

Physics 160 Exam 3.

Name Solutions

Closed book, closed notes, no calculators / cell phones / etc. Use page backs for more space.

You may be able to do later parts of a problem even if you can't do earlier parts:

Just assign a variable to the earlier answer and carry it through.

Clear thinking, as evidenced by appropriate starting equations and diagrams, will help get you part credit.

1. 40 pts. A mass $2m$ hangs from the center of a uniform rod of mass m as shown. The rod is attached to a building with a hinge. The length of the rod is L .

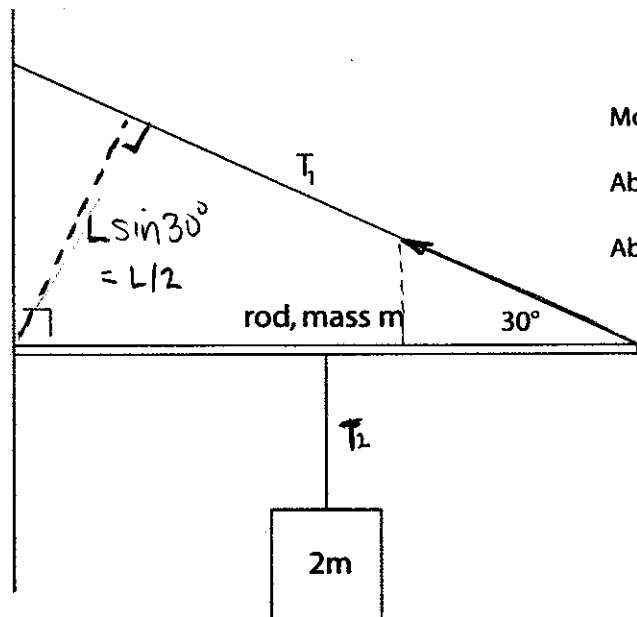
a) What is the tension in wire 1 (T_1), in terms of m , g , and L ?

b) What are the horizontal and vertical forces exerted by the hinge on the rod?

The diagonal wire breaks suddenly.

c) What is the angular acceleration of the rod around the hinge, right after the wire breaks?

d) What is the linear acceleration (direction and magnitude) of the end of the rod, right after the wire breaks?



Moment of inertia for a rod:

About its center $I = \frac{1}{12} mL^2$

About its end $I = \frac{1}{3} mL^2$

$$\begin{aligned} \text{a) } \sum \tau = 0 \quad T_1 \frac{L}{2} - 2mg \frac{L}{2} - mg \frac{L}{2} &= 0 \\ T_1 &= 3mg. \end{aligned}$$

$$\begin{aligned} \text{b) } \sum F_x = 0 \quad -T_1 \cos 30^\circ + F_{hx} &= 0 \quad F_{hx} = \frac{3\sqrt{3}}{2} mg \\ \sum F_y = 0 \quad F_{hy} + T_1 \sin 30^\circ - 2mg - mg &= 0 \quad F_{hy} = \frac{3}{2} mg. \end{aligned}$$

c) At rest, $T_2 = 2mg$. (why?)

But when falling $T_2 \neq 2mg$.

$$\textcircled{1} \quad 2mg - T_2 = 2ma$$

For the rod, $\sum \tau = I\alpha$

$$\textcircled{2} \quad mg\frac{L}{2} + T_2\frac{L}{2} = \frac{1}{3}mL^2\alpha$$

But $\alpha \times \frac{L}{2} = a$ (why?) so $\textcircled{1} \rightarrow T_2 = 2mg - mL\alpha$

Substitute $\textcircled{1}$ into $\textcircled{2}$

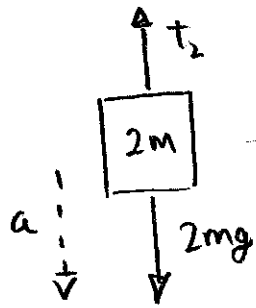
$$mg\frac{L}{2} + mg\frac{L}{2} - \frac{mL^2\alpha}{2} = \frac{1}{3}mL^2\alpha$$

$$\frac{3}{2}mg = \frac{5}{6}mL\alpha$$

$$\boxed{\frac{9g}{5L} = \alpha}$$

d) $a = \alpha r$

$$= \frac{9g}{5L} \cdot L = \frac{9}{5}g \text{ downward. (Note } > g!)$$



2. 30 pts. A hoop rolls without slipping on level ground at speed v . It then rolls up a ramp without slipping.

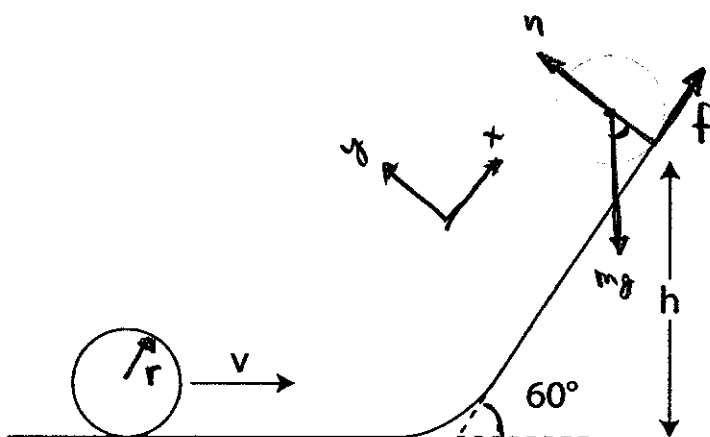
a) How high up the ramp does the hoop go, assuming it doesn't fall over? (Answer in terms of v , m , g .)

b) Draw a free-body diagram for the hoop as it rolls up the 60° ramp. (You may draw it on the sketch.)

$f =$ Friction force is uphill, to slow rotation!

c) What is the acceleration of the hoop as it moves up the ramp?

d) What is the minimum coefficient of friction required to prevent slipping?



a) Cons. energy

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{2}mr^2\left(\frac{v}{r}\right)^2 = mgh$$

$$mv^2 = mgh$$

$$\boxed{\frac{v^2}{g} = h}$$

c) $\Sigma F_x = ma$

[1] $-f + mg \sin 60^\circ = ma$ ($+a =$ downhill; consistent with ccw rolling)

Need another eqn (f, a both unknown!)

$$\Sigma \tau = I\alpha$$

[2] $+fr = mr^2 \frac{a}{r}$

\Rightarrow Sub into [1] to find

$$-ma + mg \cdot \frac{\sqrt{3}}{2} = ma$$

$$\frac{\sqrt{3}}{4}g = a$$

d) Since friction $f = ma$ ([2], above)

and normal force $n = mg \cos 60^\circ$ (from $\Sigma F_y = 0$)

$$\frac{f}{n} = \mu_{f, \min} = \frac{ma}{mg/2} = \frac{2a}{g} = \frac{\sqrt{3}}{2}$$

3. 30 pts. A large mass ($2m$) moving in the $+$ direction at speed 6 m/s collides with a small mass (m) moving in the $+$ direction at speed 3 m/s at $x=0, t=0$. There is no friction, and the collision is 1 dimensional.

- a) If the two masses stick together, where will they be 1 second after the collision?
- b) If the collision is completely elastic, where will the smaller mass be 1 second after the collision?
- c) Assume now, instead, that the collision is not direct, so that it is two-dimensional. One second after the collision the large mass is at $x=5$ m, $y=2$ m. Where is the small mass?

a) $\Delta p = 0 \quad m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f \quad v_f = 5 \text{ m/s.} \quad x = 5 \text{ m at } t = 1 \text{ s.}$

b) Can conserve p and K. Or consider in different frame!

$v_{c/G} = 3 \text{ m/s}$ will make small mass stationary; call it mass B.

$$v_{B/c} = v_{B/G} + v_{G/c} = 0$$

$$v_{A/c} = v_{A/G} + v_{G/c} = 6 - 3 = 3$$

before.

$$v_{B2} = \frac{2m_A}{m_A + m_B} v_{A1} \quad \text{EQN SHEET}$$

$$= \frac{2 \cdot 2m}{3m} \cdot 3 = 4$$

$$v_{B/G} = v_{B/c} + v_{G/G} = 4 + 3 = 7 \text{ m/s.}$$

After.
Small mass will be at $x = 7 \text{ m.}$

Could also use CM frame, where $v_{B2} = -v_{B1}$.

c) Conserve \vec{p} !

$$\vec{p}_i = 15m \left(\frac{m}{s}\right) \hat{i}$$

$$\vec{p}_f = 2m\vec{v}_{Lf} + m\vec{v}_{sf}$$

$$= 10m\hat{i} + 4m\hat{j} + m v_{sfx}\hat{i} + m v_{sfy}\hat{j}$$

So $v_{sfy} = -4$, $v_{sfx} = 5$ (m/s) and small mass is at $x = 5$, $y = -4$ m.

