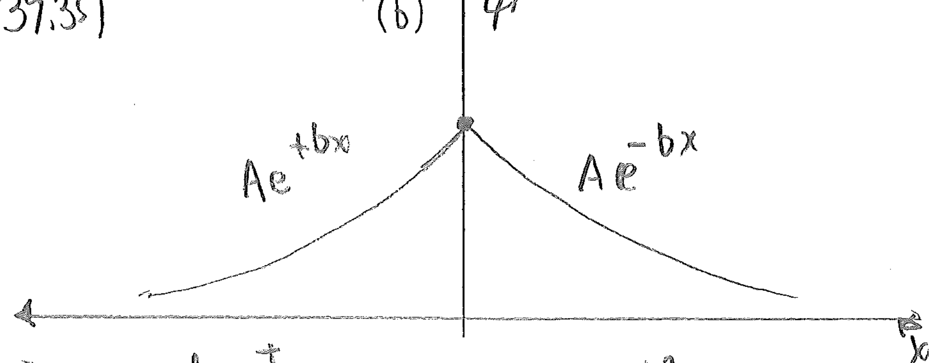
Homework 7 solutions

39.35)



(a) normalization $2\int_0^\infty |Ae^{-bx}|^2 dx = 1$

$$= 2A^2\left[\frac{1}{2b}e^{-2bx}\right]\Bigg|_\infty^0 = \frac{A^2}{b} \quad \text{so} \quad A = \sqrt{b}$$

with $b = 2\,m^{-1}$, $A = 1.414\,m^{-1/2}$.

(c)(i) $P(-50, +50\,cm)$

$$= 2\int_0^{+0.5} A^2 e^{-2bx}\,dx = 2A^2\left[\frac{1}{2b}e^{-2bx}\right]\Bigg|_{x=0.5}^{x=0} = 1 - e^{-2} = 0.865$$

(ii) $P(x<0) = 1/2$ by symmetry.

(iii) $P(x \in [0.5, 1]) = \int_{0.5}^{1} A^2 e^{-2bx}\,dx = \frac{A^2}{2b}(e^{-2} - e^{-4}) = 5.85\%$

39.36) $-\frac{\hbar^2}{2m}\frac{d^2(B\psi_1 + C\psi_2)}{dx^2} + U(B\psi_1 + C\psi_2) =$

$$-\frac{\hbar^2}{2m}B\psi_1'' + UB\psi_1 + -\frac{\hbar^2}{2m}C\psi_2'' + UC\psi_2 = \quad \left(\psi'' = \frac{d^2\psi}{dx^2}\right)$$

$$BE\psi_1 + CE\psi_2 = E(B\psi_1 + C\psi_2) \checkmark$$

39.37) If $E_1 \neq E_2$, then $BE_1\psi_1 + CE_2\psi_2 \stackrel{?}{=} E(B\psi_1 + C\psi_2)$

or $B(E_1 - E)\psi_1 + C(E_2 - E)\psi_2 \stackrel{?}{=} 0$

This can only happen if $E_1 = E_2 = E$.