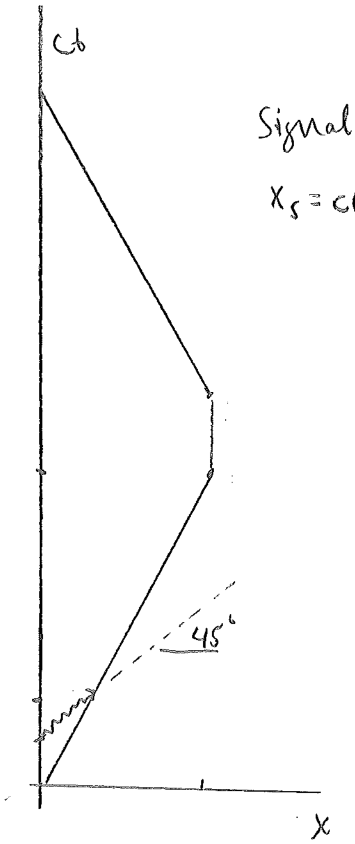
Ohanian solutions #2

(1)



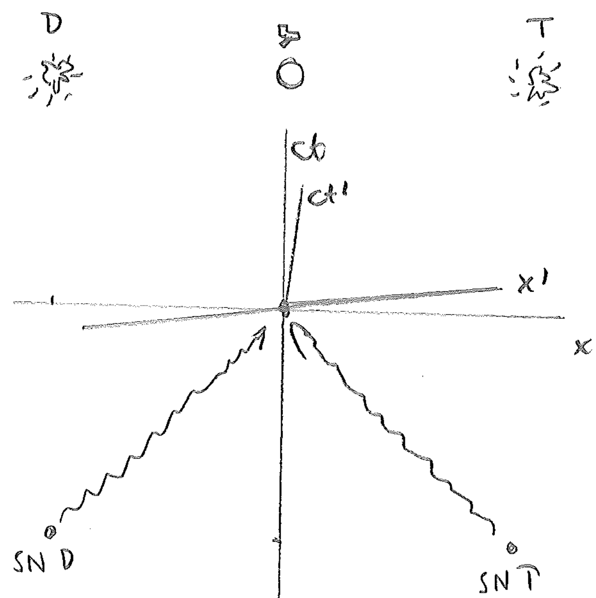
Signal reaches ship

$x_s = c(t-1) = x_{ss} = Vt = \frac{ct}{2}$

$2ct - 2c = ct$

$t = 2$ years.

(4)



Can use L.T. on two events at $ct = -1000$ ly, $x = \pm 1000$ ly

Or note that $\Delta t'$ does not depend on ct.

So $ct' = \gamma(ct - \frac{V}{c}x)$ with $t = 0$, $x = 10^3$ ly

$\gamma = 1 + \frac{1}{2}\frac{V^2}{c^2}$ (Taylor Series)

Note that we can take $\gamma \approx 1$!

The result is exactly what airplane calculates using light travel time. ⟹

$ct' = -(1 + \frac{1}{2}\frac{V^2}{c^2})\frac{V}{c}x \approx -\frac{V}{c}x = \frac{750 \times 10^3}{3600 \times 3 \times 10^8} \cdot 10^3$ ly

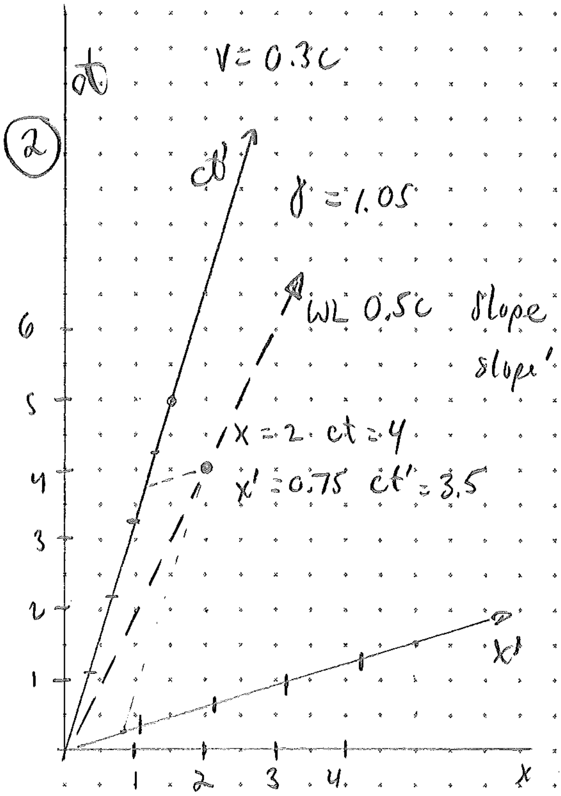
$\approx 7 \times 10^{-4}$ yrs $= 6.1$ hrs.

So Tucana is first, by 12.2 hrs.

(2)

$V = 0.3c$

$\gamma = 1.05$



WL 0.5c slope = 2 $V = c/2 = c/\text{slope}$

$\text{slope}' = \frac{3.5}{0.75} \approx 4.7$ $V' = 0.2c$

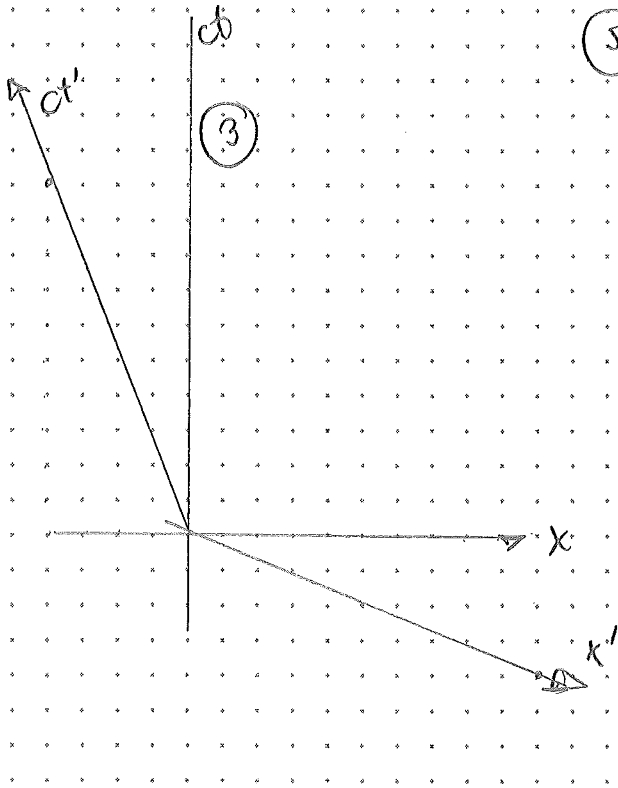
Velocity addn formula $V' = \frac{V_0 + V}{1 - V_0 V} = \frac{0.2}{0.85} = 0.24c$

Lorentz.

$x' = \gamma(x - Vt) = 1.05\cdot(2 - 0.3\cdot 4) = 0.84$

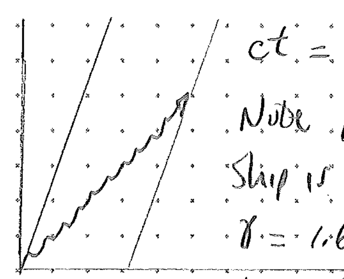
$ct' = \gamma\left(ct - \frac{Vx}{c}\right) = 1.05(4 - 0.3\cdot 2) = 3.57$

(3)



(5) a) $t = d/c = 300/3\times10^{8} = 1\,\mu s$.

b)



$ct = L_s + V_s t$

Note $L_s = L_{s0}/\gamma$

Ship is Lorentz contracted

$\gamma = 1.67$

$L_s = 180\,m$

$(c - V_s)\,t = L_s$

$t = \frac{180\,m}{0.2c} = 3\,\mu s$.

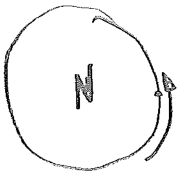
Lorentz $x' = 300\,m$

$t' = 1\,\mu s$

$t = \gamma\left(t' + \frac{Vx'}{c^2}\right) = \gamma\cdot(10^{-6} + 0.8\cdot 10^{-6})$

$= 3\,\mu s$.

(7) a)



View of N pole. Earth rotates eastward.
Eastbound signal has farther to go, since start point moves.

$t \approx \frac{2\pi r}{c} = \frac{25{,}000 \text{ mi}}{186{,}000 \text{ mi/s}} = 0.1344 \text{ s}$. During this time, Earth moves

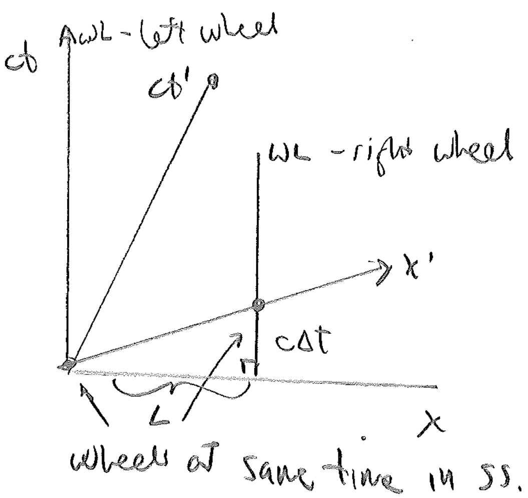
$\Delta x = \frac{25{,}000 \text{ mi}}{\text{day}} \cdot 0.1344 \text{ s} = 0.039 \text{ mi}$.

So light takes a little longer $\Delta t = \frac{\Delta x}{c} = 2.1 \times 10^{-7} \text{ s}$.

Total $\Delta t = 4.2 \times 10^{-7} \text{ s}$.

b) Earth's surface is not a purely inertial frame!

(10)



$\frac{c\Delta t}{L} = \frac{v}{c}$ from slope of x' axis.

$\Delta t = \frac{vL}{c^2}$

$\Delta\theta' = \omega\Delta t = \frac{\omega v L}{c^2}$.

(20) $\Delta s^2 = c^2\Delta t^2 - \Delta x^2 = 1.76 \times 10^{18} > 0$ timelike.
Never simultaneous.
Can be at same place, if $v = \frac{\Delta x}{\Delta t} = 2.5 \times 10^{8}$ m/s.
then $\Delta x' = 0$ so $\Delta t' = 4.42$ s.