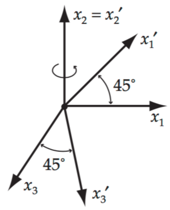
1-1.



Axes x_1' and x_3' lie in the x_1x_3 plane.

The transformation equations are:

$$x_1' = x_1 \cos 45° - x_3 \cos 45°$$

$$x_2' = x_2$$

$$x_3' = x_3 \cos 45° + x_1 \cos 45°$$

$$x_1' = \frac{1}{\sqrt{2}}x_1 - \frac{1}{\sqrt{2}}x_3$$

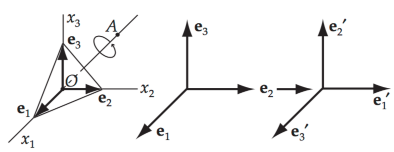
$$x_2' = x_2$$

$$x_3' = \frac{1}{\sqrt{2}}x_1 - \frac{1}{\sqrt{2}}x_3$$

So the transformation matrix is:

$$\boxed{\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}}$$

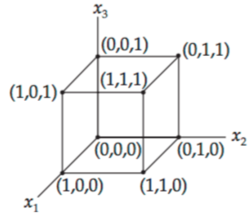
1-3.



Denote the original axes by x_1, x_2, x_3, and the corresponding unit vectors by $\mathbf{e}_1, \mathbf{e}_2$, $\mathbf{e}_3$. Denote the new axes by x_1', x_2', x_3' and the corresponding unit vectors by $\mathbf{e}_1'$, $\mathbf{e}_2'$, $\mathbf{e}_3'$. The effect of the rotation is $\mathbf{e}_1 \to \mathbf{e}_3'$, $\mathbf{e}_2 \to \mathbf{e}_1'$, $\mathbf{e}_3 \to \mathbf{e}_2'$. Therefore, the transformation matrix is written as:

$$\lambda = \begin{bmatrix} \cos(\mathbf{e}_1', \mathbf{e}_1) & \cos(\mathbf{e}_1', \mathbf{e}_2) & \cos(\mathbf{e}_1', \mathbf{e}_3) \\ \cos(\mathbf{e}_2', \mathbf{e}_1) & \cos(\mathbf{e}_2', \mathbf{e}_2) & \cos(\mathbf{e}_2', \mathbf{e}_3) \\ \cos(\mathbf{e}_3', \mathbf{e}_1) & \cos(\mathbf{e}_3', \mathbf{e}_2) & \cos(\mathbf{e}_3', \mathbf{e}_3) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

1-7.



There are 4 diagonals:

$\mathbf{D}_1$, from (0,0,0) to (1,1,1), so (1,1,1) – (0,0,0) = (1,1,1) = $\mathbf{D}_1$;

$\mathbf{D}_2$, from (1,0,0) to (0,1,1), so (0,1,1) – (1,0,0) = (–1,1,1) = $\mathbf{D}_2$;

$\mathbf{D}_3$, from (0,0,1) to (1,1,0), so (1,1,0) – (0,0,1) = (1,1,–1) = $\mathbf{D}_3$; and

$\mathbf{D}_4$, from (0,1,0) to (1,0,1), so (1,0,1) – (0,1,0) = (1,–1,1) = $\mathbf{D}_4$.

The magnitudes of the diagonal vectors are

$$|\mathbf{D}_1| = |\mathbf{D}_2| = |\mathbf{D}_3| = |\mathbf{D}_4| = \sqrt{3}$$

The angle between any two of these diagonal vectors is, for example,

$$\frac{\mathbf{D}_1 \cdot \mathbf{D}_2}{|\mathbf{D}_1||\mathbf{D}_2|} = \cos\theta = \frac{(1,1,1)\cdot(-1,1,1)}{3} = \frac{1}{3}$$

so that

$$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.5°$$

Similarly,

$$\frac{\mathbf{D}_1 \cdot \mathbf{D}_3}{|\mathbf{D}_1||\mathbf{D}_3|} = \frac{\mathbf{D}_1 \cdot \mathbf{D}_4}{|\mathbf{D}_1||\mathbf{D}_4|} = \frac{\mathbf{D}_2 \cdot \mathbf{D}_3}{|\mathbf{D}_2||\mathbf{D}_3|} = \frac{\mathbf{D}_2 \cdot \mathbf{D}_4}{|\mathbf{D}_2||\mathbf{D}_4|} = \frac{\mathbf{D}_3 \cdot \mathbf{D}_4}{|\mathbf{D}_3||\mathbf{D}_4|} = \pm\frac{1}{3}$$

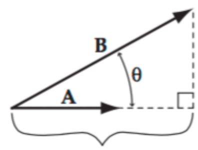
1-9. $\mathbf{A} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} \quad \mathbf{B} = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

a) $\boxed{\mathbf{A} - \mathbf{B} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k}}$

$$|\mathbf{A} - \mathbf{B}| = \left[(3)^2 + (-1)^2 + (-2)^2\right]^{1/2}$$

$$\boxed{|\mathbf{A} - \mathbf{B}| = \sqrt{14}}$$

b)



component of **B** along **A**

The length of the component of **B** along **A** is $B \cos \theta$.

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta$$

$$B \cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{A} = \frac{-2 + 6 - 1}{\sqrt{6}} = \frac{3}{\sqrt{6}} \text{ or } \frac{\sqrt{6}}{2}$$

The direction is, of course, along **A**. A unit vector in the **A** direction is

$$\frac{1}{\sqrt{6}}(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$$

So the component of **B** along **A** is

$$\boxed{\frac{1}{2}(\mathbf{i}+2\mathbf{j}-\mathbf{k})}$$

c) $$\cos\theta=\frac{\mathbf{A}\cdot\mathbf{B}}{AB}=\frac{3}{\sqrt{6}\sqrt{14}}=\frac{\sqrt{3}}{2\sqrt{7}};\ \theta=\cos^{-1}\frac{\sqrt{3}}{2\sqrt{7}}$$

$$\boxed{\theta\simeq 71°}$$

d) $$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 2 & -1\\ -2 & 3 & 1\end{vmatrix}=\mathbf{i}\begin{vmatrix}2 & -1\\ 3 & 1\end{vmatrix}-\mathbf{j}\begin{vmatrix}1 & -1\\ -2 & 1\end{vmatrix}+\mathbf{k}\begin{vmatrix}1 & 2\\ -2 & 3\end{vmatrix}$$

$$\boxed{\mathbf{A}\times\mathbf{B}=5\mathbf{i}+\mathbf{j}+7\mathbf{k}}$$

e) $$\mathbf{A}-\mathbf{B}=3\mathbf{i}-\mathbf{j}-2\mathbf{k}\qquad \mathbf{A}+\mathbf{B}=-\mathbf{i}+5\mathbf{j}$$

$$(\mathbf{A}-\mathbf{B})\times(\mathbf{A}+\mathbf{B})=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 3 & -1 & -2\\ -1 & 5 & 0\end{vmatrix}$$

$$\boxed{(\mathbf{A}-\mathbf{B})\times(\mathbf{A}+\mathbf{B})=10\mathbf{i}+2\mathbf{j}+14\mathbf{k}}$$

1-10. $\mathbf{r}=2b\sin\omega t\,\mathbf{i}+b\cos\omega t\,\mathbf{j}$

a) $$\boxed{\begin{aligned}&\mathbf{v}=\dot{\mathbf{r}}=2b\omega\cos\omega t\,\mathbf{i}-b\omega\sin\omega t\,\mathbf{j}\\ &\mathbf{a}=\dot{\mathbf{v}}=-2b\omega^2\sin\omega t\,\mathbf{i}-b\omega^2\cos\omega t\,\mathbf{j}=-\omega^2\mathbf{r}\end{aligned}}$$

$$\text{speed}=|\mathbf{v}|=\left[4b^2\omega^2\cos^2\omega t+b^2\omega^2\sin^2\omega t\right]^{1/2}$$

$$=b\omega\left[4\cos^2\omega t+\sin^2\omega t\right]^{1/2}$$

$$\boxed{\text{speed}=b\omega\left[3\cos^2\omega t+1\right]^{1/2}}$$

b) At $t=\pi/2\omega$, $\sin\omega t=1$, $\cos\omega t=0$

So, at this time, $\mathbf{v}=-b\omega\ \mathbf{j}$, $\mathbf{a}=-2b\omega^2\ \mathbf{i}$

So, $\boxed{\theta\simeq 90°}$

1-20.

a) Consider the following two cases:

When $i \neq j$ $\quad \delta_{ij} = 0$ $\quad$ but $\varepsilon_{ijk} \neq 0$.

When $i = j$ $\quad \delta_{ij} \neq 0$ $\quad$ but $\varepsilon_{ijk} = 0$.

Therefore,

$$\boxed{\sum_{ij} \varepsilon_{ijk}\, \delta_{ij} = 0}$$

b) We proceed in the following way:

When $j = k$, $\varepsilon_{ijk} = \varepsilon_{ijj} = 0$.

Terms such as $\varepsilon_{j11}\, \varepsilon_{\ell 11} = 0$. Then,

$$\sum_{jk} \varepsilon_{ijk}\, \varepsilon_{\ell jk} = \varepsilon_{i12}\, \varepsilon_{\ell 12} + \varepsilon_{i13}\, \varepsilon_{\ell 13} + \varepsilon_{i21}\, \varepsilon_{\ell 21} + \varepsilon_{i31}\, \varepsilon_{\ell 31} + \varepsilon_{i32}\, \varepsilon_{\ell 32} + \varepsilon_{i23}\, \varepsilon_{\ell 23}$$

Now, suppose $i = \ell = 1$, then,

$$\sum_{jk} = \varepsilon_{123}\, \varepsilon_{123} + \varepsilon_{132}\, \varepsilon_{132} = 1 + 1 = 2$$

for $i = \ell = 2$, $\sum_{jk} = \varepsilon_{213}\, \varepsilon_{213} + \varepsilon_{231}\, \varepsilon_{231} = 1 + 1 = 2$. For $i = \ell = 3$, $\sum_{jk} = \varepsilon_{312}\, \varepsilon_{312} + \varepsilon_{321}\, \varepsilon_{321} = 2$. But $i = 1$, $\ell = 2$ gives $\sum_{jk} = 0$. Likewise for $i = 2,\ \ell = 1$; $i = 1,\ \ell = 3$; $i = 3,\ \ell = 1$; $i = 2,\ \ell = 3$; $i = 3,\ \ell = 2$.

Therefore,

$$\boxed{\sum_{j,k} \varepsilon_{ijk}\, \varepsilon_{\ell jk} = 2\delta_{i\ell}} \tag{2}$$

c) $\sum_{ijk} \varepsilon_{ijk}\, \varepsilon_{ijk} = \varepsilon_{123}\, \varepsilon_{123} + \varepsilon_{312}\, \varepsilon_{312} + \varepsilon_{321}\, \varepsilon_{321} + \varepsilon_{132}\, \varepsilon_{132} + \varepsilon_{213}\, \varepsilon_{213} + \varepsilon_{231}\, \varepsilon_{231}$

$$= 1 \cdot 1 + 1 \cdot 1 + (-1) \cdot (-1) + (-1) \cdot (-1) + (-1) \cdot (-1) + (1) \cdot (1)$$

or,

$$\boxed{\sum_{ijk} \varepsilon_{ijk}\, \varepsilon_{ijk} = 6} \tag{3}$$

1-22. To evaluate $\sum_k \varepsilon_{ijk}\, \varepsilon_{\ell mk}$ we consider the following cases:

a) $i = j: \sum_k \varepsilon_{ijk}\, \varepsilon_{\ell mk} = \sum_k \varepsilon_{iik}\, \varepsilon_{\ell mk} = 0$ for all i, ℓ, m

b) $i = \ell: \sum_k \varepsilon_{ijk}\, \varepsilon_{\ell mk} = \sum_k \varepsilon_{ijk}\, \varepsilon_{imk} = 1$ for $j = m$ and $k \neq i, j$

$= 0$ for $j \neq m$

c) $i = m: \sum_k \varepsilon_{ijk}\, \varepsilon_{\ell mk} = \sum_k \varepsilon_{ijk}\, \varepsilon_{\ell ik} = 0$ for $j \neq \ell$

$= -1$ for $j = \ell$ and $k \neq i, j$

d) $j = \ell: \sum_k \varepsilon_{ijk}\, \varepsilon_{\ell mk} = \sum_k \varepsilon_{ijk}\, \varepsilon_{jmk} = 0$ for $m \neq i$

$= -1$ for $m = i$ and $k \neq i, j$

e) $j = m: \sum_k \varepsilon_{ijk}\, \varepsilon_{\ell mk} = \sum_k \varepsilon_{ijk}\, \varepsilon_{\ell jk} = 0$ for $i \neq \ell$

$$= 1 \text{ for } i = \ell \text{ and } k \neq i, j$$

f) $\ell = m: \sum_k \varepsilon_{ijk}\, \varepsilon_{\ell mk} = \sum_k \varepsilon_{ijk}\, \varepsilon_{\ell\ell k} = 0$ for all i, j, k

g) $i \neq \ell$ or m: This implies that $i = k$ or $i = j$ or $m = k$.

Then, $\sum_k \varepsilon_{ijk}\, \varepsilon_{\ell mk} = 0$ for all i, j, ℓ, m

h) $j \neq \ell$ or m: $\sum_k \varepsilon_{ijk}\, \varepsilon_{\ell mk} = 0$ for all i, j, ℓ, m

Now, consider $\delta_{i\ell}\, \delta_{jm} - \delta_{im}\, \delta_{j\ell}$ and examine it under the same conditions. If this quantity behaves in the same way as the sum above, we have verified the equation

$$\sum_k \varepsilon_{ijk}\, \varepsilon_{\ell mk} = \delta_{i\ell}\, \delta_{jm} - \delta_{im}\, \delta_{j\ell}$$

a) $i = j: \delta_{i\ell}\, \delta_{im} - \delta_{im}\, \delta_{i\ell} = 0$ for all i, ℓ, m

b) $i = \ell: \delta_{ii}\, \delta_{jm} - \delta_{im}\, \delta_{ji} = 1$ if $j = m$, $i \neq j, m$

$$= 0 \text{ if } j \neq m$$

c) $i = m: \delta_{i\ell}\, \delta_{ji} - \delta_{ii}\, \delta_{j\ell} = -1$ if $j = \ell$, $i \neq j, \ell$

$$= 0 \text{ if } j \neq \ell$$

d) $j = \ell: \delta_{i\ell}\, \delta_{\ell m} - \delta_{im}\, \delta_{\ell\ell} = -1$ if $i = m$, $i \neq \ell$

$$= 0 \text{ if } i \neq m$$

e) $j = m: \delta_{i\ell}\, \delta_{mm} - \delta_{im}\, \delta_{m\ell} = 1$ if $i = \ell$, $m \neq \ell$

$$= 0 \text{ if } i \neq \ell$$

f) $\ell = m: \delta_{i\ell}\, \delta_{j\ell} - \delta_{il}\, \delta_{j\ell} = 0$ for all i, j, ℓ

g) $i \neq \ell, m: \delta_{i\ell}\, \delta_{jm} - \delta_{im}\, \delta_{j\ell} = 0$ for all i, j, ℓ, m

h) $j \neq \ell, m: \delta_{i\ell}\, \delta_{jm} - \delta_{im}\, \delta_{i\ell} = 0$ for all i, j, ℓ, m

Therefore,

$$\boxed{\sum_k \varepsilon_{ijk}\, \varepsilon_{\ell mk} = \delta_{i\ell}\, \delta_{jm} - \delta_{im}\, \delta_{j\ell}}$$

Using this result we can prove that

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$$

First $(\mathbf{B}\times\mathbf{C})_i = \sum_{jk} \varepsilon_{ijk} B_j C_k$. Then,

$$\begin{aligned}
\left[\mathbf{A}\times(\mathbf{B}\times\mathbf{C})\right]_\ell &= \sum_{mn} \varepsilon_{\ell mn} A_m (B\times C)_n = \sum_{mn} \varepsilon_{\ell mn} A_m \sum_{jk} \varepsilon_{njk} B_j C_k \\
&= \sum_{jkmn} \varepsilon_{\ell mn}\, \varepsilon_{njk}\, A_m B_j C_k = \sum_{jkmn} \varepsilon_{\ell mn}\, \varepsilon_{jkn}\, A_m B_j C_k \\
&= \sum_{jkm} \left(\sum_n \varepsilon_{lmn}\, \varepsilon_{jkn} \right) A_m B_j C_k \\
&= \sum_{jkm} \left(\delta_{jl}\delta_{km} - \delta_{k\ell}\delta_{jm} \right) A_m B_j C_k \\
&= \sum_m A_m B_\ell C_m - \sum_m A_m B_m C_\ell = B_\ell \left(\sum_m A_m C_m \right) - C_\ell \left(\sum_m A_m B_m \right) \\
&= (\mathbf{A}\cdot\mathbf{C}) B_\ell - (\mathbf{A}\cdot\mathbf{B}) C_\ell
\end{aligned}$$

Therefore,

$$\boxed{\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C}}$$