Useful Equations

$$\boxed{\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta}$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{p} = m\vec{v}$$

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{\tau} = \frac{d\vec{L}}{dt}$$

$$L = I\omega \qquad I = \int r^2 dm$$

$$F_f = \mu N$$

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1. 30 pt. A toy car crosses a large turntable at constant speed v (with respect to the turntable.) The turntable is rotating at constant angular speed ω. At t=0 the car passes through the axis of rotation.

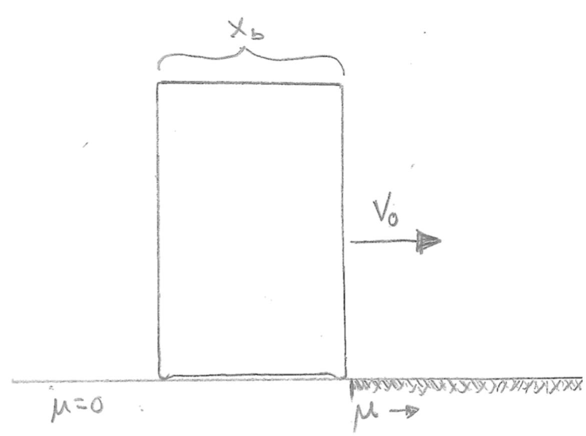
a) What is the vector acceleration of the car at time t, in polar coordinates? Express your answer in terms of the parameters given and the polar unit vectors ($\hat{r},\hat{\theta}$).

b) If the coefficient of friction is μ, find the time at which the car begins to skid. Express your answer in terms of the parameters given.

2. 18 pt. A swimmer who swims 1 m/s in still water wishes to cross a river that flows due south with a current of 0.5 m/s. The swimmer wishes to land on the western shore directly opposite her departure point.

a) In what direction should she swim?
b) If the river is 86.6 m across, how long will it take her to reach the opposite shore?

3. 18 pt. A block of uniform density slides across a frictionless surface at speed v_0, and then hits a rough patch where the coefficient of friction is μ (see figure. It might help to note that the block rides on the front and back edges, as shown.) What is the tallest the block can be without falling over? Your answer *might* contain v_0, g, μ, x_b.



4. 34 pt. A frictionless railroad car, mass M, is at rest on a very slight grade, angle α with respect to horizontal. At time t=0, the car is released. Also at time t=0, sand is released from a stationary hopper into the car at k kg/s. (You may neglect the distance the sand falls.)

a) Write a differential equation for the speed of the railroad car as it rolls down the grade while being filled with sand. Your equation *might* contain v, dv/dt, M, k, g, t, α. (You do *not* need to "separate variables.")

b) Find the speed of the railroad car at time t by a different method. (Or, if you prefer, solve the differential equation in part a.)