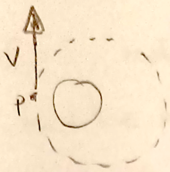
8.18



$V_p = 8500\ \mathrm{m/s}$

$r_p = R_e + 250\,\mathrm{km} = 6.65\times10^6\ \mathrm{m}$

$\vec{l} = \vec{r}\times\vec{p}$ so $l = r_p V_p = 5.6525\times10^{10}\ \frac{m^2}{s}\times M_{sat}$

$\alpha = \dfrac{l^2}{\gamma\mu}$, $\gamma = G M_{sat} M_e$ so $\alpha = \dfrac{3.195\times10^{21}\ \cancel{M_{sat}^2}}{G M_e\ \cancel{M_{sat}\mu}}$ ← $\mu \approx M_{sat}$

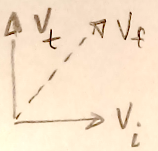
Now $G M_e = g R_e^2 = 9.8\ \mathrm{m/s^2}\cdot(6.4\times10^6\ \mathrm{m})^2 = 4.014\times10^{14}$

so $\alpha = 7.96\times10^6\ \mathrm{m}$.

$r_p = \dfrac{\alpha}{1+\varepsilon} \Rightarrow \varepsilon = \dfrac{\alpha - r_p}{r_p} = 0.197$

Apogee $r_a = \dfrac{\alpha}{1-\varepsilon} = 9.913\times10^6\ \mathrm{m}$ Height $= 3.513\times10^6\ \mathrm{m}$.
351 km.

8-9.



$\vec{V}_f = \vec{V}_i + \vec{V}_t$ ← thrust velocity added

$|V_f| = \sqrt{2}\,|V_i| \quad T_f = 2T_i$

Since initial orbit is circular with $T_i = \frac{1}{2}|U|$, after thrust $T_f = |U|$ so $T_f + U = E_f = 0$. Parabolic orbit.

Angular momentum is unchanged, since thrust provides no torque

b) E is conserved, so $E(r) = 0$.

$U(r) = -\dfrac{G M_e M_s}{r}$ and $T(r) = +\dfrac{G M_e M_s}{r}$

$V(r) = U(r) + \dfrac{l^2}{2\mu r^2} \quad l = \mu r_0 V$ so $V(r) = \dfrac{-GMm}{r} + \dfrac{m r_0 V}{2r^2}$

Since $E = 0$, motion is parabolic.
Can you find the perihelion of the parabola? (The satellite doesn't get there, since it is before the thrusters fired)

8-14. $r = k\theta^2$. Since $\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{\ell^2} F(r)$, we have

$$\frac{d}{d\theta}\left(\frac{1}{k\theta^2}\right) = \frac{-2}{k}\frac{1}{\theta^3} \qquad \frac{d^2}{d\theta^2}\left(\frac{1}{k\theta^2}\right) = \frac{6}{k}\frac{1}{\theta^4} = \frac{6k}{r^2}$$

so

$$\frac{6k}{r^2} + \frac{1}{r} = -\frac{\mu r^2}{\ell^2} F(r) \qquad F(r) = -\frac{\ell^2}{\mu}\left[\frac{6k}{r^4} + \frac{1}{r^3}\right].$$

Note that the force depends on ℓ, so not <u>all</u> orbits are spirals $r = k\theta^2$.

8-19. The semimajor axis of an orbit is defined as one-half the sum of the two apsidal distances, $r_{\max}$ and $r_{\min}$ [see Eq. (8.44)], so

$$\frac{1}{2}\left[r_{\max} + r_{\min}\right] = \frac{1}{2}\left[\frac{\alpha}{1+\varepsilon} + \frac{\alpha}{1-\varepsilon}\right] = \frac{\alpha}{1-\varepsilon^2} \tag{1}$$

This is the same as the semimajor axis defined by Eq. (8.42). Therefore, by using Kepler's Third Law, we can find the semimajor axis of Ceres in astronomical units:

$$\frac{a_C}{a_E} = \left[\frac{\dfrac{k_C}{4\pi^2\mu_C}\tau_C^2}{\dfrac{k_E}{4\pi^2\mu_E}\tau_E^2}\right] \tag{2}$$

where $k_c = \gamma M_s m_c$, and

$$\frac{1}{\mu_c} = \frac{1}{M_s} + \frac{1}{m_c}$$

Here, M_s and m_c are the masses of the sun and Ceres, respectively. Therefore, (2) becomes

$$\frac{a_C}{a_E} = \left[\frac{M_s + m_c}{M_s + m_e}\left[\frac{\tau_c}{\tau_E}\right]^2\right]^{1/3} \tag{3}$$

from which

$$\frac{a_C}{a_E} = \left[\frac{333,480 + \dfrac{1}{8,000}}{333,480 + 1}(4.6035)^2\right]^{1/3} \tag{4}$$

so that

$$\boxed{\frac{a_C}{a_E} \cong 2.767} \tag{5}$$

The period of Jupiter can also be calculated using Kepler's Third Law:

$$\frac{\tau_J}{\tau_E} = \left[\frac{\dfrac{4\pi^2\mu_J}{k_J}a_J^3}{\dfrac{4\pi^2\mu_E}{k_E}a_E^3}\right]^{1/2} = \left[\frac{M_s + m_E}{M_s + m_J}\left[\frac{a_J}{a_E}\right]^3\right]^{1/2} \tag{6}$$

from which

$$\frac{\tau_J}{\tau_E} = \left[\frac{333,480 + 1}{333,480 + 318.35}(5.2028)^3\right]^{1/2} \tag{7}$$

Therefore,

$$\boxed{\frac{\tau_J}{\tau_E} \cong 11.862} \tag{8}$$

The mass of Saturn can also be calculated from Kepler's Third law, with the result

$$\boxed{\frac{m_s}{m_e} \cong 95.3} \tag{9}$$

8-27. By conservation of angular momentum

$$mr_a v_a = mr_p v_p$$

$$or \qquad v_a = \frac{r_p v_p}{r_a}$$

Substituting gives

$$\boxed{v_a = 1608 \text{ m/s}}$$

8-35. If we write the radial distance r as

$$r = \rho + x, \qquad \rho = \text{const.} \tag{1}$$

then x obeys the oscillatory equation [see Eqs. (8.88) and (8.89)]

$$\ddot{x} + \omega_0^2 x = 0 \tag{2}$$

where

$$\omega_0 = \sqrt{\frac{3g(\rho)}{\rho} + g'(\rho)} \tag{3}$$

The time required for the radius vector to go from any maximum value to the succeeding minimum value is

$$\Delta t = \frac{\tau_0}{2} \tag{4}$$

where $\tau_0 = \dfrac{2\pi}{\omega_0}$, the period of x. Thus,

$$\Delta t = \frac{\pi}{\omega_0} \tag{5}$$

The angle through which the particle moves during this time interval is

$$\phi = \omega \Delta t = \frac{\pi \omega}{\omega_0} \tag{6}$$

where ω is the angular velocity of the orbital motion which we approximate by a circular motion. Now, under the force $F(r) = -\mu g(r)$, ω satisfies the equation

$$\mu \rho \omega^2 = -F(r) = \mu g(\rho) \tag{7}$$

Substituting (3) and (7) into (6), we find for the apsidal angle

$$\phi = \frac{\pi \omega}{\omega_0} = \frac{\pi \sqrt{\dfrac{g(\rho)}{\rho}}}{\sqrt{\dfrac{3g(\rho)}{\rho} + g'(\rho)}} = \frac{\pi}{\sqrt{3 + \rho \dfrac{g'(\rho)}{g(\rho)}}} \tag{8}$$

Using $g(r) = \dfrac{k}{\mu} \dfrac{1}{r^n}$, we have

$$\frac{g'(\rho)}{g(\rho)} = -\frac{n}{\rho} \tag{9}$$

Therefore, (8) becomes

$$\boxed{\phi = \pi / \sqrt{3 - n}} \tag{10}$$

In order to have the closed orbits, the apsidal angle must be a rational fraction of 2π. Thus, n must be

$$n = 2, -1, -6, \ldots$$

n = 2 corresponds to the inverse-square-force and n = –1 corresponds to the harmonic oscillator force.