HW Solution 12

November 19, 2010

40.52

(a) $\psi_1(x) = A_1 2xe^{-\alpha^2x^2/2}$

Therefore

$\frac{d\psi_1(x)}{dx} = -\alpha^2x^2A_1e^{-\alpha^2x^2/2} + 2A_1e^{-\alpha^2x^2/2}$,

$\frac{d^2\psi_1(x)}{dx^2} = -2A_1\alpha^2 2xe^{-\alpha^2x^2/2} - 2A_1\alpha^2x^2(-\alpha^2x)e^{-\alpha^2x^2/2} + 2A_1(-\alpha^2x)e^{-\alpha^2x^2/2} = [-3\alpha^2 + \alpha^4x^2]\psi_1(x)$

Equation 40.22:

$-\frac{\hbar}{2m}\frac{d^2\psi_1(x)}{dx^2} + \frac{k'x^2}{2}\psi(x) = E\psi(s)$

$-\frac{\hbar}{2m}\frac{d^2\psi_1(x)}{dx^2} + \frac{k'x^2}{2}\psi(x) = -\frac{\hbar}{2m}[-3\alpha^2 + \alpha^4x^2]\psi_1(x) + \frac{k'x^2}{2}\psi_1(x)$

$\because\ \alpha = m\omega/\hbar,\ \omega = \sqrt{\frac{k'}{m}}$

$-\frac{\hbar}{2m}\frac{d^2\psi_1(x)}{dx^2} + \frac{k'x^2}{2}\psi(x) = \frac{3\hbar\omega}{2}\psi_1(x)$

for $n = 1$, $E\psi(x) = \hbar\omega(1 + \frac{1}{2})\psi_1(x) = \frac{3\hbar\omega}{2}\psi_1(x)$

$\therefore\ \psi_1(x)$ is a solution of $-\frac{\hbar}{2m}\frac{d^2\psi_1(x)}{dx^2} + \frac{k'x^2}{2}\psi(x) = E\psi(s)$ when $n = 1$.

(b) $A_1 = \frac{1}{\sqrt{2}}(\frac{m\omega}{\hbar\pi})^{1/4}$

(c) $P(x) = |\psi_1(x)|^2 = 2(\frac{m\omega}{\hbar\pi})^{1/2}x^2e^{-m\omega x^2/\hbar} = 0\ \Rightarrow\ x = 0,\ \pm\frac{1}{\alpha}$

$\frac{dP}{dx} = A_1^2 8xe^{-\alpha^2x^2} - A_1^2 8x^3\alpha^2e^{-\alpha^2x^2}$

$\frac{d^2P}{dx^2} = A_1^2 8e^{-\alpha^2x^2} - A_1^2 40x^2\alpha^2e^{-\alpha^2x^2} - A_1^2 16x^4\alpha^4e^{-\alpha^2x^2}$

$\because\ \frac{dP(0)}{dx} = 0,\ \frac{d^2P(0)}{dx^2} > 0 \therefore$ it is the maximum.

$\because\ \frac{dP(\pm\frac{1}{\alpha})}{dx} = 0,\ \frac{d^2P(0)}{dx^2} < 0 \therefore$ they are the minimum.

40.56

(a) $\psi(x) = Asinkx$

(b) For $x > L$, the wave function must have the form of $\frac{d\psi(s)}{dx} = \frac{2m}{\hbar^2}(U_0 - E)\psi(x)$.

For the wave function to remain finite as $x \to \infty,\ C = 0.\ \kappa^2 = 2m(U_0 - E)/\hbar^2$.

(c)$AsinkL = A'sin\kappa L,\ kAsinkL = \kappa A'sin\kappa L$

$\therefore kcotkL = -\kappa.$