8. Unstable particles, such as muons, decay by a probabilistic law: the probability that a particle decays within a time t is $1 - e^{-t/\tau}$, and the probability that the particle does not decay is $e^{-t/\tau}$, where τ is the **mean lifetime** of the particle. For muons, the mean lifetime is $\tau = 2.20 \times 10^{-6}$ s. Suppose that the impact of a cosmic ray on the upper atmosphere creates a muon. The muon is initially at a height of 40 km, and it has a downward speed of $0.99c$.

(a) If there were no time dilation, what would the probability be that this muon reaches the ground at sea level?

(b) Taking time dilation into account, find the actual probability.

9. Cosmonauts want to travel from the Earth to the center of our Galaxy, at a distance of 25,000 light-years. By exploiting time dilation, the cosmonauts plan to complete this trip in 10 years of their own proper time.

(a) At what constant speed, relative to the Earth, must they travel?

(b) When they reach the center of the Galaxy, how much time has elapsed on the Earth?

11. Figure 3.20 shows the worldlines of two identical clocks that move from A to B. The first clock has a straight worldline $x = vt$, where v is a constant. The second clock has the worldline $x = \frac{1}{2}at^2$, where a is a constant. For each clock, find the elapsed proper time between A and B. Which clock shows the longer proper time?

16. The light clock in Section 3.1 used a light signal bouncing between two mirrors up and down along the y' axis. Suppose that instead we use a light signal bouncing back and forth along the x' axis, between $x' = 0$ and $x' = L$. What is the time in the spaceship frame and what is the time in the laboratory frame for the forward trip? for the backward trip? for one complete round trip? Is this result in agreement with the expected time dilation?

19. Suppose you send a light signal of frequency ν toward a mirror that is approaching you at speed v. What is the frequency of the reflected light that the mirror returns to you?

21. A spaceship passes by the Sun at a speed of $0.8c$. Assume that the average wavelength of the light emitted by the Sun is 550 nm. What is the wavelength of the sunlight that the observers on the spaceship see when they are far away, approaching the Sun? when the Sun is abeam, at the point of closest approach? when they are far away, receding from the Sun?

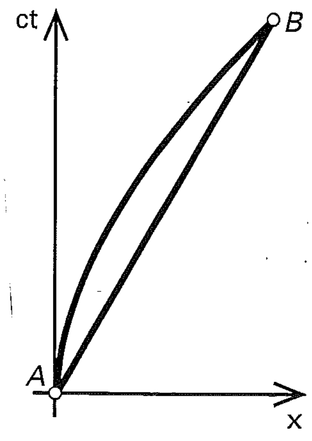

Fig. 3.20 Two worldlines connecting A and B.