Useful Equations & Numbers

$$\frac{1}{4\pi\varepsilon_0} = k = 9\times10^9 \text{ Nm}^2/\text{C}^2$$

1 nanoCoulomb = 1 nC = 10^{-9} C

$$\varepsilon_0 = 8.85\times10^{-12} \text{ C}^2/\text{Nm}^2$$

$$F = \frac{k|qq_0|}{r^2} = q_0E$$

$$U = \frac{kqq_0}{r} = q_0V$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$V = -\int \vec{E}\cdot d\vec{l}$$

$$E_x = -\frac{\partial V}{\partial x}$$

$$Q = CV$$

$$C_{pp} = \varepsilon A/d$$

$$\varepsilon = K\varepsilon_0$$

$$U_{cap} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}Q^2/C$$

$$u = \tfrac{1}{2}\varepsilon E^2$$

$$C_{par} = C_1 + C_2$$

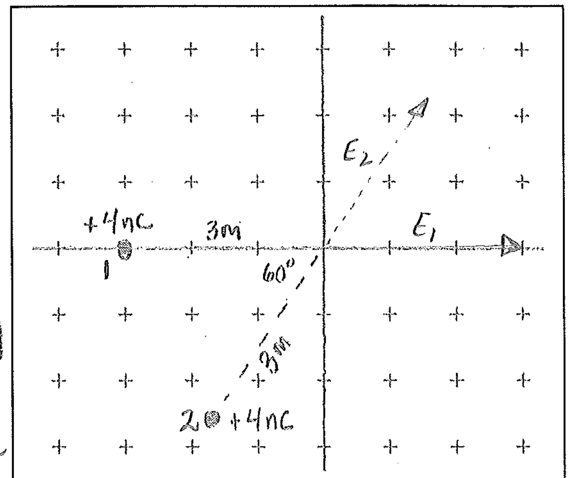
$$1/C_{ser} = 1/C_1 + 1/C_2$$

Round answers to the nearest integer; round 0.5 to the even integer.
(e.g. 4.5 rounds to 4, 5.5 rounds to 6)

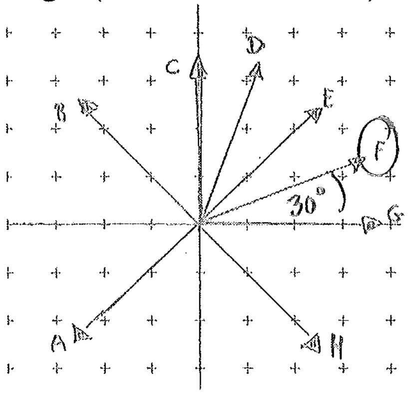
1. Which are **true**? Electric field lines:
I) are tangent to the field at all points
II) are closer together where the field is stronger
III) are trajectories of charged particles in the field

A) only I
B) only II
C) only III
D) I & II
E) II & III
F) I & III
G) all are true
H) none are true

2. The figure at right is for Q2-Q5. What is the magnitude of the electric field at the origin, to the nearest N/C? Choose 9 for 9 or greater.



3. What is the direction of the electric field at the origin (choose closest arrow)?



$|E_1| = \frac{kQ}{3^2} = 4\ N/C.$

$= |E_2|.$

$E_{2x} = \frac{|E_2|}{2} \quad E_{2y} = \frac{\sqrt{3}}{2}|E_2|$

$E_x = 4 + \frac{4}{2} = 6\ N/C$

$E_y = \frac{\sqrt{3}}{2} \cdot 4 = 3.46\ N/C \quad E = (E_x^2 + E_y^2)^{1/2} = 6.93\ N/C. \approx 7\ N/C.$

4. What is the electric potential at the origin, in V (take V=0 at infinity)? Choose nearest answer.

A] -8
B] -4
C] 0
D] 4
E] 8
F] 12
G] 16
H] 20
I] 24
J] 32

$V = \frac{2kQ}{3} = 24\ V.$

5. A point charge of +4 nC is brought to the origin. Then all charges are released. All charges have equal mass m. What happens?
A] the charges orbit around each other in erratic orbits
B] the charges orbit around each other in stable orbits
C] the charges move off to infinity, each with different kinetic energy (at infinity)
D] the charges move off to infinity, each with 8 nJ of kinetic energy
E] the charges move off to infinity, each with 12 nJ of kinetic energy
F] the charges move off to infinity, each with 24 nJ of kinetic energy
G] the charges move off to infinity, each with 36 nJ of kinetic energy
H] the charges move off to infinity, each with the same kinetic energy, which cannot be determined without knowing mass m.

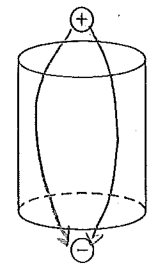
$U = U_{12} + U_{23} + U_{13} = K_{1f} + K_{2f} + K_{3f}$

$U_{12} = \frac{kQ^2}{r} = \frac{9\times10^{+9} \cdot (4\times10^{-9})^2}{3} = 48\ nJ.$

Correct answer not given, everybody gets +1 pt.

6. What is the sign of the flux through the top and bottom surface of the cylinder shown? (The charges are symmetrically placed.)

A] top +, bottom -
B] top +, bottom 0
C] top +, bottom +
D] top 0, bottom –
E] top 0, bottom 0
F] top 0, bottom +
G] top -, bottom –
H] top -, bottom 0
I] top -, bottom +



7. What is the sign of the total electric flux through all surfaces of the cylinder?

A] - B] 0 C] +

8. What is the flux through one side face of the cube shown? The edge length is L. An infinite line of charge runs through the center of the top and bottom faces and has linear charge density λ. The line is insulating; charges cannot move.

A] $\dfrac{\lambda L}{4\varepsilon_0}$

B] $\dfrac{\lambda L^2}{4\varepsilon_0}$

C] $\dfrac{\lambda}{4L\varepsilon_0}$

D] $\dfrac{\lambda}{\pi L\varepsilon_0}$

E] $\dfrac{\lambda L}{\varepsilon_0}$

F] $\dfrac{4\lambda L^2}{\varepsilon_0}$

G] $\dfrac{\lambda L^2}{6\varepsilon_0}$

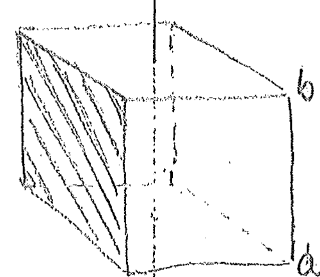
H] 0

I] The answer is not given.

J] There is insufficient symmetry to determine.

$\oint E\cdot dA = \dfrac{\lambda L}{\varepsilon_0} = \Phi_{tot}$.

$\Phi = \dfrac{\Phi_{tot}}{4}$



9. What is the magnitude of the electric field at the center of the face shown? Choose from the answers to Q8 above.

Use Gaussian cylinder $E\cdot 2\pi r L = \dfrac{\lambda L}{\varepsilon_0}$ $E = \dfrac{\lambda}{2\pi r\varepsilon_0}$ $r = \dfrac{L}{2}$

$E = \dfrac{\lambda}{\pi L\varepsilon_0}$

$\boxed{D}$

10. Suppose now that, the bottom of the cube sits on an infinite insulating plane of surface charge density σ. What is the potential difference between the corners a and b?

A] $\dfrac{\lambda}{4\varepsilon_0}+\dfrac{\sigma}{2\varepsilon_0}$

B] $\dfrac{\lambda L}{4\varepsilon_0}+\dfrac{\sigma L}{2\varepsilon_0}$

C] $\dfrac{\lambda L^2}{\varepsilon_0}+\dfrac{\sigma L}{2\varepsilon_0}$

D] $\dfrac{\sigma\lambda L^2}{2\varepsilon_0}$

E] $\dfrac{\sigma\lambda L}{2\varepsilon_0}$

F] $\dfrac{\sigma L}{2\varepsilon_0}$

G] The potential difference cannot be defined, because potential is infinite in this problem

Line doesn't matter since a & b are same distance.

$\Delta V = E\cdot L = \dfrac{\sigma}{2\varepsilon_0}\cdot L$

11. An electric field is given by $E_x = -x/2$, with x in meters and E in N/C. E_y and E_z are unknown. What is the potential difference between the origin and (x,y,z) = (4 m, 0, 0) (in V)? Enter ~~zero if~~ insufficient information has been given.

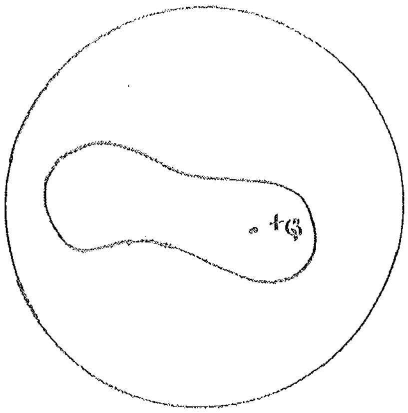
$V = -\int \vec{E}\cdot d\vec{l} = \int \frac{x}{2}\,dx = \frac{x^2}{4}\Big|_0^4 = \boxed{4\text{ V.}}$

12. An electrical potential is given by V = 3x + 2, x in meters and V in volts. What is the magnitude of the electric field at the origin?

$E_x = -\dfrac{\partial V}{\partial x} = 3\,\dfrac{V}{m}$ $E_y = E_z = 0.$ $\boxed{3\text{ V/m.}}$

13. An electrically neutral conducting metal sphere has a peanut-shaped hole in it. Inside the hole is a point charge of Q = +4 C. The surface area of the peanut hole is A_{in}. The radius of the sphere is R. What is the charge density on the inner surface?

A) 0
B) it is the same everywhere, and equal to Q/A_{in}
C) it is the same everywhere, and equal to $-Q/A_{in}$
D) it varies, but averages to Q/A_{in}
E) it varies, but averages to $-Q/A_{in}$
F) cannot determine



14. What is the magnitude of the electric field **just outside** the metal sphere?

A) 0 everywhere
B) it is the same everywhere, and equal to kQ/R^2
C) it is the same everywhere, and equal to $2kQ/R^2$
D) it varies, but averages to kQ/R^2
E) it varies, but averages to $2kQ/R^2$
F) cannot determine

15. A spherical balloon has a surface charge Q, uniformly distributed. It is inflated from radius a to b. What is the charge density when the balloon is at radius b, compared to the charge density when it is at radius a?

A) they are the same
B) $\frac{\sigma_a}{\sigma_b} = \frac{a}{b}$
C) $\frac{\sigma_a}{\sigma_b} = \frac{a^2}{b^2}$
D) $\frac{\sigma_a}{\sigma_b} = \frac{a^3}{b^3}$
E) $\frac{\sigma_a}{\sigma_b} = \frac{b}{a}$
F) $\frac{\sigma_a}{\sigma_b} = \frac{b^2}{a^2}$
G) $\frac{\sigma_a}{\sigma_b} = \frac{b^3}{a^3}$
H) none are correct

$Q = \sigma A$

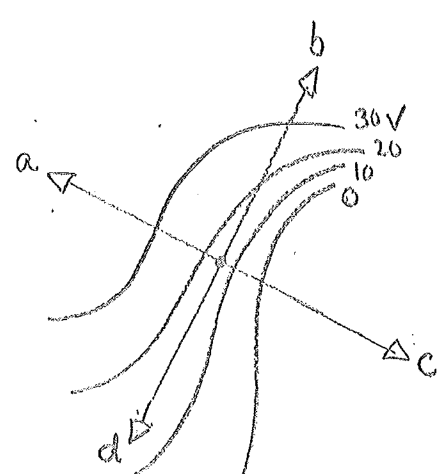
$\frac{\sigma_a}{\sigma_b} = \frac{A_b}{A_a} = \frac{b^2}{a^2}$

16. What happens to the field at a distance b from the center of the balloon as the balloon is inflated (but before it reaches b)?

A) it increases as the balloon surface gets closer
B) it decreases as the balloon surface gets closer
C) it is unchanged, even though the balloon surface gets closer.

17. What happens to the field just above the surface of the balloon?

A) it is unchanged as the balloon is inflated
B) it decreases as the balloon is inflated
C) it increases as the balloon is inflated
D) it's always infinite
E) it's always zero



18. A graph of equipotential lines is shown.
What is the direction of the electric field at point P? (choose a, b, c, or d)

$\boxed{C}$

19. A dielectric slab with K= 7 is inserted halfway into an isolated parallel plate capacitor which has charge Q. By what factor does the capacitance increase, to the nearest integer? (Choose 9 for 9 or greater.)

$C_E = C_1 + C_2 = \frac{C_0}{2} + \frac{KC_0}{2} = \frac{(1+K)}{2} C_0 = 4C_0$ (4)

20. When the dielectric slab is inserted, is there are force on it from the capacitor?

A) No B) Yes, pulling it in C) Yes, pushing it out

21. An air-spaced parallel plate capacitor has a capacitance of 1 microfarad and a plate separation of 10^{-3} m. The charge on the capacitor is 6 nC.

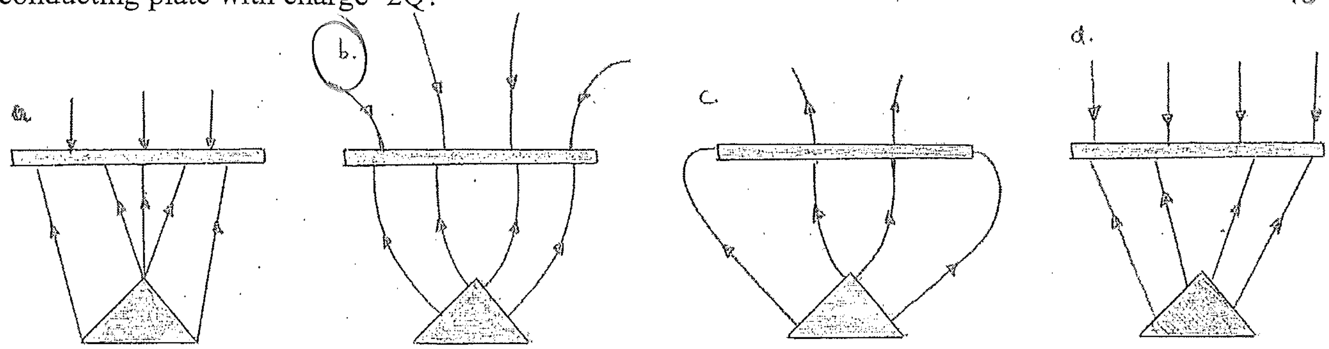
What is the force between the plates in N? (Hint: use energy conservation.)

$U = \frac{1}{2}\frac{Q^2}{C} = F\cdot d$

$F = \frac{1}{2}Q^2/dC = \frac{1}{2}\cdot 36\times10^{-9}$ N

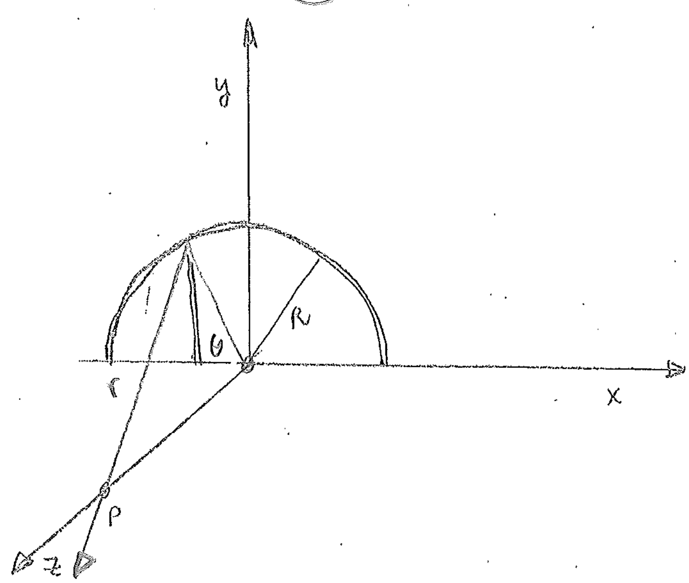
$= 18\times10^{-9}$ N.

22. What graph could show the field lines between a conducting pyramid with charge +Q and a conducting plate with charge -2Q?



23. Consider a point P located a distance z above a semicircle (radius R) of charge density λ, as shown. What field components at P are zero by symmetry?

A) none B) E_x C) E_y D) E_z E) E_x and E_y



24. What integral would you need to solve to find E_z at P (in Q23)? Choose from integrals on following page.

$\int \frac{k\,dq}{r^2}\cdot\left(\frac{z}{r}\right)$ ← z-component but r is same, so is z so $\int dq = Q$. $\boxed{A}$

25. What integral would you need to solve to find E_y at P? (in Q23)?

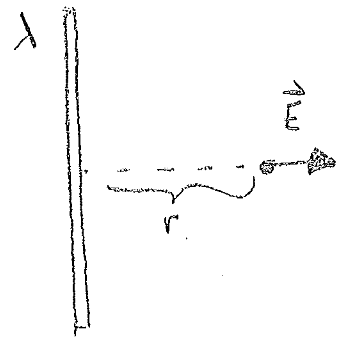
$\int \frac{k\,d\ell}{r^2}\cdot\left(\frac{y}{r}\right) = \frac{k}{r^3}\int y\,d\ell = \frac{k}{r^3}\int R\sin\theta\,d\theta\cdot\lambda$ $\boxed{B}$

↳ y-component

26. The field from a finite line of charge of length $2a$ at a distance r from its center is

$$E = \frac{kQ}{r\sqrt{r^2 + a^2}}.$$

What is the electric field from a square of side length 2a, with uniform charge density, a distance y above the center of the square? Choose the appropriate integral from the table.



Integrals to choose from for Q24, Q25, Q26
In the following, a and b are arbitrary constants.

A) $\int dq$

B) $\int \sin x dx$ or $\int \cos x dx$

C) $\int \frac{dx}{x^2 + a^2}$

D) $\int \frac{dx}{(x^2 + a^2)^{1/2}}$

E) $\int \frac{dx}{(x^2 + a^2)^{3/2}}$

F) $\int \frac{dx}{(x^2 + a^2)\sqrt{x^2 + b^2}}$

G) $\int \frac{xdx}{(x^2 + a^2)\sqrt{x^2 + b^2}}$

H) $\int \frac{x\, dx}{(x^2 + a^2)^{3/2}}$

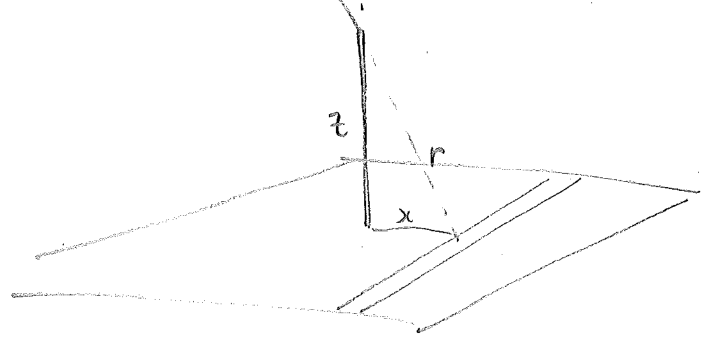
I) $\int \frac{x^2\, dx}{x^2 + a^2}$

J) $\int \frac{(x^2 + a^2)dx}{(x^2 + b^2)^{1/2}}$

$dE = \frac{k\,dq}{r\sqrt{r^2+a^2}}$

Need z-component

$dE_z = \frac{z}{r}\, dE$



So

$$E = \int dE_z = \int_{-a}^{a} \frac{\overbrace{k\cdot 2a\cdot\sigma\cdot dx}^{dq}\cdot z}{r^2\sqrt{r^2+a^2}}$$

b/c $r = z^2 + x^2$

So

$$|E| = \int_{-a}^{a} \frac{k\cdot 2a\cdot\sigma\cdot dx\cdot z}{(x^2+z^2)\sqrt{x^2+z^2+a^2}}$$ z = const.

Integral $\boxed{F}$