Physics 303 Fall 2016 Final Exam

1. A bead slides down a stationary frictionless wire shaped according to $y = \sqrt{kx}$ pulled by gravity. k = 1 m. The bead is released from x = 1 m, y = 1 m.
a) Sketch the wire and the path of the bead.

b) How long does it take to reach the origin? You may leave your answer in the form of an integral that contains only a dummy variable. Take g = 10 m/s^2.

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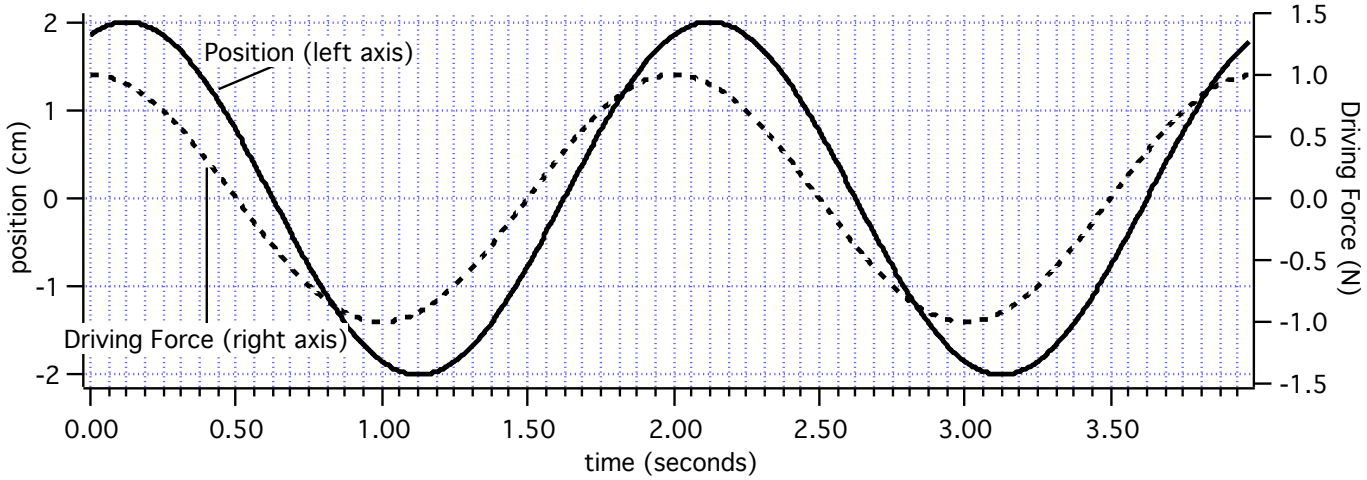
2. A mass-spring system has been driven with a Force F = F_0 cos ωt for a long time. The response is shown. The damping is β = 1.95 s^{-1}.



a. What is ω? (Figure it out from the graph.) What is the undamped resonant frequency of the oscillator?

b. What is the mass, in kilograms?

c. What is the spring constant k, in N/m?

d. The driving force is removed at t = 1.5 seconds. Add to the graph the response of the system.

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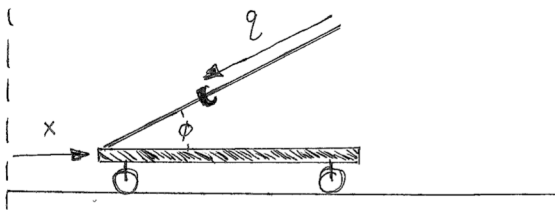
3. A bead of mass m slides without friction down a straight wire held on a cart with frictionless wheels. The mass of the cart and wire is M. The wire is at angle ϕ with respect to the horizontal.

a. Using generalized coordinates x for the cart and q for the bead, write down the kinetic energy of the system.

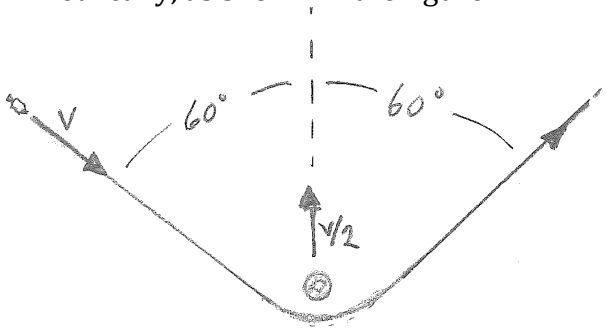
b. Write down the Lagrangian.

c. Write down and simplify the equations of motion for x and q.



d. If the wire is of length L, what is the maximum speed the cart could have when the bead reaches the bottom of the wire? (Choose whatever masses give the maximum speed.)

4. A spaceship is to be accelerated by passing behind a moving planet of mass M. In the planet's rest frame, the spaceship approaches at 60° to the direction of the planet's motion and leaves symmetrically, as shown in the figure.



a. What is the eccentricity of this hyperbolic orbit?

b. The spaceship's speed, far from the planet, is v (in the planet's frame). What is the distance of closest approach to the planet's center, in terms of G, M, and v? (Hint: what is the energy of this orbit?)

c. The planet's speed is v/2 with respect to the solar system. What is the initial speed of the spaceship with respect to the solar system? What is the final speed of the spaceship with respect to the solar system? Draw a sketch showing initial and final velocities.

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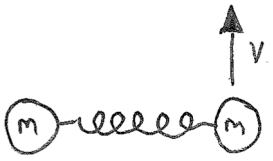
5. Two identical masses m are attached by an ideal, massless spring with unstretched length L. They are at rest on a frictionless table. The right mass is given a small initial horizontal velocity v, perpendicular to the spring, as shown at right (view from above).



a. What is the angular momentum about the center of mass of the system?

b. What is the reduced mass of this system?

c. Is there a stable separation (r) for these masses with this angular momentum? If so, what is it? (Hint: since the angular momentum is small, r will be close to L, i.e. $r = L + \delta$. Expand and keep only lowest order terms.)

d. What is the amplitude of the oscillations in r ?

e. What is the angular frequency ω_R of the small oscillations in r?