21.22. IDENTIFY: Apply Coulomb's law to calculate each force on $-Q$.

SET UP: Let $\vec{F}_1$ be the force exerted by the charge at $y=a$ and let $\vec{F}_2$ be the force exerted by the charge at $y=-a$. The distance between each charge q and Q is $r=\left(a^2+x^2\right)^{1/2}$. $\cos\theta=\dfrac{|x|}{\left(a^2+x^2\right)^{1/2}}$.

EXECUTE: (a) The two forces on $-Q$ are shown in Figure 21.22a.

(b) When $x>0$, F_{1x} and F_{2x} are negative. $F_x=F_{1x}+F_{2x}=-2\dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{(a^2+x^2)}\cos\theta=\dfrac{1}{4\pi\epsilon_0}\dfrac{-2qQx}{(a^2+x^2)^{3/2}}$. When $x<0$, F_{1x} and F_{2x} are positive and the same expression for F_x applies. $F_y=F_{1y}+F_{2y}=0$.

(c) At $x=0$, $F_x=0$.

(d) The graph of F_x versus x is sketched in Figure 21.22b.

EVALUATE: The direction of the net force on $-Q$ is always toward the origin.

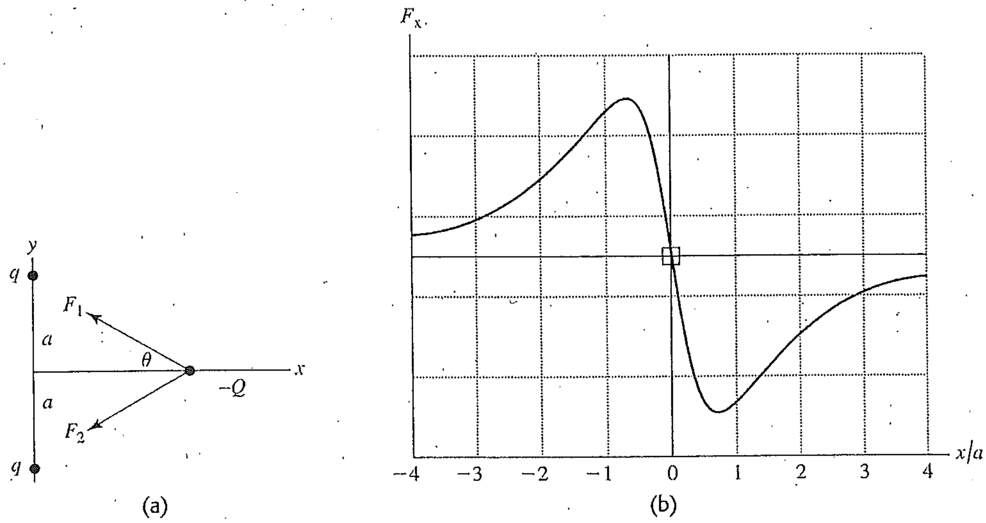
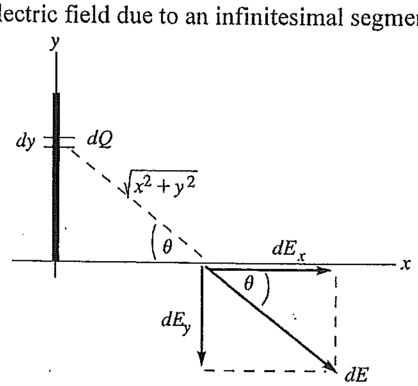


Figure 21.22

21.90. IDENTIFY: Use Eq. (21.7) to calculate the electric field due to a small slice of the line of charge and integrate as in Example 21.11. Use Eq. (21.3) to calculate $\vec{F}$.

SET UP: The electric field due to an infinitesimal segment of the line of charge is sketched in Figure 21.90.



$$\sin\theta=\frac{y}{\sqrt{x^2+y^2}}$$

$$\cos\theta=\frac{x}{\sqrt{x^2+y^2}}$$

Figure 21.90

Slice the charge distribution up into small pieces of length dy. The charge dQ in each slice is $dQ=Q(dy/a)$. The electric field this produces at a distance x along the x-axis is dE. Calculate the components of $d\vec{E}$ and then integrate over the charge distribution to find the components of the total field.

EXECUTE: $dE=\dfrac{1}{4\pi\epsilon_0}\left(\dfrac{dQ}{x^2+y^2}\right)=\dfrac{Q}{4\pi\epsilon_0 a}\left(\dfrac{dy}{x^2+y^2}\right)$

$$dE_x=dE\cos\theta=\frac{Qx}{4\pi\epsilon_0 a}\left(\frac{dy}{\left(x^2+y^2\right)^{3/2}}\right)$$

$$dE_y=-dE\sin\theta=-\frac{Q}{4\pi\epsilon_0 a}\left(\frac{ydy}{\left(x^2+y^2\right)^{3/2}}\right)$$

$$E_x = \int dE_x = -\frac{Qx}{4\pi\epsilon_0 a}\int_0^a \frac{dy}{(x^2+y^2)^{3/2}} = \frac{Qx}{4\pi\epsilon_0 a}\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]_0^a = \frac{Q}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}}$$

$$E_y = \int dEy = -\frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{ydy}{(x^2+y^2)^{3/2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^a = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right)$$

(b) $\vec{F} = q_0\vec{E}$

$$F_x = -qE_x = \frac{-qQ}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}};\ F_y = -qE_y = \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right)$$

(c) For $x >> a$, $\frac{1}{\sqrt{x^2+a^2}} = \frac{1}{x}\left(1+\frac{a^2}{x^2}\right)^{-1/2} = \frac{1}{x}\left(1-\frac{a^2}{2x^2}\right) = \frac{1}{x}-\frac{a^2}{2x^3}$

$$F_x \approx -\frac{qQ}{4\pi\epsilon_0 x^2},\ F_y \approx \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{x}+\frac{a^2}{2x^3}\right) = \frac{qQa}{8\pi\epsilon_0 x^3}$$

EVALUATE: For $x >> a$, $F_y << F_x$ and $F \approx |F_x| = \frac{qQ}{4\pi\epsilon_0 x^2}$ and $\vec{F}$ is in the $-x$-direction. For $x >> a$ the charge distribution Q acts like a point charge.

21.104. IDENTIFY: Apply Eq.(21.11) for the electric field of a disk. The hole can be described by adding a disk of charge density $-\sigma$ and radius R_1 to a solid disk of charge density $+\sigma$ and radius R_2.

SET UP: The area of the annulus is $\pi(R_2^2 - R_1^2)\sigma$. The electric field of a disk, Eq.(21.11) is $E = \frac{\sigma}{2\epsilon_0}\left[1-1/\sqrt{(R/x)^2+1}\right]$.

EXECUTE: (a) $Q = A\sigma = \pi(R_2^2 - R_1^2)\sigma$

(b) $\vec{E}(x) = \frac{\sigma}{2\epsilon_0}\left(\left[1-1/\sqrt{(R_2/x)^2+1}\right]-\left[1-1/\sqrt{(R_1/x)^2+1}\right]\right)\frac{|x|}{x}\hat{i}$. $\vec{E}(x) = \frac{-\sigma}{2\epsilon_0}\left(1/\sqrt{(R_1/x)^2+1}-1/\sqrt{(R_2/x)^2+1}\right)\frac{|x|}{x}\hat{i}$.

The electric field is in the $+x$ direction at points above the disk and in the $-x$ direction at points below the disk, and the factor $\frac{|x|}{x}\hat{i}$ specifies these directions.

(c) Note that $1/\sqrt{(R_1/x)^2+1} = \frac{|x|}{R_1}(1+(x/R_1)^2)^{-1/2} \approx \frac{|x|}{R_1}$. This gives $\vec{E}(x) = \frac{\sigma}{2\epsilon_0}\left(\frac{x}{R_1}-\frac{x}{R_2}\right)\frac{|x|^2}{x}\hat{i} = \frac{\sigma}{2\epsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)x\hat{i}$.

Sufficiently close means that $(x/R_1)^2 << 1$.

(d) $F_x = qE_x = -\frac{q\sigma}{2\epsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)x$. The force is in the form of Hooke's law: $F_x = -kx$, with $k = \frac{q\sigma}{2\epsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{q\sigma}{2\epsilon_0 m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}.$$

EVALUATE: The frequency is independent of the initial position of the particle, so long as this position is sufficiently close to the center of the annulus for $(x/R_1)^2$ to be small.