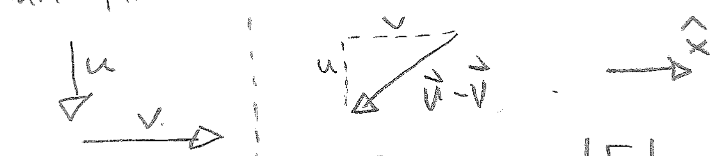
1. Car in cross wind.

Plan: find wind in car's frame, then force. Take x-component of force

a).



Earth frame : Car frame $\quad |F_d| = b|(\vec{u}-\vec{v})^2| = b(u^2+v^2)$

the $\hat{x}$ component is $F_x = -b(u^2+v^2)\dfrac{v}{\sqrt{u^2+v^2}}$ by sim. triangles.

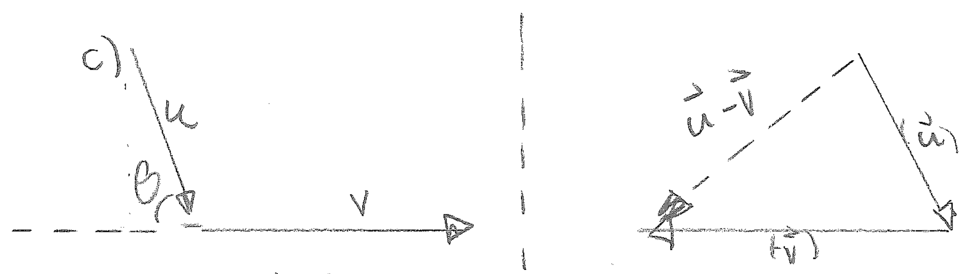
$$F_x = -b\sqrt{u^2+v^2}\cdot v$$

So gas mileage goes down even if wind is 90° to travel!

b) $|F_d| = b(u^2+v^2)^{1/2}$. Low Reynolds #, $F \propto |\vec{v}|$

$$F_x = -b(u^2+v^2)^{1/2}\cdot\frac{v}{\sqrt{u^2+v^2}} = -bv. \quad \text{Independent of crosswind!}$$

c)



Earth frame $\qquad$ Car frame

Split $\vec{u}$ into components, $u_x = u\cos\theta$, $u_y = u\sin\theta$

then $\vec{u}' = (u\cos\theta - v)\hat{x} + (u\sin\theta)\hat{y}$

$u'^2 = u^2c^2 - 2uvc + v^2 + u^2s^2 \qquad s = \sin\theta,\ c = \cos\theta.$

$\quad = u^2 - 2uvc + v^2$

The x-component of the force is $b u'^2 \cdot \dfrac{u'_x}{u'} = b(u^2 - 2uvc + v^2)^{1/2}(uc - v)$

We desire the angle where this force $= bv^2$, the same as no wind

Rename $u \to u/v$, square both sides, divide by v^4 to get

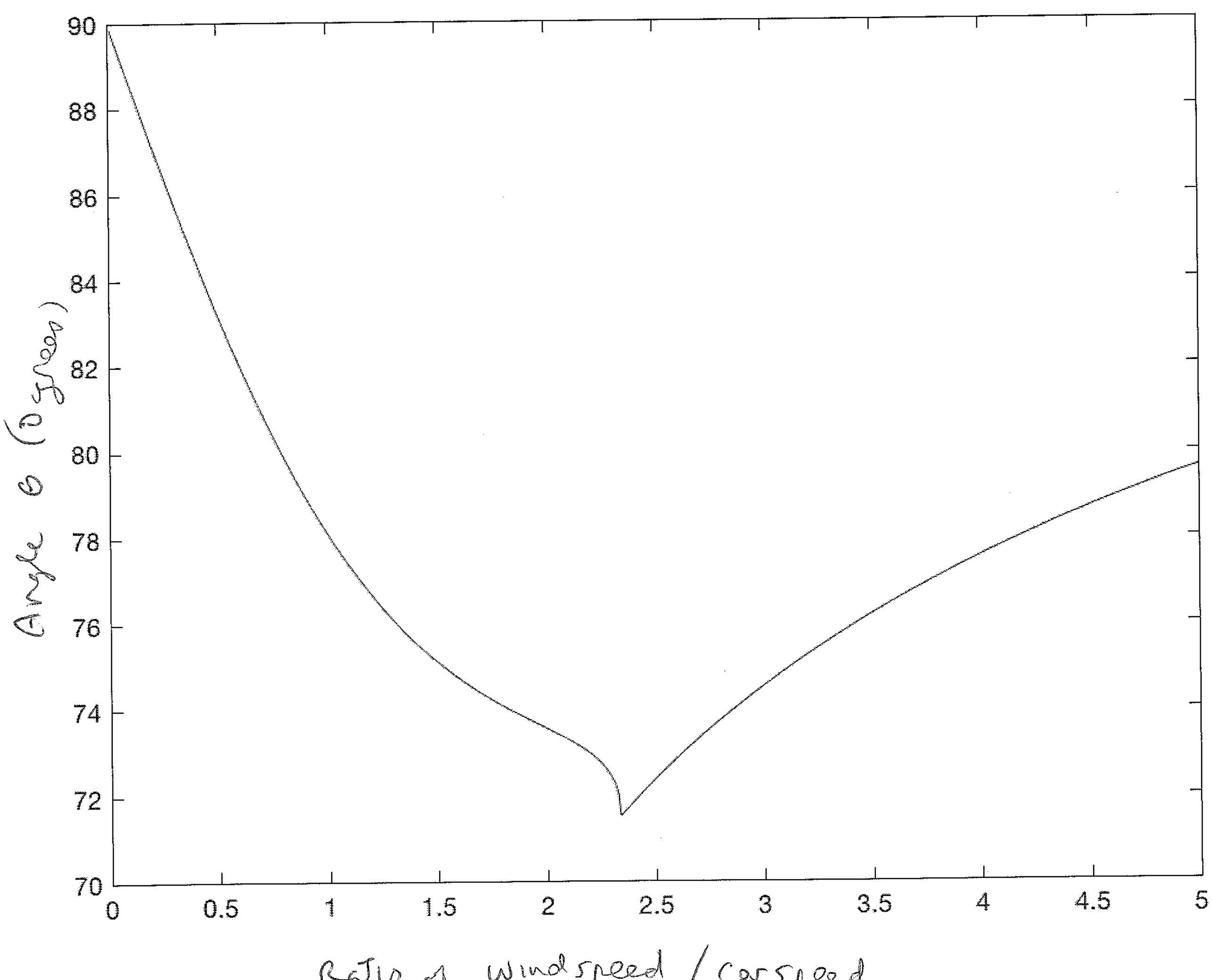
$$(u^2 - 2uc + 1)(uc - 1)^2 = 1$$

$$(u^2 - 2uc + 1)(u^2c^2 - 2uc + 1) = 1$$

$$-2u^3c^3 + 4u^2c^2 + u^2c^2 + u^4c^2 + -2u^3c - 2uc - 2uc + u^2 + \cancel{1} = \cancel{1}\ 0$$

$$(-2u^3)c^3 + (u^4 + 5u^2)c^2 - (2u^3 + 4u)c + u^2 = 0$$

use matlab to solve for $c = \cos\theta$, then find θ.



Projectile - Linear Resistance

$$m\ddot{x} = -b\dot{x} \quad \xrightarrow{\text{or}} \quad m\dot{v}_x = -bv_x$$

$$\int m\frac{dv}{v} = \int -b\,dt$$

$$\ln v = -\frac{t}{\tau} + c \qquad v = v_{0x}e^{-t/\tau} \qquad \tau \equiv m/b.$$

$$\frac{dx}{dt} = v_{0x}e^{-t/\tau} \rightarrow x = -v_{0x}\tau e^{-t/\tau} + c$$

Take $x=0$ at $t=0$, $\quad x = v_{0x}\tau\left(1-e^{-t/\tau}\right)$ ①

$$m\ddot{y} = -mg - b\dot{y}$$

Define $v_t = \frac{mg}{b}$. Then $m\dot{v}_y = -b(v_y - v_t)$

terminal speed

$$v_y - v_t = Ce^{-t/\tau}.$$

When $t=0$, $v_y = v_{0y}$, so $C = v_{0y} - v_t$.

$$v_y = v_{0y}e^{-t/\tau} + v_t\left(1-e^{-t/\tau}\right)$$

Find $y(t)$ by integration $y = (v_{0y}+v_t)\tau\left(1-e^{-t/\tau}\right) - v_t t$. ②

Solve ① for t: $\frac{x}{v_{0x}\tau} = 1 + e^{-t/\tau} \rightarrow e^{-t/\tau} = 1 - \frac{x}{v_{0x}\tau}$

$$-\frac{t}{\tau} = \ln\left(1-\frac{x}{v_{0x}\tau}\right)$$

Then substitute in y ② to get $y(x)$.

$$y = (v_{0y}+v_t)\tau\frac{x}{v_{0x}\tau} + v_t\tau\ln\left(1-\frac{x}{v_{0x}\tau}\right).$$

Range $R = x$ when $y = 0$.

$$0 = \frac{(V_{oy} + V_t)}{V_{ox}} R + V_t \tau \ln\left(1 - \frac{R}{V_{ox}\tau}\right)$$

If resistance is small, $V_{ox}\tau$ is large compared with R.
(large $\tau \equiv$ small resistance)

$$\ln(1-\epsilon) = -\epsilon - \frac{1}{2}\epsilon^2 - \frac{1}{3}\epsilon^3 \qquad \epsilon = \frac{R}{V_{ox}\tau}. \qquad \text{Taylor series}$$

$$\left[\frac{V_{oy} + \cancel{V_t}}{V_{ox}}\right] R - V_t \tau \left[\cancel{\frac{R}{V_{ox}\tau}} + \frac{1}{2}\left(\frac{R}{V_{ox}\tau}\right)^2 + \frac{1}{3}\left(\frac{R}{V_{ox}\tau}\right)^3\right] = 0$$

Cancel

Divide by R (R=0 is a trivial solution)

$$\frac{V_{oy}}{V_{ox}} - V_t \tau \left[\frac{1}{2}\frac{R}{(V_{ox}\tau)^2} + \frac{1}{3}\frac{R^2}{(V_{ox}\tau)^3}\right] = 0$$

Isolate R

$$R = \frac{2}{V_t \tau}\frac{V_{oy}}{V_{ox}} \cdot (V_{ox}\tau)^2 - \frac{1}{3}\frac{R^2}{V_{ox}\tau} \times 2.$$

Now $\frac{\tau}{V_t} = \frac{m}{b} \cdot \frac{b}{mg} = \frac{1}{g}$

so

$$R = \frac{2 V_{ox} V_{oy}}{g} - \frac{2R^2}{3 V_{ox}\tau} \qquad R_{vac} = \frac{2 V_{ox} V_{oy}}{g}$$

Now $\frac{2R^2}{3V_{0x}T}$ is the small correction to the vacuum range.

So use R_{vac} in this term to get

$$R = R_{vac} - \frac{2}{3V_{0x}T} R_{vac}^2$$

$$= R_{vac}\left(1 - \frac{4}{3}\frac{V_{y0}}{V_t}\right)$$

If V_{y0} is anything close to V_t, this approximation is bad!

3. $F_{net} = 500(e^{-t/20} - e^{-t/10})$ $\quad m = 1000\text{ kg}$

(t in sec.) $\quad F = m\frac{dV}{dt}$ so $\int F\,dt = m\Delta V$

$$= 500\int_0^t (e^{-t/20} - e^{-t/10})\,dt = 1000 \cdot V(t)$$

$$= 5\left(-20e^{-t/20} + 10e^{-t/10}\right)\Big|_0^t = V(t)$$

$$= 5e^{-t/10} - 10e^{-t/20} + 5 = V(t)$$

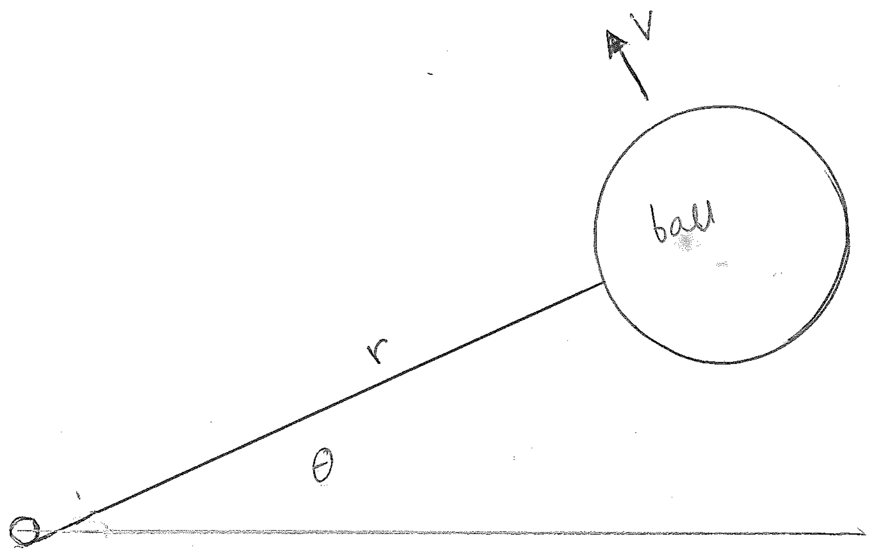
then $\quad x = \int v\,dt = -50e^{-t/10} + 200e^{-t/20} + 5t\Big|_0^t$

$$x(t) = 200e^{-t/20} - 50e^{-t/10} + 5t - 150. \quad \text{(in meters)}$$

Team A wins, of course.

$V_f = 5$ m/s. But we ignore friction and assume it takes no force to move at constant speed, through air and over ground.

4.



a) If ∡ momentum conserved $rV = \text{constant}$ ∴ $V \propto \frac{1}{r}$.
if kinetic energy is conserved $V = \text{constant}$.
They can't both be true.

b) yes, there is a torque $= -Tb$ b = radius of pole.

c) Centripetal force $= \frac{mV^2}{r} = m\omega^2 r$ $T = -m\omega^2 r b$.

$$L = I\omega = mr^2\omega$$

$$\dot{L} = \cancel{m}[2r\dot{r}\omega + r^2\dot{\omega}] = -\cancel{m}\omega^2 rb.$$

Now $r = r_0 - \theta b$... the string gets shorter as it wraps.

So $\dot{r} = -\dot{\theta} b = -\omega b$.

Use to eliminate b from R.H.S.

$$2r\dot{r}\omega + r^2\dot{\omega} = +\omega r\dot{r}$$

$$r^2\dot{\omega} = -r\dot{r}\omega$$

$$\frac{\dot{\omega}}{\omega} = \frac{-\dot{r}}{r}$$

$$\int\frac{d\omega}{\omega} = \int -\frac{dr}{r} \Rightarrow \frac{r}{r_0} = \frac{\omega_0}{\omega}.$$

$r\omega = \text{constant} = V$.
Kinetic Energy is conserved

Note: I tried to solve using $L = mvr$, $\dot{L} = m(\dot{v}r + v\dot{r})$.
That may work, but you may not write $\dot{r} = -\omega b$ or $\dot{r} = -\frac{v}{r}b$. That would be true for circular motion, but not for the in-spiraling ball.

In other words, $\dot{\theta} \neq \frac{v}{r}$ for the spiral.

4.11 Hard way to solve $F = \frac{dp}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$

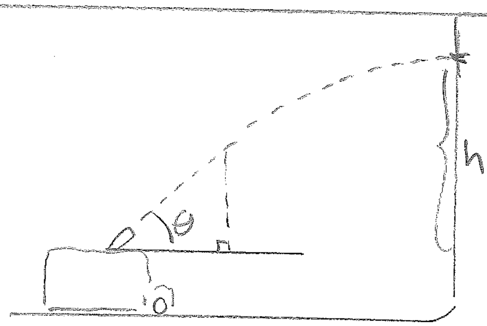
$\frac{dm}{dt} = b$ (constant), $m = M_0 + bt$.

Separate variables, integrate, solve.

Easier. Since F = const., $F\Delta t = \Delta p$

$$Ft = (M_0 + bt)V \qquad V = \frac{Ft}{M_0 + bt}$$

4.21



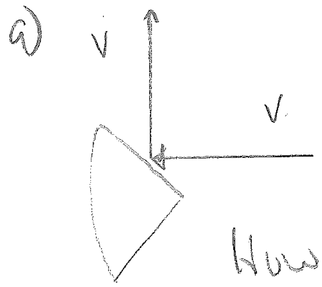
The x & y motions are separable.
Applying $v^2 = 2ax$ to the y motion,
I find $v_{y_0} = \sqrt{2gh} = V_0 \sin\theta$

so $V_0 = \frac{\sqrt{2gh}}{\sin\theta}$.

$$F = \frac{dp}{dt} = V_0\frac{dm}{dt} = \frac{\sqrt{2gh}}{\sin\theta}\cdot K.$$ Direction to left, θ below horizontal.

4.26 In rocket frame

a)



Elastic collision sends particle at 90°.

$$\frac{dp}{dt} = mv \quad \text{per particle.}$$

How many particles do we hit in time t?

All particles in a cylinder, radius R, length vt will be hit. $N_t = \pi R^2 \cdot v \cdot t \cdot N$

$$\left.\frac{dp}{dt}\right|_{TOTAL} = \left.\frac{dp}{dt}\right|_{1\ ptcl} \times \pi R^2 v N = m \cdot \pi R^2 N v^2 \quad (b = m\pi R^2 N).$$

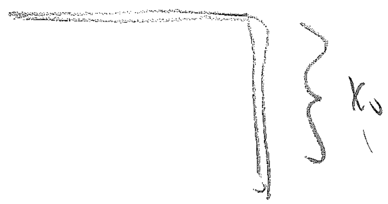
b)

$$F = M\frac{dv}{dt} = -bv^2$$

$$\int \frac{dv}{v^2} = \int -Mb\, dt$$

$$\frac{1}{v} - \frac{1}{v_0} = Mbt.$$

• Falling Rope



$F = ma \qquad \lambda = \text{density of rope}$

$$\cancel{\lambda} x\, g = L \cancel{\lambda} \ddot{x}$$

$$\ddot{x} - \frac{g}{L} x = 0 \qquad \tau^2 = \frac{L}{g}$$

$$\ddot{x} - \frac{x}{\tau^2} = 0 \qquad x = A_\pm e^{\pm t/\tau}.$$

At $t = 0$, $x = x_0$, so $x_0 = A_+ + A_-$.

At $t = 0$, $v = 0$, so $\frac{A_+}{\tau} - \frac{A_-}{\tau} = 0 \quad A_+ = A_- = \frac{x_0}{2}$.

$$\therefore x = \frac{x_0}{2}\left(e^{+t/\tau} + e^{-t/\tau}\right).$$

If rope is coiled around the hole, $F = \frac{dp}{dt}$ $\boxed{\lambda x g = \lambda x \ddot{x} + \dot{x} \lambda \dot{x}}$

Initially, this would have acceleration g, since $\dot{x}_0 = 0$.
The rope above has initial acceleration $\frac{x_0}{L} \cdot g < g$. So this rope falls faster at first.

Later, though, this rope has $|\vec{a}| \to \frac{g}{3}$ (as done in class,) while rope above has $|\vec{a}| \to g$.