Name EXAM 2 SOLUTIONS

Physics 161 Exam 2 Spring 2010 No notes, calculators OK.

pts. Problem 1. Two parallel plate capacitors are wired together as shown. The capacitors each have capacitance C. A total charge +Q is put on the top wire, -Q on the bottom wire.

5 a) What is the potential difference between points S and T? (Answer in terms of Q, C)

5 b) What is the energy stored in the capacitors? (Give your answer in terms of Q, C, ε_0)

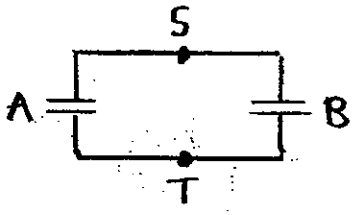
A dielectric, with dielectric constant K = 3, is now added to capacitor A, and fills the gap.

6 c) What is the potential difference between points S and T?

6 d) What is the charge on each capacitor?

6 e) What is the energy stored in the capacitor A, in capacitor B, and the total?

2 f) If your answer to (e) (total) differs from your answer to (b), explain where the energy went/ came from.



$C_{eq} = C + C = 2C$ (in parallel)

a) $V = \frac{Q}{C_{eq}} = \frac{Q}{2C}$.

b) $U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} 2C \cdot \frac{Q^2}{4C^2} = \frac{Q^2}{4C}$.

c) $C_A = 3C \quad C_B = C \quad C_{eff} = 4C \quad V = \frac{Q}{4C}$

d) $Q_A = C_A V = 3C \cdot \frac{Q}{4C} = \frac{3}{4} Q$

$Q_B = C_B V = C \cdot \frac{Q}{4C} = \frac{1}{4} Q$

e) $U_A = \frac{1}{2} \frac{Q_A^2}{C_A} = \frac{1}{2} \cdot \frac{9}{16} \frac{Q^2}{3C} = \frac{3}{32} \frac{Q^2}{C}$.

$U_B = \frac{1}{2} \frac{Q_B^2}{C_B} = \frac{1}{2} \frac{1}{16} \frac{Q^2}{C} = \frac{1}{32} \frac{Q^2}{C}$

Total $U = \frac{1}{8} \frac{Q^2}{C}$ < ans. b.

f) Dielectric is pulled in. $W = F \cdot x$.

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Problem 2. A **perfectly insulating spherical shell** has a total charge of -Q distributed uniformly throughout. The inner radius of the shell is R/2; the outer radius is R.

pts

8 a) Find the electric field for points in the cavity (r<R/2) and outside the shell (r>R). (Your answer *may* include r, R, Q, ε_0) Inside, $E=0$ by Gauss. Outside $E = \frac{1}{4\pi\varepsilon_0}\frac{-Q}{r^2}$ (toward c)

Now, a point charge +Q is placed at point P, just at the surface of the shell.

4 b) What is the electric field at the center of the shell, point C?

Superposition ⇒ only E_p contributes $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$ to the left.

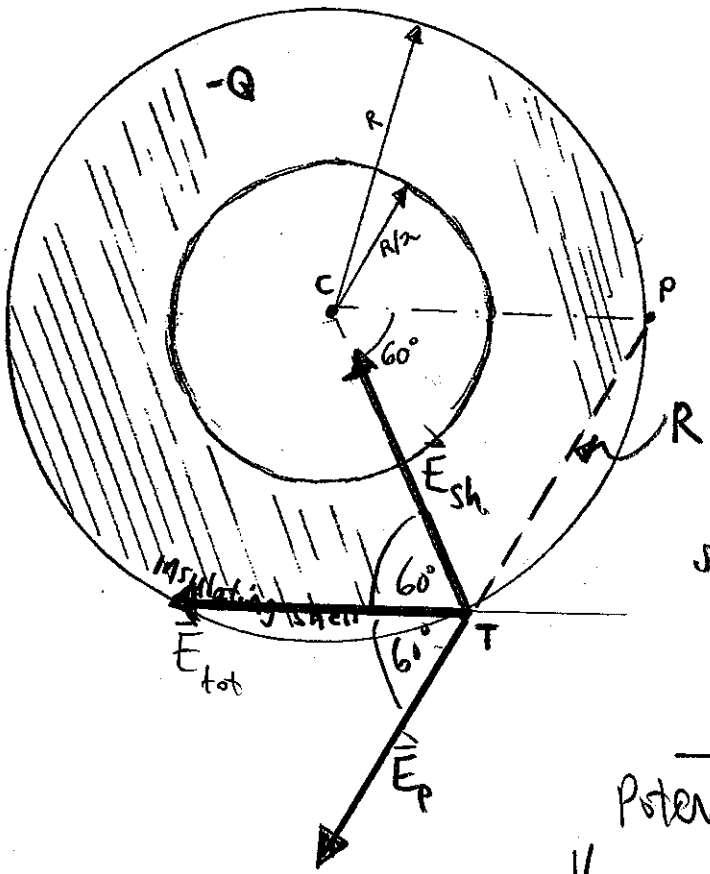
12 c) What is the electric field at point T? Show it on the diagram.

Superposition. Add the two fields shown below.

8 d) What is the electric potential at point T? (Take V=0 at ∞) See below.

3 e) A high energy photon causes an electron to be emitted from the surface at point T. The electron leaves T with a speed of 100 m/s. When the electron is very far from the sphere, what is its speed? (Your answer *may* contain the mass of the electron, m_e and the charge on the eletron –e.) Ans. $V_f = V_i = 0V$, so $K_f = K_i$

100 m/s.



$|E_p| = \frac{1}{4\pi\varepsilon_0}\frac{+Q}{R^2}$

$|E_c| = \frac{1}{4\pi\varepsilon_0}\frac{|-Q|}{R^2}$

y Components cancel

x Components $= E\cos 60°$

$= \frac{1}{2}E$

So Total $= \frac{1}{2}E + \frac{1}{2}E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

to the left.

Potential

$V_{sh} = \frac{1}{4\pi\varepsilon_0}\frac{-Q}{r}$ outside shell because E is same as for pt. charge.

By superposition, $V_{sh} + V_p = 0$ at T.

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Problem 3. A charge of +Q is situated some distance above an uncharged **conducting metal cube** as shown on the following page. There is a cubical hole in the metal cube. (The hole is completely surrounded by metal: the picture shows a cross section.)

12 a) Sketch E field lines on the diagram. Use a reasonable number of lines, say six or eight.

6 b) Show on the diagram where there is charge density (excess + or excess -) on or in the metal cube. Use + and – signs for the excess charge.

8 c) Given that point S has a potential of +4 V and point P has a potential of +2 V, sketch *(approximately)* equipotential lines for +4V, +3V, +2V, +1V and (if there are any) 0V, -1V, etc. Label each with its potential. (Use the usual convention that V=0 at infinity.)

6 d) Treating point P as the origin, the potential near P (just above the cube) is V=3y + 2, where V is in volts and y in meters. What is the electric field at P? Show on the sketch.

$$\vec{E} = -\vec{\nabla}V \Rightarrow E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

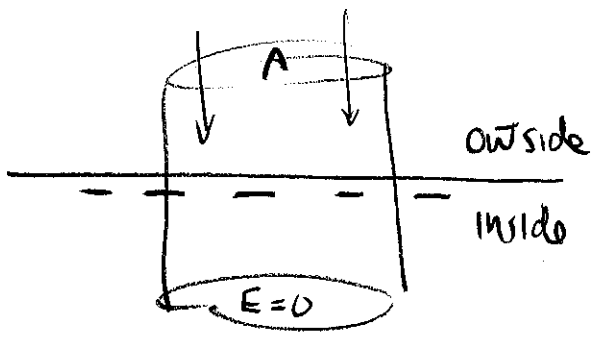
$$\vec{E} = -3\hat{j}$$

2 e) What is the surface charge density on the cube right below P? Your answer may contain ε_0.

$$\oint E\,dA = \frac{q_{enc}}{\epsilon_0} \qquad EA = \frac{q_{enc}}{\epsilon_0} \qquad \epsilon_0 E = \sigma \qquad \sigma = -3\epsilon_0$$

1 f) What is the field at point T in the hole?

0. Use Gauss' Law in metal



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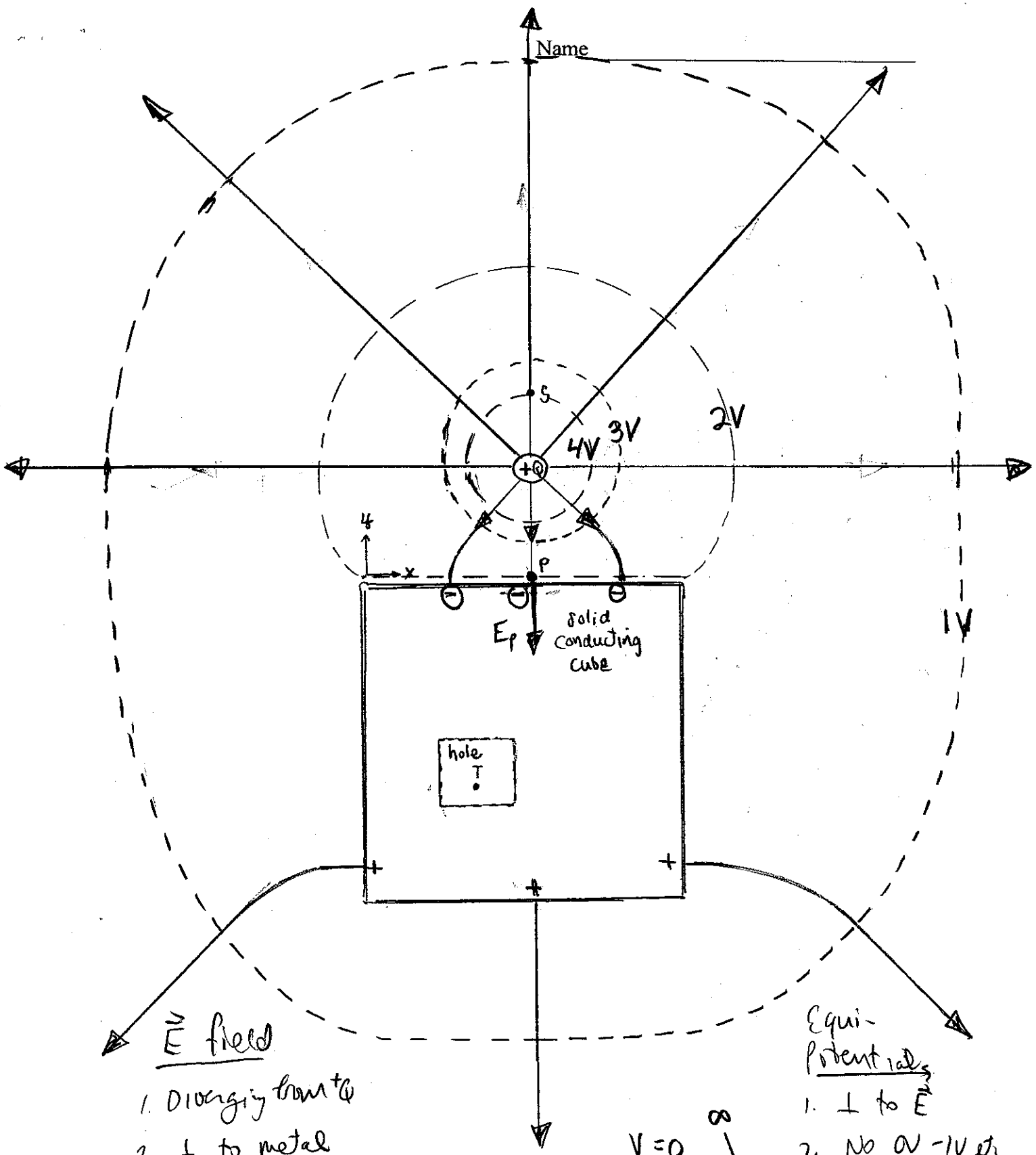

$\vec{E}$ field

1. Diverging from +Q
2. ⊥ to metal
3. Lines leave metal
4. None in metal or hole

Equi-Potentials

1. ⊥ to $\vec{E}$
2. No 0V, -1V etc
3. At most one V in metal
4. V=3 closer to 4 than 2V.

V=0 ∞