle$ denotes averaging performed by the detector and subsequent electronics. Using the identity $2 cos\omega_1 t \, cos\omega_2 t) = cos(\omega_1 - \omega_2)t + cos(\omega_1 + \omega_2)t$ we can write the signal as

$$S = A^2 \langle cos^2\omega_1 t \rangle + A^2 \langle cos^2\omega_2 t \rangle + A^2 \langle cos(\omega_1 - \omega_2)t \rangle + A^2 \langle cos(\omega_1 + \omega_2)t \rangle \quad (2.2)$$

Typically, in particular if we work with light, the frequencies $\omega_{1,2}$ cannot be resolved directly and the detection system averages the terms 1,2, and 4 in Eq. (2.2). The time dependence of the signal (the 3rd term in (2.2)) is then characterized by a frequency

$$\omega_{beat} = | \omega_1 - \omega_2 |$$

This modulation is also referred to as a *beat frequency* or a *beat note* after the analogous acoustic phenomenon.[†] When two nearly identical tones are played together, the result is a `waa-waa-waa' variation of the loudness with a periodicity equal to the inverse of the frequency difference of the two tones.

So the heterodyne technique eliminates problem 1 above, as long as the Doppler shift is not too large. But what of problem 2? The stability of a small He-Ne laser, over a time span of a few seconds, is certainly no better than a several MHz. Thus you should not be able to measure speeds < 2 m/s ~ 10 km/hr, right? Wrong, as can be seen by comparing the coherence time of such a laser (tens of ns), the time delay between the signal and reference beams for the experimental arrangements used here (< 1 ns), and the Schawlow-Townes linewidth of a laser (~ Hertz). Qualitatively, what **would** be the limit of such a measurement? (Note: It is worth devoting a few lines to this in your report.)

The theory of the Doppler effect is covered in most basic physics texts, and will not be discussed here. The arrangement in the first part of this experiment is identical to a Michelson interferometer, and the expected frequency shift of the light can be derived by considering the object to be equivalent to the translating mirror. From this point of view, the beat note signal is just the rate at which fringes pass the detector. In the second (fast wheel) part, the measured frequency is the quotient of the transverse speed of the wheel and the fringe spacing, which is determined by the wavelength of the light used and the angle of intersection of the two beams. For the third (water flow) part of the experiment, the angle of intersection of the

beams and the refractive index of water must be taken into consideration. The necessary derivations are left to the student and a summary thereof should be included in the lab write-up.

All three parts of this experiment involve measuring the light scattered from a large number of randomly placed scattering centers. Given that the characteristic frequency of interest is the same for all of the scatterers, the signal from each will be randomly phased so one might expect any modulation at that frequency to average out. Furthermore, since the collected light is due to scattering, the total intensity can vary wildly at frequencies unrelated to the one in which we are interested. Considering such a small signal to noise ratio, how can we expect to measure anything? (Hint: The net signal from an ensemble of randomly phased sources is a problem analogous to the drunkard's walk. Also, the beauty of using spectrum analysis is that it measures the total amplitude of signals over only a small range of frequencies. Thus the appropriate figure of merit is the ratio S/N where S is the signal amplitude at a frequency of interest and N is the amplitude of the noises whose frequencies are within one resolution bandwidth of the signal .

Experimental Procedures

This experiment is divided into three parts: a) Doppler shift measurement of the speed of a train and a rotating wheel; b) Transverse velocity measurement of the speed of a different wheel using scattering from a fringe pattern; c) Doppler shift measurement of the velocity profile of water flowing through a tube.
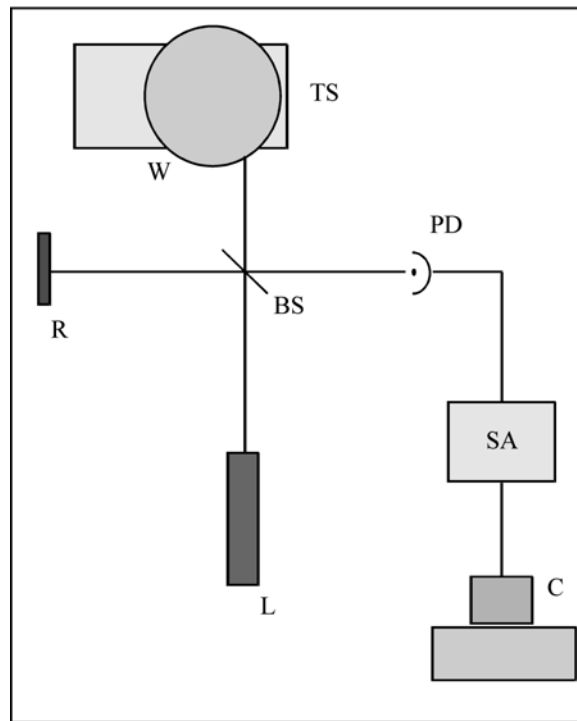

Figure 1. Experimental setup for the slow wheel Doppler Shift measurements. *L* is the laser source, *BS* is a beam splitter, *W* is a motor driven wheel with reflective tape on its rim, *R* is a reflector (also tape covered), and *PD* is the optical detector. The electrical signal is sent to the 3585A spectrum analyzer *SA* for analysis and then to the computer *C* for recording. The translation stage *TS* is provided for positioning the wheel.

Doppler shift from a train and a rotating wheel. The experimental setup for the wheel is sketched in Fig. 1. Note the equivalence of this arrangement to a Michelson interferometer. The beam from laser *L* is split by beam splitter *BS* and directed onto the edge of the slowly rotating wheel *W* and reference reflector *R*, both of which are covered with retroreflecting tape (of the type used by runners, bicyclists, etc., to avoid getting hit by cars). This tape is composed of small beads embedded in a substrate with a matched refractive index such that light entering the bead is refracted to the back of the bead where it is internally reflected back toward the source. This eliminates the need for the precise alignment required when flat mirrors are used. The retroreflected beams are then recombined at the beamsplitter and continue to detector *PD*. The electrical signal from *PD* is sent to a spectrum analyzer *SA* for measurement.

A spectrum analyzer (HP 3585A) is used to measure the beat frequency may be measured directly. You can transfer the spectra to a computer - see Appendix for instructions. First, calculate the expected Doppler shift from an estimate of the wheel rotation rate and set the analyzer accordingly. Before connecting the detector signal to the spectrum analyzer, look at it with an oscilloscope; what is the amplitude of the signal, and does its period correspond to your calculated estimate? Measurements are to be made over a range of wheel positions at roughly one centimeter intervals, for which translation stage *TS* is provided. The range chosen should be such that one edge is accessible. Note: You must be sure that you are looking at the right peak on the spectrum analyzer trace. Determine this by blocking either the reference or shifted beams (the signal will then disappear) and by translating the wheel (the signal frequency should then shift). Once the data are taken, you will also need the diameter of the wheel, a direct measurement of its rotation rate, and the wavelength of the laser. From these you should be able to determine the rotation rate of the wheel and to estimate the precision of your measurements. Discuss the precision of your results and the important error sources.

Transverse speed measurement. As pointed out earlier, the Doppler technique is applicable only for measuring longitudinal velocities. (There is in fact a transverse Doppler effect, but it goes as v^2/c^2 and so is observable only for relativistic particles.) However, it is possible to measure the transverse velocity of an object using light scattering. The setup is as shown in Fig. 2. The laser beam is split and reflected by *BS* and mirror *M* onto the edge of the fast rotating wheel *W*, intersecting at its surface at an angle a to form a fringe pattern of spacing δL. This spacing should be too small to be visible with the unaided eye but much greater than the wavelength of light (i.e., between 5 and 50 µm). As it is determined by the beam crossing angle α, you must position *BS* and *M* appropriately. The most accurate way to find this angle is to measure the separation of the beams some distance from their intersection; take several such measurements at different positions to obtain an accurate value for α.

The scattered light may be viewed from the side, as shown in Fig. 2. A lens *L* is used to collect the scattered light and focus it onto the detector *PD*. As before, the detector signal is sent to the spectrum analyzer. The signal frequency of interest may be discerned by blocking either of the interfering beams - it should disappear. Position the wheel transversely to maximize the scattering frequency (i.e., so that the scattering comes from the foremost point on the wheel). Adjust the spectrum analyzer to fully resolve the signal frequency and measure its value. Also measure the diameter of the wheel and estimate its rotation rate observing the detector signal on an oscilloscope - though it may be quite noisy, it should be periodic over one rotation.
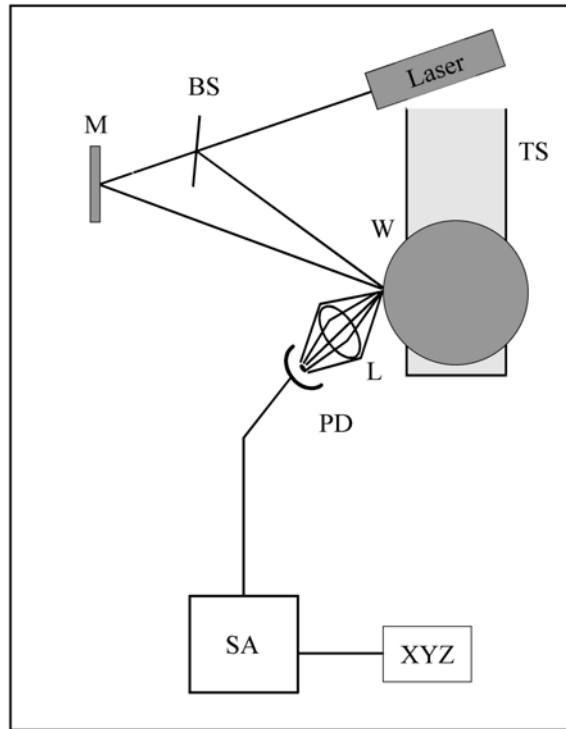
Figure 2. Experimental setup for the transverse speed measurements. Most labeled parts are as in Fig. 1, except that *L* is a collecting lens (optional) *M* is a flat mirror, and *XYZ* is a PC to which the spectra can be transferred.}

Report the measured frequency and its bandwidth. Calculate α and δL, and the rotation rate. How does this compare with the oscilloscope estimate? Which would you expect to be more accurate and why? What are the major sources of error in this experiment, and in particular what is the effect of the bandwidth of the measured scattering frequency?

Velocity profile of flowing water. This is probably the most difficult part of the experiment. It is diagrammed in Fig. 3. A small portion of the laser beam is split off by the microscope slide *BS* (reflectance ~ 4-5% per face). This reference beam is directed through the flow tube *FT* perpendicular to its faces and through aperture *A* and lens *L* onto detector *PD*. The rest of the laser beam is reflected at mirror *M* to intersect the reference beam at a point inside *FT*, at an angle of $10^{\circ} - 35^{\circ}$. (This angle is measured as in the transverse velocity measurements above - but be sure to account for the differing indices of water and air.) The tube is filled with water in which a colloidal substance has been suspended - CoffeeMate nondairy creamer works nicely - and is connected to a pump. The suspended particles serve to scatter some of the strong signal beam into the detector where it will be mixed with the reference, any frequency shift will then be observable with the spectrum analyzer. Note the similarity of this arrangement to a Mach-Zehnder interferometer. We should expect the signal beam to be Doppler shifted by an amount determined by the projection of the flow velocity onto the direction of the signal beam (the derivation of the expected shift is left to the student).
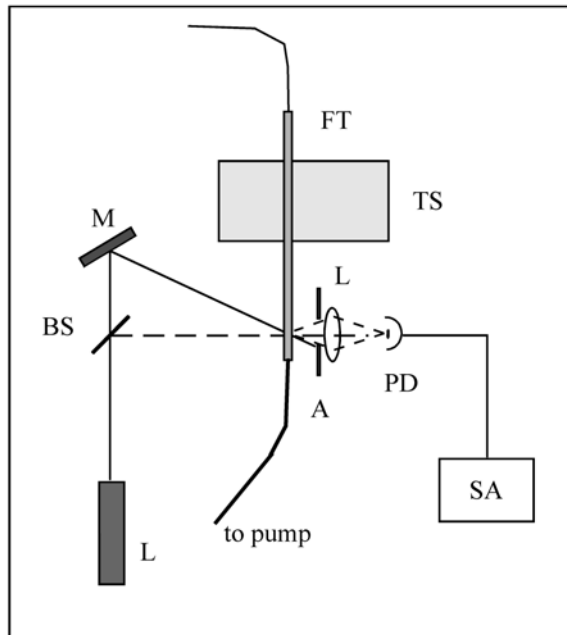
Figure 3. Experimental setup for the flow profile measurements. *BS* is a microscope slide, *FT* is the flow tube, and *A* is an aperture.

Several important factors must be considered for this experiment to work properly. First, it is essential that the flow be laminar - a turbulent flow will give Doppler shifts over a wide range of frequencies, blurring the spectrum analyzer peak until it becomes unrecognizable. Second, we must limit the size of the particular scattering region at which we are looking for each measurement - if the volume is large, there will again be a range of frequencies in the spectrum. This is the function of the lens: it images a small region within the intersection volume onto the detector. Third, we must restrict the range of scattered wave vectors - light scattered off at different angles will have different Doppler shifts, again broadening the frequency peak. This is the function of the aperture *A*: although the lens will image any light scattered from the volume of interest onto the detector, the aperture blocks all but that light propagating along the direction of the reference beam. (A vertical slit could be used instead. Why?)

Special care must be taken to obtain a signal which is sufficiently clean for taking measurements. The detector should be placed so that it receives the reference beam directly. The position of the lens is particularly critical. Initially determine the approximate height and transverse position by centering the lens on the reference. Then find the longitudinal position in the following manner: Translate the tube so that the beams intersect at the face nearest the detector. Block the beams with a thin sheet of paper and adjust the positions of *L* and *PD* to image the beam-spot onto the detector element. Then remove the paper and check that the reference beam is not deflected away from the detector - if it is, repeat the positioning steps. When this procedure is completed, translate the flow tube *FT* so that the beam intersection is in the middle of the tube. Adjust the spectrum analyzer until a signal is found (start by estimating the expected Doppler shift). The true beat note signal will disappear when either of the beams is blocked. Check to see that the frequency varies with the pump speed. When you have found the beat note, adjust the position of the various components to maximize the signal amplitude. If the signal is sufficiently above the background (> 10 dB), you will be able to take the necessary data by computer as with the slow wheel. Take measurements at ~1 mm intervals starting at one side and continuing until the other side is reached. In analyzing your data, plot the profile and qualitatively compare

it with what would be expected for a laminar flow (look it up). As usual discuss experimental problems, error sources, etc. Note: On occasion this experiment does not work out very well. If such is the case for you, at least obtain a measurement of the speed at the center of the tube and explain the problems encountered.

Summary of Procedures

Upon completion of this experiment you should have sufficient data to obtain the following:

- The rotation rate of the slow wheel using the Doppler effect.
- The speed of the toy train (compare with direct measurement).
- The rotation rate of the fast wheel using the interference-scattering technique.
- The velocity profile of a laminar flow, via the Doppler effect. \end{itemize}.

Food for thought

Doppler velocity with the Michelson arrangement. It is not really necessary to invoke the Doppler Effect. Find an alternate explanation (it should lead to the same numerical expression)

Transverse velocimetry measurement (fast wheel). This can be interpreted also through the Doppler effect…

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APPENDIX: HP 3585A Spectrum Analyzer

 The Hewlett-Packard 3585A is a digital spectrum analyzer with a bandwidth of 20 Hz - 40 MHz. It can be operated manually by the front panel controls, or by computer via the IEEE-488 interface. The manual method is the subject of this appendix.

The entry keys of the front panel are divided into eight groups: INPUT, ENTRY, SWEEP, TRACE, MARKER/CONTINUOUS ENTRY, RBW-VBW-ST, STATUS, and TRIGGER. The pertinent keys in each of these groups will be discussed below. Most keys are designed so that they will be lit, or a corresponding entry on the CRT display highlighted, when on or in effect.

Before discussing the key panel, look at the CRT display. The horizontal axis is in units of frequency, and either the START and STOP frequencies or the CENTER frequency and frequency SPAN will be given below the bottom grid line. The vertical scale gives the power of the signal in logarithmic units (dBm, dBV, or dB) with the power corresponding to the top grid line given by REF, the values for the other lines decreasing from this value by the amount given in dB/DIV, both displayed in the upper left corner. (Remember that a signal ratio of 10 dB implies an electrical power ratio of 10, while a ratio of 20 dB implies a {\it voltage} ratio of 10. For most optical detectors, it is the output voltage that is proportional to the optical power in.) The RANGE determines the noise level due to the internal amplifiers. The upper right corner usually gives the MARKER position, but may display other information, and will be discussed later. The meaning of the entries on the bottom line will likewise be discussed below. The CRT is also used to display various messages, which will generally be accompanied by a warning tone. For example, every two minutes or so the instrument will execute a self-calibration routine and ``CALIBRATING" will briefly appear on the screen; similarly, if front panel operation is attempted while the device is in the remote mode, the message ``HP-IB OPERATION ONLY" will be displayed.
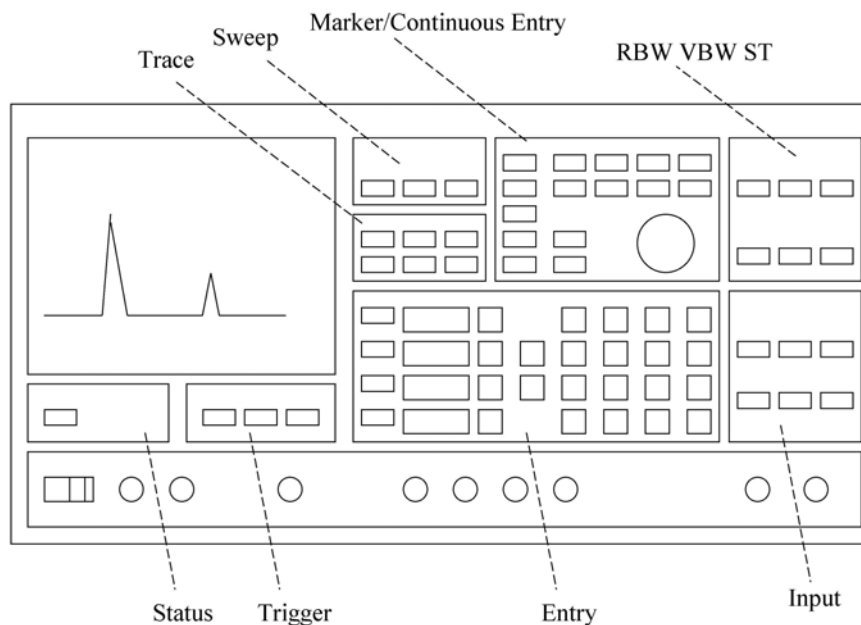

Figure 1. Layout of the HP 3585A Spectrum Analyzer front panel

The Eight Panel Groups

1. *INPUT*. Proper settings and connections of the INPUT group are required to obtain a trace, and to prevent overloading. For low frequency applications, the IMPEDANCE should be set to 1~MΩ; the 50 Ω and 75 Ω inputs need to be used only when investigating frequencies of >10 MHz and when the source is capable of driving such an impedance. The signal source must be connected to the input jack corresponding to the active IMPEDANCE setting. When AUTORANGE is on, the input amplifier RANGE will be adjusted to suit the input signal; with REF LEV TRACK on the power level corresponding to the top of the display grid will follow any adjustments of the RANGE (whether this is due to AUTORANGE, or to any manual STEP KEY or numerical entry)

2. *ENTRY*. This group is the most important of the eight. In particular, the REFERENCE LEVEL, dB/DIV, START FREQ/STOP FREQ and/or CENTER FREQUENCY/FREQUENCY SPAn may be set using the STEP, numerical, and unit keys. To change the value of any of these parameters, press the corresponding key. The corresponding CRT entry will then be highlighted, and the value may be changed either incrementally using the STEP keys (which usually change the parameter by a factor of 2 or in a 1-2-5-10 or 1-3-10 sequence) or by numerical entry from the keypad. If the new value is not allowed a bell will sound and possibly an error message will be displayed; the parameter will remain highlighted awaiting an acceptable entry. A change implemented by the STEP keys will be effected immediately, while a numerical entry will not take effect until an appropriate unit is entered. In either case, the CRT entry will remain highlighted until a units key has been pressed.

3. *SWEEP*. The SWEEP (i.e., the scanning of the internal reference oscillator through the appropriate range of frequencies) may be either CONTinuous (a new sweep beginning as soon as the last one is completed) or SINGLE (sweeping stops when the STOP FREQ is reached). In either case the SWEEPING light will be lit and the CRT trace will be continually updated while the sweep is in progress. The MANUAL ENTRY mode is rarely useful.

4. *TRACE*. The 3585A can store two traces: A, which is continually updated during sweeping, and B, which can be used for permanent storage. A trace must first be obtained in the A trace, then it can be transferred to B by pressing the STORE A→B key. Either or both can be viewed at any time, as can their algebraic difference A - B, by activating the appropriate key. The B trace is particularly useful for storing a good trace for later plotting, or for storing baseline noise trace which can then be subtracted from the A signal trace.

5. *MARKER/CONTINUOUS ENTRY*. The MARKER is the highlighted pixel on the spectrum trace which can be positioned by the CONTINUOUS ENTRY knob. The frequency and signal amplitude at its position on the trace will be given in the upper right corner of the CRT when it is active. When the COUNTER feature is active the trace will stop for $\approx$ 0.1 s when the marker frequency is reached and the device will measure the frequency (to an accuracy of 0.1 Hz) of the strongest signal within one resolution bandwidth (RBW - see below) of the marker frequency. The other features in this group are of little interest for our purposes.

6. *RBW-VBW-ST*. The RESolution Bandwidth is the frequency range allowed through the input filters to the system amplifiers, it is thus the displayed width of any input signal of bandwidth less than the RBW value displayed at the lower left corner of the CRT. It can be reduced to

resolve closely spaced signal or increased to speed up the sweep rate if the extra resolution is not needed. The VIDEO BandWidth (VBW - bottom center of the CRT) is, qualitatively speaking, the inverse of the time over which a particular element of the display is average; as such, it can be reduced to average fluctuations in the noise level which might mask weak signals, or increased to reduce the SWEEP TIME. With the COUPLES TO SPAN feature active, the SWEEP TIME (ST - lower right corner of the display) will be automatically set to the minimum value for which the instrument can still take calibrated measurements; if the ST is too small for the RBW and VBW, the UNCAL LED will light, and the SWEEP TIME increased until it goes off. The RBW and VBW can be varied in a 1-3-10 sequence only; the ST can be set to any multiple of 0.2 s.

7. *STATUS*. When the 3585A is being controlled via the IEEE-488 [also know as the HP-IB (Hewlett-Packard Interface Bus) and GPIB (General Purpose Interface Bus)] interface on the rear of the instrument, the REMOTE LED will be lit and the front panel will be disabled. The other three LEDs indicate the status of the instrument with respect to the bus. REMOTE control may be overridden and front panel operation restored by pressing the local key.

8. *TRIGGER*. The FREE RUN mode should be used.

Plotting. Though not an entry key group, the capability to generate a hard copy plot of the displayed frequency spectrum is a useful feature of the 3585A. To obtain a plot, an X-Y recorder must be connected to the X and Y PLOTTER OUTPUT connectors on the back panel of the instrument (if the recorder provides a remote pen lift feature, this should be connected to the Z output - note however that there may be some compatibility problems). The plot is initiated by pressing the RECALL (on) key [lower left corner of the ENTRY block] then the 8 (plot1) key. An analog copy of the trace will be sent to the output jacks at a rate which a typical recorder will be able to follow. Output voltages will range from 0 V (START FREQUENCY) to +10 V (STOP FREQUENCY) for X, and from 0 V (lower grid line) to +10.64 (REFERENCE LEVEL = 10.4 V) for Y. Set the recorder inputs ranges accordingly. The outputs are set to the high limits after each plot, so zero the recorder at the upper right corner of the page.

A simple demonstration - ``Watching the radio''. To demonstrate the operation of the 3585A, connect a short piece of unshielded cable to the 1MΩ input. Tune the START and STOP frequencies to locate the AM radio band (0.5 - 1.6 MHz). Position the marker on one of the stations and measure the frequency of the station using the COUNTER. Watch the frequency and amplitude of the signal over several measurements. Can you see why these are `amplitude modulation' stations rather than `frequency modulation'? Can you find the FM stations?

As mentioned in the former subsection, the Hewlett-Packard 3585A spectrum analyzer may be remotely operated by computer via the IEEE-488 interface [also known as the HP-IB (Hewlett-Packard Interface Bus, or GPIB (General Purpose Interface Bus)]. The program DOPPLER.EXE has been written to drive the 3585A and to collect the appropriate data for this experiment. Use of this program is the subject of this portion. Throughout this section anything that is enclosed in quotes " ") is what you will be expected to type from the keyboard; names of special keys which you must type will be enclosed in Dirac brackets (e.g., <esc>, <enter>, etc.).

Setting up. Turn on the computer, and the 3585A. Get into the optics lab directory by typing "CD/OPTLAB<enter>". Obtain a signal trace on the analyzer and move the marker to the top of the peak. Once this is done you are ready to initiate the program by typing "DOPPLER<enter>".

Initializing the program. The program will begin by displaying a summary of its operation, which will remain on screen until you press <enter>. The screen will then be cleared and you will be asked for the name of a file in which to store your data. Enter this as a root name of up to eight letters followed by a period followed by a three letter suffix (########.###, e.g., MYDATA.NEW or GEORGE.DAT). The computer will then take control of the 3585A and take a datum as an operational test; so long as the system does not hang up and no bells are sounded, everything is OK. You will then be asked to type ``T<enter>'' when you are ready to begin data taking or ``A<enter>'' if for some reason you wish to abort the program.

Data collection. You will next be prompted for the initial position reading of the translation stage. Enter this in centimeters or millimeters, but always be consistent - use one or the other throughout the session. The computer will then turn the 3585A COUNTER feature on, trigger a trace, wait for the analyzer to measure the frequency, then request the value of the measurement. It will then display (on the computer CRT) the number of data points taken thus far, the current position of the translation stage, and the current frequency measurement. It will also move the spectrum analyzer MARKER to the measured frequency so that you can determine whether the counted frequency corresponds to the Doppler shift signal. You will then be asked whether to keep the datum or reject it. The datum should be rejected if the reported does not correspond to the reading from the translation stage (in which case you moved to the peak of the Doppler shift signal (in which case you should move the MARKER manually to that position using the CONTINUOUS ENTRY knob on the 3585A front panel - this will prepare the instrument for the next measurement). After keeping/discarding the datum you will be prompted with "Type <enter> to continue, <space><enter> to change position, or STOP to quit". Respond accordingly. If the first option is chosen, a new trace will be triggered and a new measurement taken. If you want to 'change position' you will be prompted for the new position; move the translator, type in the new reading, and press <enter> to indicate a new measurement. Respond with "STOP" if you are finished taking data or if you wish to quit the session; the output data file will be closed and the 3585A will be returned to manual control.

Suggestions. You will want to take several data points at each position to get an idea of the accuracy of your measurements. Also, you may need to change the CENTER/SPAN or START/STOP frequencies to keep the signal on the trace. Similarly, the RBW, VBW, and dB/DIV may need to be adjusted to suit the signal. If you have problems with spurious signals, the A-B feature is very useful.

LAB 4: The Wavemeter

J-C. Diels and W. Rudolph

Purpose:

Familiarize the student with the Michelson interferometer, gaussian beams and the transformation of gaussian beams by lenses.

Reading Assignment

As referenced in text, and Chapter 4 of *Building Scientific Apparatus*, 3rd edition, by John Moore et al. (Perseus Books, Cambridge MA, 2003).

Background

The goal of this experiment is to utilize the sensitivity of the Michelson interferometer, used here in a wavemeter, to precisely determine the frequency and wavelength of an "unknown" laser. Due to the wavelike nature of light, the basic Michelson interferometer, as depicted in Fig.1, produces dark and bright fringes at the photodetector (PD) as the length of one arm is continuously varied by an amount Δ. These fringes are the result of constructive and destructive interference of the fields from each arm of the interferometer. For a more detailed description of the Michelson interferometer, see Hecht.

Theory

The intensity at the output of a Michelson interferometer, see Fig.1, that is recorded by the photodiode is given by:

$$I = (I_o/2) \left[1 + cos\{2\pi(d_2 - d_1)/\lambda\}\right]$$

The photodetector will see one fringe go by for a displacement of one half wavelength of M_1. If we translate M_1 by a known distance d, and count the number of fringes N on the photodetector, we can then determine λ from the relationship

$$d = N\lambda/2$$

by measuring N and d. If for instance $d = 30$ cm and $N = 10^6$ fringes, then $\lambda = 0.6$ μm. There are, however, two limiting factors on the precision with which we can determine λ. 1) How well we know N, and 2) how accurately we can determine d. For instance, even if d is known with absolute accuracy, simple fringe counting would only yield an accuracy of 10^{-6} for the determination of the wavelength, limited by the lack of information from rounding off to the nearest fringe integer. A higher precision is possible by recording the interference fringes and calculating the position between fringes after the last full fringe has been counted, such that N is known to a certain number of decimal places. N is in fact the round-off of $arccos(2I/I_o)$:

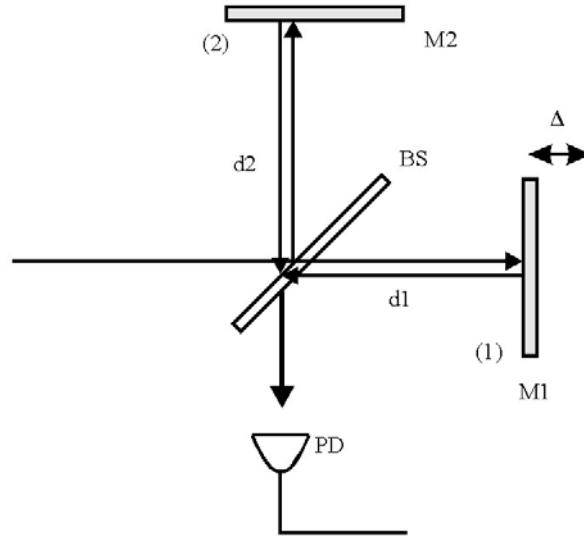$$N = INT \left[arccos(2I/I_o) - 1/2\right]$$

Figure 1. Michelson Interferometer

Determining *N* with a higher precision will help, but in reality the distance *d* cannot be measured with the required accuracy. One could try to translate M_1 uniformly and to record accurately the fringe count rate. Knowing the speed of translation, one could determine the wavelength from the fringe counting rate (or frequency). The problem with that approach is in achieving the required uniformity of motion. One could use synchronous motors (the frequency stability of the 60 Hz network is fairly good). However, the error in the reducing gears and lead screw of the translation stage make it impossible to achieve uniform translation speed. In fact, a chart recording of the fringes of an interferometer using a synchronous motor driven translation stage will indicate periodic changes in "wavelength", which can be used to identify the gear sizes and defects in the reduction box. Therefore, a simple way to determine the quality of such a drive mechanism is to record closely spaced fringes (chart the motor on a low speed relative to the periodicity of the interferences), and copy them on a transparency. The "Moire" pattern observed when the transparency is superimposed on the original recording is an indication of the speed variations in the motor drive.

So how then can we determine *d* with the required precision? The answer seems almost circular: use the laser. It is usually the frequency of a laser which is known with the most precision. The absolute frequency of many well stabilized lasers has been determined by beating its frequency down, in a long chain of complicated nonlinear mixing devices, to a range in which it can be compared with the current definition of time, the ground state hyperfine splitting of cesium, which is 9 192 631 770 Hz. As of 1983, the speed of light has been *defined* to be 299 792 458 m/s. Thus by knowing the precise frequency of the laser, and using the defined value for the speed of light to determine the wavelength (in vacuum), one can calculate the distance *d* from the equation for λ above. This is in fact how the meter is currently defined, through the definition of c and the second. This technique will then require two lasers, the frequency of one of them is well known. *The fringe count of the known laser frequency is used to calculate the distance d, which is in turn used - knowing the fringe count for the other laser - to calculate the unknown*

wavelength. Several arrangements are possible. One of them is the Mach-Zehnder interferometer. The other is the double Michelson sketched in Fig. 2.

One should be aware that this method provides a wavelength ratio in the laboratory environment. Depending on how successful your experiment is, you may have to correct for the dispersion of air. The appropriate constants for that correction can be found in the *Handbook of Chemistry and Physics* (Chemical Rubber Co.)

Two other limitations are the bandwidth and stability of the reference laser, as well as that of the laser to be measured. These (bandwidth and stability) are two different factors, which might - but need not - be related.

Experimental Setup

Attempt to make a Michelson interferometer using an air rail as one arm, see Fig. 2. This interferometer will be used as a wavemeter by combining two lasers, a known reference (red He-Ne) and an unknown source (green He-Ne, this is the least you should determine!). The polarization of the lasers should be orthogonal to each other. Using a polarizing beamsplitter
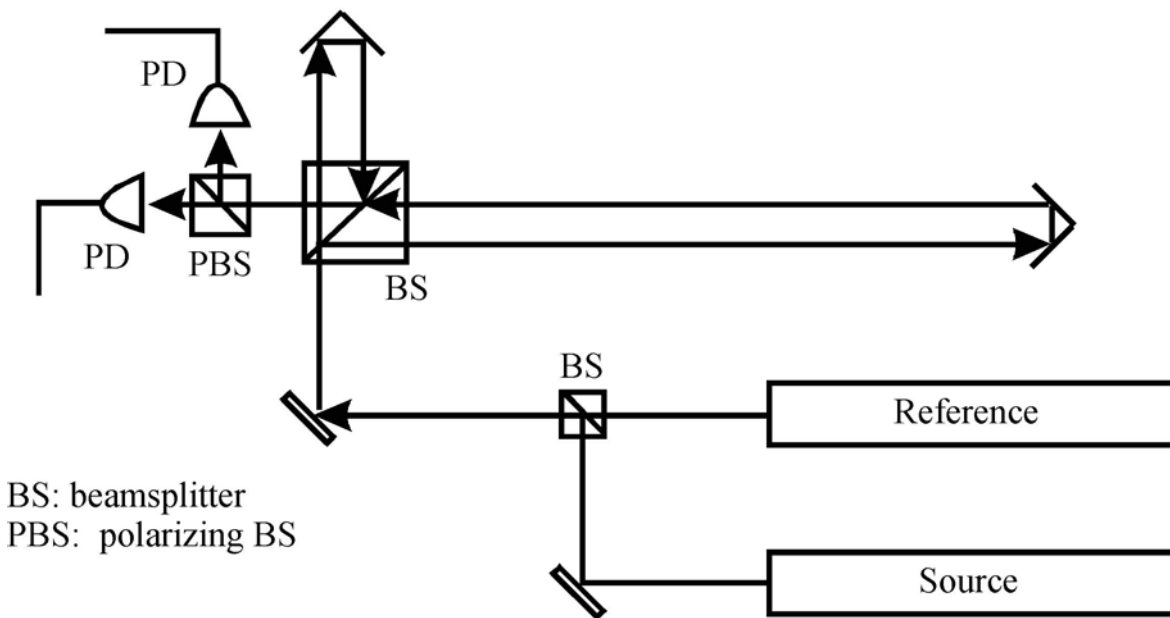

Figure 2. Wavemeter

(PBS), the beams from the two lasers can be superimposed on each other and aligned through the wavemeter. This allows the two beams to propagate the same optical path length through the same interferometer. The beams are then separated at the exit of the interferometer and sent to two different photodetectors. If necessary, use color filters to further suppress the unwanted beam at the detectors. We then can obtain data from "two" interferometers. You will notice two practical problems with this arrangement:

1. The translation axis of the variable arm has to be lined up with the beam with great accuracy;

2. Due to diffraction, it is difficult to observe uniform extinction (and therefore good fringe contrast) over the cross section of the beam.

Both problems can be alleviated by changing the beam waist of the Gaussian beam emitted by the He-Ne laser. Indeed, the beam path in the long arm can be as much as 1 m longer than in the short arm. In order to produce interference fringes with the best possible contrast, the two beams must have the same intensity and therefore the same cross-section. This condition, for a delay variation of 2 m, implies some minimum dimension for the beam waist. What is this minimum size?

Expand the beam with an appropriate choice of lenses. You will notice that, while correcting problem (2), you have alleviated to some extent problem (1). It may also help to use a polarizer in front of the He-Ne laser. Observe the He-Ne interference fringes with and without a polarizer. The standard recording instrument used with a wavemeter is a frequency counter. That will be the first technique to use. By measuring the ratio of the fringe frequency from each laser as the mirror on the air rail is moved at a constant velocity, one should be able to calculate the frequency of the unknown laser. The alternative technique that will be used is to record all the data, store them, and make appropriate manipulations to determine the number of fringes passed from each laser. This is the more direct approach. The details on how to take data from the two photodiodes, D_s and D_r, and store it on the computer, are given in the Appendix. With this data, you can count the number of fringes, including the partial fringe, to calculate the frequency of the unknown laser.

Summary

In this lab you should have learned how the Michelson interferometer can be used as a wavemeter to make precision measurements of a laser's frequency and wavelength. You should also try to identify some of the factors which limited the precision of your measurements.

Additional questions

1. How is the precision of this technique affected by the distance which mirror 1 is translated? What does this tell you about the physical dimensions of a high-precision wavemeter?

2. Corner cube reflectors: The metallic corner cube reflectors have an accuracy of 2" of arc. Will that be sufficient to keep the laser beams aligned?

3. Fringe visibility and diffraction: Given as a condition that the beam diameter should not vary more than 10% over the entire range, in order to provide good fringe visibility, what should the initial waist of the beam be for each laser? (Hint: This requires some basic knowledge of Gaussian beam propagation.) You should estimate the size of the initial beam waist from each laser by measuring its far-field divergence angle.

REFERENCES

1. Eugene Hecht, *Optics* (Addison, New York, 2002).

APPENDIX: Computer-based Data Acquisition

The wavemeter lab involves taking data with a computer. For our purposes, the data acquisition board in the computer is a DASH-16 model offered by Keithley Metrabyte.

The DASH-16 consists of a set of A/D converters (fast AD 774 12 bit converters), and relatively fast D/A drivers. It can be operated with 8 differential or 16 single ends inputs, and can be set in unipolar or bipolar modes (i.e. strictly positive or both positive and negative voltage inputs). The maximum throughput conversions is 100,000/sec.

The A/D conversion may be triggered three ways, by software command (which is generally used), by an on-board programmable timer, or by an external trigger pulse. The transfer of the data can also be accomplished in three ways, by program control, by interrupt service routine, or by direct memory access (DMA). All modes are selectable in software.

The DASH-16(F) has a wide variety of available input ranges which are selected by DIP switch on the board. Selectable ranges include +1v, +2v, +5v, +10v, and +0.5v, +1v, +2.5v, +5v, +10 volts. Other user selectable input ranges can be obtained by simply installing a single gain setting resistor.

The software supporting DASH-16 is STREAMER, which is a high speed data acquisition software routine. The STREAMER allows analog input data from a DASH-16 to be directly stored on hard disk, at sample rates of up to 60,000 samples per second. The STREAMER utilizes a DMA transfer, to transfer data from the I/O board to the computer's memory. The software is also transferring data from the computer memory to the hard disk. This capability allows continuous high speed data transfers over long periods of time, limited only by the capacity of the disk.

The STREAMER program is accessed via a batch file. The relevant batch file is called TAKESDATA in the directory \OPTLAB\ WAVMETER. It first accesses STREAMER which acquires the data in a binary form. After STREAMER is exited, the batch file calls utilities that convert the data to a useful ASCI format. The data (usually two columns worth for most lab purposes) is then indexed, meaning that an additional first column consisting of the sample or index number is added (for plotting purposes). After the batch file has completed operation, subsequent data analysis in plotting programs such as GRAPHER, ORIGIN, or EZPLOT (to name a few) can be done.

LAB 5: Holographic Interferometry

J-C. Diels and W. Rudolph

Purpose:

Familiarize the student with holography, holographic interferometry, and diffraction gratings.

Reading Assignment

As referenced in text, and Chapter 4 of *Building Scientific Apparatus*, 3rd edition, by John Moore et al. (Perseus Books, Cambridge MA, 2003).

Background

The word hologram was coined from the Greek word "holos" which means whole, because the recorded hologram contains all the information of the image field . Unlike a photograph, which records the square of the electric field, a hologram contains both the amplitude and phase information from the image. Although originally invented by Dennis Gabor in 1948 for the purpose of improving electron microscope images, holography did not attract much interest until the discovery of the laser in 1960.[6] Since then, both artistic and scientific communities have explored the field enthusiastically, finding numerous applications in holographic optical elements (HOE's), optical storage, imaging, and of course, visual arts.

On-axis holography

In Gabor's original work, two waves are superposed on a recording medium as shown in Fig.1.
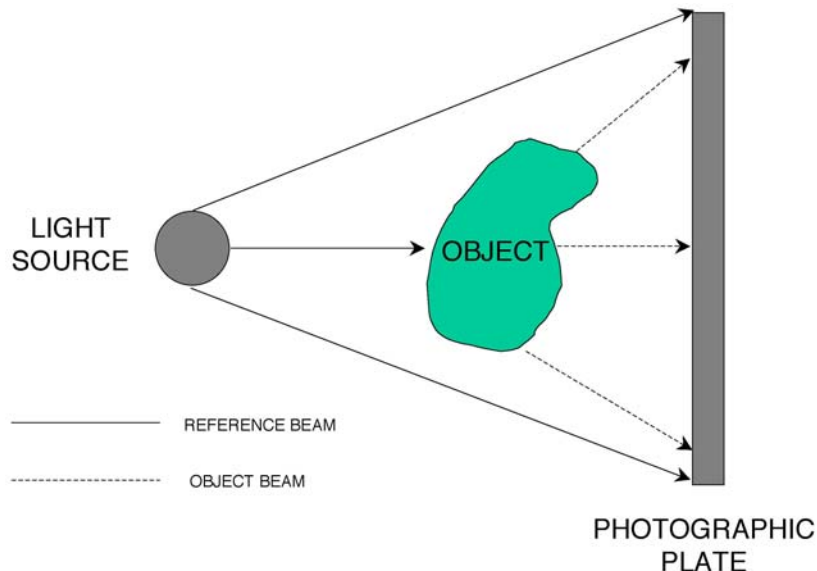

Figure 1. Gabor's scheme for recording a hologram.

A small semi-transparent object is illuminated by a light source. The object scatters the light and creates a second wave which superposes on the recording medium with the reference wave

originating from the source. The field on the recording plate can be written as $\mathbf{E}_{total} = \mathbf{E}_r + \mathbf{E}_o$ where where $\mathbf{E}_r$ is the electric field of the reference wave and $\mathbf{E}_o$ the field of the scattered or object wave. In terms of amplitudes and phases, $\mathbf{E}_{total}$ can be represented in the following form:

$$\boldsymbol{E}_{total} = [\boldsymbol{E}_r + \boldsymbol{E}_o exp[i(\Psi_o - \Psi_r)]\ exp\{i\Psi_r\}$$

Thus the recorded intensity is

$$I = \left| E_{total} \right|^2 = \{E_r^2 + E_o^2 + E_r E_o\ exp[i(\Psi_o - \Psi_r)] + E_r E_o\ exp[-i(\Psi_o - \Psi_r)]\}$$

If the exposure time is T, the amplitude transmittance t of the developed plate is $t = t_o - \beta T I$ where to is the amplitude transmission of the unexposed film. The term E_o^2 contributes to a noise in the reconstruction. This noise is small compared to the reference background E_r^2. If we remove the object and illuminate the developed plate with the original reference beam, the transmitted field will be:

$$\boldsymbol{E}_t = \boldsymbol{E}_r t\ exp\{i\Psi_r\}$$

The field $\boldsymbol{E}_t$ is proportional to:

$$exp\{i\Psi_r\}\ \{E_r + E_o\ exp[i(\Psi_o - \Psi_r)] + E_o\ exp[-i(\Psi_o - \Psi_r)]\}$$

We recognize in the first term the wave which originally illuminated the plate; the second term is identical to the wave scattered by the object. Such a plate which has the property of keeping the amplitude and phase of a wave is called a hologram. The third term is identical to the second except for the phase term $(\Psi_o - \Psi_r)$ which corresponds to a second reconstruction, located on the opposite side of the plate. This conjugate wave which appears during reconstruction, is not seen when we focus on the primary reconstruction. However, its presence degrades the primary wave and prevents getting a high quality hologram. More complete discussions of the theory are available.[2, 6, 7]

Off-axis holography

The technique of holography was improved in 1964 when Leith and Upatnieks proposed the off-axis holography. The experimental set-up for off axis holography is shown in Fig. 2. The beam emitted by a laser is split into two beams. One beam is expanded and sent onto a high resolution recording medium. The second beam is used to illuminate an object. The light scattered by the object interferes in a complex fashion with the reference light. The interferences are recorded on a photosensitive plate. After development of the plate, an object wave can be reconstructed by illuminating the plate with the reference wave at the same angle as for the recording. With this off-axis hologram it is possible to detect the object wave without being disturbed by the reference beam which propagates along another direction. Off-axis holography requires the use of a laser.

Amplitude and phase holograms. A hologram on a photographic plate (with the recording medium of silver halide) is often called an amplitude hologram. The diffraction efficiency (percentage of light in the reconstruction) of an amplitude hologram is very low. It is possible to obtain a transparent hologram with about 10 times higher efficiency by bleaching the plate.
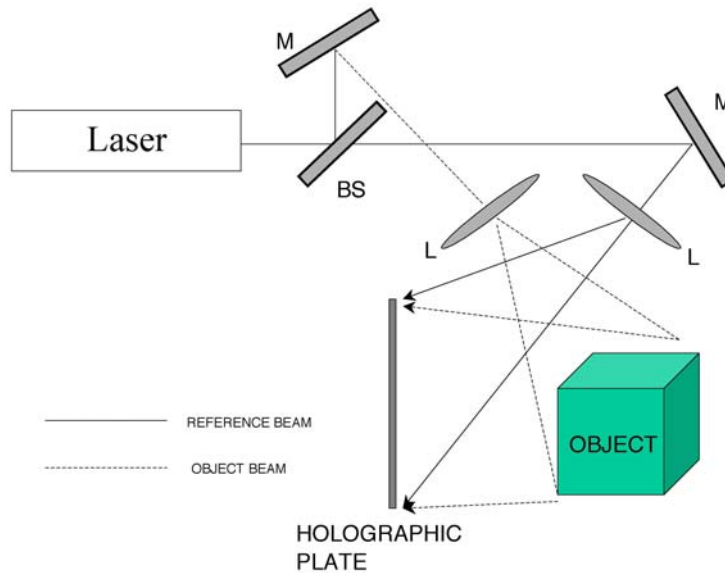
Figure 2. Experimental setup for off-axis holography. (L=lens; M=mirror; BS=beam splitter)

Bleaching is a chemical process which converts the silver image (dark) into a silver salt which is transparent.

Holographic recording. The photographic film consists of a base material (glass, acetate or paper) coated with a suspension of silver halides in gelatin. When the film is exposed to light and developed, the grains of silver halides are changed into metallic silver and the transmittance of the film is altered. If *d* is a quantity proportional to the amount of the metallic silver, the intensity transmittance (ratio of transmitted light to the incidence light) is given by $\tau = t^2 = Ae^{-d}$ *or d = log(1/τ) + log A*.

Film development.

Developing the film is probably the most critical step, and a myriad of developing techniques and recipes exist. The following one is that of T.H. Joeng from Lake Forest College:

Solution A: In 750ml of distilled water (at 100 F) add the following chemicals in order: 20g of Catechol, 10g of Ascorbic Acid, 10g Sodium Sulfite, 75g Urea. When all chemicals have been dissolved add cold water to bring the final solution up to 1 liter.

Solution B: Add 60g of Sodium Carbonate to 750ml of distilled water at room temperature. When disolved, add water to bring the total solution up to 1 liter.

Bleach: In 750ml of distilled water (at 100 F) add the following chemicals in order: 17g Copper Sulfate, 55g Potassium Bromide, 2g Succinic Acid. When all chemicals have been dissolved add cold water to bring the final solution up to 1 liter.

When ready to develop the hologram, mix equal parts of solutions A and B in a shallow tray. This is the developer and will last for up to 8 hours. In a second tray, place some of the bleach. Put the film in the developer for 2 minutes, and constantly agitate the mixture by rocking the tray

back and forth. Next, wash the film in running water for 3 minutes. After this, bleach the film (in the bleach solution) until the film is transparent. Then leave it in the bleach solution for an additional 30 seconds. And finally, wash the film again in running water for 3 minutes. At this point it is also a good idea to give the hologram a final rinse in Kodak Photo-Flo (follow the directions on the bottles). This will help the hologram dry evenly without streaking. Before viewing, the hologram must be completely dry. Use a hair dryer to speed up the process.

Experiments

The first requirement for a good hologram is a film with high enough resolution to record the pattern created by the interference of the reference and object beam. You will use Agfa 8E75HD which can resolve 5000 lines per millimeter. The film should be first cut to size in a dark room and then placed between two plates of glass in the provided frame holder. The second requirement is a light source (laser beam) that uniformly illuminates the object to be imaged. Use a spatial filter to clean up and expand the laser beam.

Simple hologram. Produce a hologram of a small object of your choice and read out the image after development. Use the off-axis geometry.

Holographic grating. In this experiment you will produce a holographic diffraction grating. The experimental layout is shown in Fig.3. Two laser beams produce an interference pattern consisting of parallel lines in the plane of the film. Two polarizers in one arm allow you to adjust the laser power while keeping the polarization constant. Measure the intersection angle of the two beams and calculate the line spacing (grating constant) *d*. Write a grating with the two beams having nearly equal intensity. Write a second grating after attenuating one beam by about 50% with the same exposure time. After developing the hologram evaluate the diffraction pattern produced by the gratings using the unexpanded beam from the HeNe laser. Measure the diffraction angle and compare it to theory. Measure the diffraction efficiency into 1st and 2nd order for both gratings and discuss.
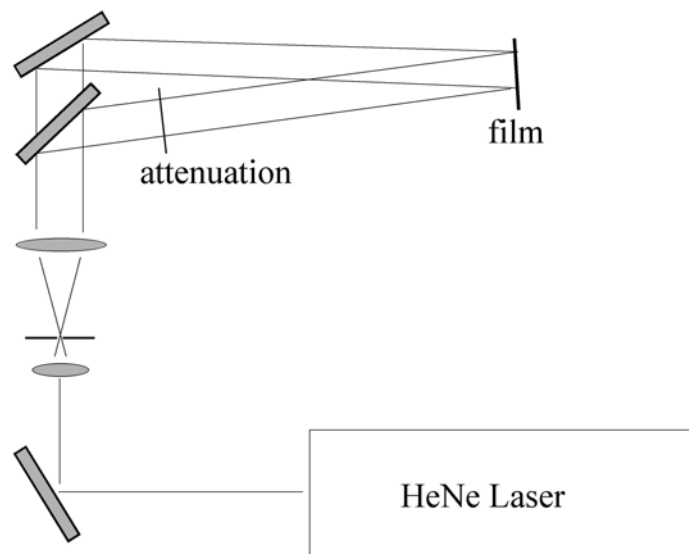

Figure 3. Experimental setup to write a holographic grating

Holographic interferometry. Because holography offers the unique potential to capture the complete electrical field, it is possible to interfere two reconstructed images with each other. This is what is done in a double exposure hologram. In this lab a single exposure will first be taken of an aluminum bar. Then without moving anything, a small weight (20-30 g) will be added to the bar and a second exposure will be made. Since the phase information from both exposures is stored in the hologram, any changes in phase introduced by a small bending (on the order of microns) of the bar will show up as fringes in the reconstructed image. The setup for making such a double exposure hologram is shown in Fig.4, and more information can be found in references.[8]
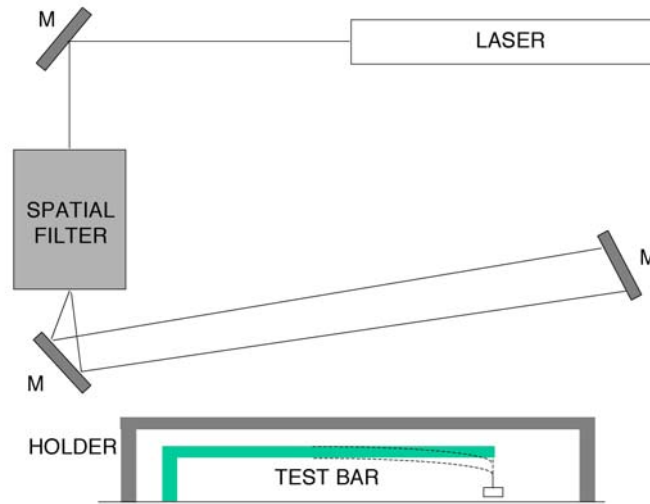

Figure 4. Apparatus for a double exposure hologram.

Upon viewing the developed hologram, one should see a series of fringes crossing the bar (fringes perpendicular to the length of the bar). As might be expected, dark fringes occur where the change in optical path length between the two exposures is equal $n\lambda/2$, where n is an odd integer. If one can assume that for each point along the length of the bar, the displacement due to the extra mass is approximately perpendicular to the surface of the bar, then it is possible to show (you should do this for your report) that fringes will be observed when the displacement Δy is given by:

$$\Delta y = (n\lambda) / \{2(\cos a + \cos \beta)\}$$

Here α and β are the incident angle of the laser light and the angle of observation respectively, both of which are measured from the normal of the surface of the bar, see Figure 5.

Because of the limited coherence length (5 cm) of the laser, you should place the bar as close to the film as possible. Give the entire apparatus some time (2 min) to "settle". This will allow any small vibrations to damp out, and trapped air to escape from between the two plates. When all is calm, expose the film for about 1 second with the He-Ne laser at full current. (You should verify the correct exposure time by measuring the energy flux at the film with a powermeter and comparing it with the film specifications accompanying this description). Then quickly remove

the weight from the bar, wait a few seconds for vibrations to settle and expose the film again for the same period of time.

After developing the film, you will want to carefully measure the spacing between the fringes and use the equation for Δy to calculate the bending of the bar along its length. Compare your results with the theoretical predictions for the bending of a rigid bar due to a force at one end. A description of this situation can be found, for example, in Stephenson's book.[2]
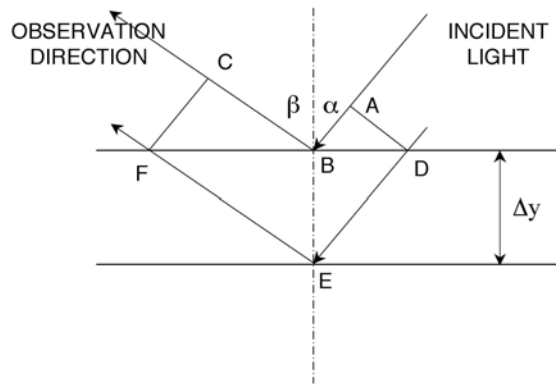

Figure 5. The displacement of the bar due to the additional weight changes the optical path length from ABC to DE. At each small segment of the bar, the displacement is assumed to be perpendicular to the surface of the bar.

Additional Experiments (Optional)

If you have enjoyed your holographic experiment thus far, you may want to continue by making a double exposure hologram of another stressed object. An aluminum can may easily be deformed between exposures by tightening a screw against its side, for example. Another type of experiment involves studying the normal modes of a vibrating object (a handbell driven with a small electromagnet would be a good candidate). In this case only one exposure is necessary, and even though the object is moving during the exposure fringes are still observed. This is because on average the oscillating surface spends most of its time at the two turning points. One then obtains the equivalent of a hologram with an exposure taken at each extremum. Unfortunately, with the more complex geometry an exact analysis of the resulting fringe pattern is not always possible. Another interesting experiment is to produce a Fresnel lens. You can use the basic setup of Fig. 3. Place a lens in one arm so that in the film plane you have a superposition of a parallel and a converging beam. Readout the developed hologram with the expanded parallel beam and search for the focus spot produced by the lens. Measure the focal length and explain.

Summary
1. Produce a simple hologram of a small object.
2. Produce two holographic gratings. Measure and discuss the diffraction efficiency and the diffraction angle.

3. Investigate the bending of a metal bar under the influence of a weight using holographic interferometry. Compare your results qualitatively to theory.

4. Produce a hologram of a vibrating object.

REFERENCES

1. G. Saxby, *Practical Holography* (Wiley, New York, 1988).

2. R. J. Stephenson, *Mechanics and Properties of Matter* (Wiley, New York, 1969).