Physics 161 Midterm Exam 1 Name Solutions

Closed book Closed Notes, Calculators OK, 50 minutes. Use backs or extra sheets to **SHOW YOUR WORK.**

$Q = mc\Delta T$

$Q = mL$

$pV = nRT = Nk_BT$

$\Delta U = Q - W$

$\Delta U = nC_v\Delta T$

$C_v = \frac{3}{2}R$ monatomic ideal gas

$W_{isoT} = nRT\ln(V_f/V_i)$

$pV^\gamma = const.$

$TV^{\gamma-1} = const.$

$\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v}$

$e = \frac{W}{Q_{added}}$

$e_C = 1 - \frac{T_c}{T_H}$

$dS = \frac{dQ}{T}$

$\Delta S = mc\ln(T_f/T_i)$

Question 1. (30 pts.) Unusual water is discovered on Mars.
It melts at 0°C, but the specific heat is 4 J/g°C;
the specific heat of Martian ice is 2 J/g°C;
and the latent heat of fusion is 20 J/g.
(All at Martian atmospheric pressure.)

100 g of martian ice at -80°C is mixed with 160 g of martian water at 20°C (all at martian pressure) in a perfectly insulated vessel.

a) What is the final temperature and composition?

b) What is the overall change in entropy? Is this process reversible?

(a) ice warms / water cools

$mc\Delta T = 16{,}000\,J$ $\qquad mc\Delta T = -12{,}800\,J$

water → 0°. 3200 J still to add to ice to reach 0°C

---

ice warms more / water freezes.

3200 J → 0° $\qquad mL_f = 3200\,J$

---

final state all ice at 0°C.

(b) $\Delta S_{ice} = mc\ln\frac{T_f}{T_i} = 200\ln\frac{273}{193}$ J/K $= 69.35$ J/K

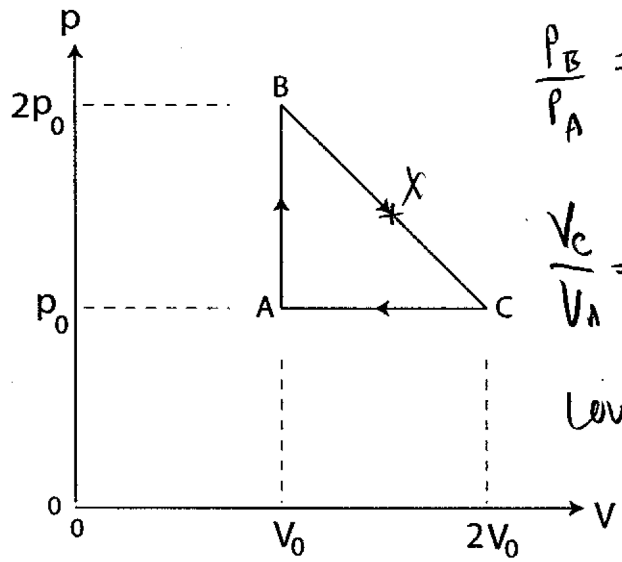
$\Delta S_{water} = mc\ln\frac{T_f}{T_i} + \frac{Q_f}{T_f}$ Freezing! $Q_f < 0.$

$= 640\ln\frac{273}{293} - \frac{3200}{273} = -56.97$ J/K.

$\Delta S_T = 12.38$ J/K irreversible

2. (70 pts) A monatomic ideal gas is reversibly taken through the cycle shown. The temperature at A is T_0.

a) What is the temperature at point B and at point C? What are the highest and lowest temperatures reached anywhere in this cycle? Express your answers in terms of T_0.



$$\frac{p_B}{p_A} = \frac{T_B}{T_A} \qquad T_B = 2T_0$$

$$\frac{V_C}{V_A} = \frac{T_C}{T_A} \qquad T_C = 2T_0$$

Lowest $T = T_0$

Highest $T = T_x$ at $p = \frac{3}{2}p_0, V = \frac{3}{2}V_0$.

$$pV = \frac{3}{2}p_0 \frac{3}{2}V_0 = \frac{9}{4}p_0V_0 = \frac{9}{4}nRT_0 = nRT$$

$$T_H = \frac{9}{4}T_0.$$

b) What is the work is done by the gas from A to B? What is the heat added to the gas from A to B? Express each in terms of p_0 and V_0.

$$W_{AB} = 0$$

$$Q_{AB} = nC_v\Delta T$$

$$= n \cdot \frac{3}{2}R \cdot T_0 = \frac{3}{2}p_0V_0 \quad (\text{Use } p_0V_0 = nRT_0)$$

c) What is the work is done by the gas from C back to A? What is the heat added to the gas from C back to A? Express each in terms of p_0 and V_0.

$$W = -p_0 V_0.$$

$$Q = nC_p\Delta T = -n\cdot\frac{5}{2}R\cdot T_0 = -\frac{5}{2}p_0V_0$$

d) What is the work done by the gas from B to C? What is the total work done in the cycle? Express in terms of p_0 and V_0.

$$W = p_0V_0 + \frac{1}{2}p_0V_0 = \frac{3}{2}p_0V_0$$

$$W_T = \frac{1}{2}p_0V_0 \quad (= W_{BC} + W_{CA})$$

e) What is the heat added to the gas from B to C? Express in terms of p_0 and V_0.

$$\Delta U = 0 \text{ so } Q = W = \frac{3}{2}p_0V_0$$

<u>OR</u> $Q_T = W_T = \frac{1}{2}p_0V_0 = Q_{AB} + Q_{CA} + Q_{BC}$

$$= \frac{3}{2}p_0V_0 - \frac{5}{2}p_0V_0 + Q_{BC}$$

$$Q_{BC} = \frac{3}{2}p_0V_0$$

f) What is the efficiency of this engine? What is the efficiency of a Carnot engine operating between the same highest and lowest temperatures? If this engine is below Carnot efficiency, explain why you couldn't run it backwards as a fridge, hook it up to the Carnot engine, and violate the 2nd law of thermodynamics.

$$e = \frac{W}{Q_{add}} = \frac{\frac{1}{2} p_0 V_0}{\frac{6}{2} p_0 V_0} = \frac{1}{6}$$

$$e_c = 1 - \frac{T_c}{T_H} = 1 - \frac{T_0}{\frac{9}{4} T_0} = 1 - \frac{4}{9} = \frac{5}{9} > e.$$

- this engine doesn't operate with only 2 temps.
- intermediate temps used.

g) What is the entropy change in the gas for the process BC? What is the total entropy change in the universe for process BC?

$\Delta S = \frac{Q_{in}}{T}$ not isothermal, but isothermal end points

$$Q_{iso} = W_{iso} = p_0 V_0 \ln 2$$

so $$\Delta S = \frac{p_0 V_0 \ln 2}{2 T_0} = nR \frac{\ln 2}{2} = 0.35\, nR.$$