1. Firetruck / Rocket $\displaystyle\int \frac{dm}{m} = -\int \frac{dV}{V_{rel}}$

Range of projectile $\Rightarrow$ $y = V_{0y}t - \frac{1}{2}gt^2 = 0$ $\quad t = \frac{2V_{0y}}{g}$

$x = V_{0x}t$

so $R = \frac{2V_{0x}V_{0y}}{g} = d$ in the question

At 45°, $V_{0x} = V_{0y} = \frac{V}{\sqrt{2}}$.

So $d = \frac{V^2}{g}$ so $V = -V_{rel} = \sqrt{gd}$ $\quad \ln \frac{M_f}{M_i} = \frac{-\Delta V}{\sqrt{gd}}$

$\frac{M_f}{M_i} = \frac{1}{2}$ so $V_f = \sqrt{gd}\,\ln 2.$

Position: need $V(t)$ first. $\ln\left(\frac{M_0}{M_0 - kt}\right) = \frac{V(t)}{\sqrt{gd}}$

So $\displaystyle\int \frac{dx}{\cancel{dt}} = \sqrt{gd}\int \ln\left(\frac{M_0}{M_0-kt}\right) dt$

$\displaystyle x = \sqrt{gd}\left[-\int \ln\left(1 - \frac{kt}{M_0}\right) dt\right]$

let $u = \frac{kt}{M_0}$ $\quad du = \frac{k}{M_0}dt$ $\quad \displaystyle = \frac{M_0\sqrt{gd}}{k}\left[-\int_0^{u_f} \ln(1-u)\,du\right]$

$\displaystyle x = \frac{M_0\sqrt{gd}}{k}\left[+u + (1-u)\ln(1-u)\right]\Big|_0^{u_f}$

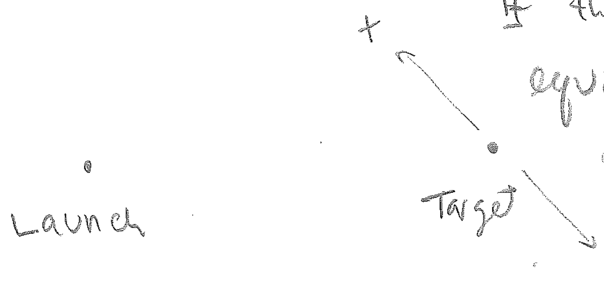
$\displaystyle x = \frac{M_0\sqrt{gd}}{k}\left[u_f + (1-u_f)\ln(1-u_f)\right] = \frac{M_0\sqrt{gd}}{k}\left[\frac{kt}{M_0} - \left(1-\frac{kt}{M_0}\right)\ln ''\right]$

When all the water is gone $kt = M_0/2$,

So $\displaystyle X = \frac{M_0\sqrt{gd}}{k}\cdot\left(\frac{1}{2} - \frac{1}{2}\ln\frac{1}{2}\right) = \frac{M_0\sqrt{gd}}{2k}\left[1 + \ln 2\right].$

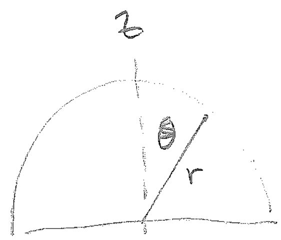
2. Center of Mass must continue on same path as before, since external force (gravity) is the same.

Along a line, if pieces land at the same time and one is 70 m short, the other must land 70 m long, ie. 270 m away.



If they land at same time, and are equal mass, they must land equally far from target in opposite directions from tb.

3



$\sin\theta\, d\theta = d(\cos\theta)$

$$z_{cm} = \frac{\int z\, dm}{M} = \frac{\int z\rho r^2 dr\; d(\cos\theta)\; d\phi}{M}$$

Now $z = r\cos\theta$ so $z_{cm} = \int_0^{2\pi} d\phi \int_0^R dr \underbrace{\int_0^1 d(\cos\theta)\cos\theta}_{\int_0^1 u\,du}\; r^3 \rho / M$

$$z_{cm} = 2\pi \cdot \frac{R^4}{4} \cdot \frac{1}{2} \cdot \rho / M$$

$$M = \frac{1}{2} \cdot \frac{4}{3}\pi R^3 \cdot \rho$$

so $z_{cm} = \frac{3}{8} R.$

4. $\tau = \frac{dL}{dt} = rF_2$ wheel 2.

$= RF_1$ wheel 1. $F_1 = -F_2$.

$\frac{\tau_1}{\tau_2} = -\frac{R}{r} = \frac{dL_1}{dL_2}$ $\quad dL_1 = I_1 d\omega_1$, $dL_2 = I_2 d\omega_2$ $\quad$ but $I_1 = I_2$

so $-\frac{R}{r} = \frac{d\omega_1}{d\omega_2} = \frac{\Delta\omega_1}{\Delta\omega_2}$

No slip means $v_1 = v_2$ at contact.

$r\omega_{2f} = R\omega_{1f}$.

$\left.\begin{array}{l}\Delta\omega_1 = \omega_{1f} - \omega_0 \\ \Delta\omega_2 = \omega_{2f}\end{array}\right\}$ $\quad -\frac{R}{r} = \frac{\omega_{1f} - \omega_0}{\frac{R}{r}\omega_{1f}}$.

$-\frac{R^2}{r^2}\omega_{1f} = \omega_{1f} - \omega_0 \qquad \omega_{1f} = \frac{\omega_0}{1 + R^2/r^2}$.

$\omega_{2f} = \frac{(R/r)\,\omega_0}{1 + R^2/r^2}$.

$= \frac{\omega_0}{\frac{r}{R} + \frac{R}{r}}$.

Interestingly, the maximum ω_2 is obtained when $R = r$!

---

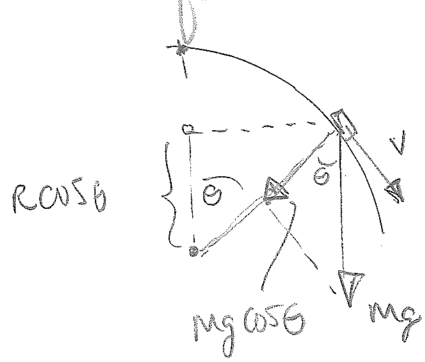
5. Conserve momentum + energy: $mgR = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$

and $mv_1 = Mv_2$. $\quad v_2 = \frac{m}{M}v_1$.

Then $\cancel{m}gR = \frac{1}{2}\cancel{m}v_1^2 + \frac{1}{2}\cancel{M}\frac{m^{\cancel{2}}}{M^{\cancel{2}}}v_1^2$

$2gR = v_1^2\left(1 + \frac{m}{M}\right) \qquad v_1 = \sqrt{\frac{2gR}{1 + \frac{m}{M}}}$

6. Block loses contact when normal force = 0; or when the
(5.6) component of mg directed toward center is insufficient for required centripetal acceleration.



$mg\cos\theta < \frac{mv^2}{R}$. Block comes off.

Now, conserve energy:

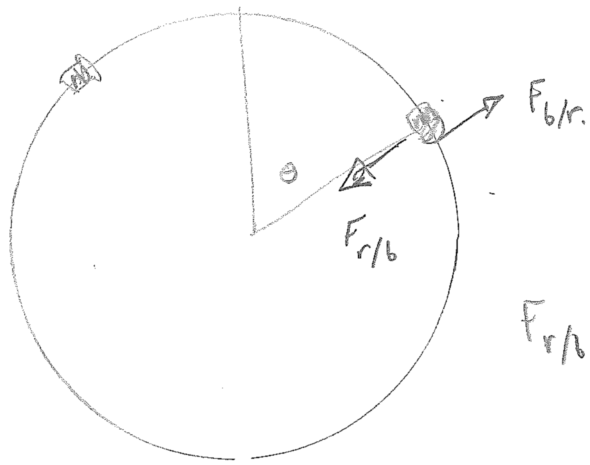
$\frac{1}{2}mv^2 = mg(R - R\cos\theta)$.

Combining:

$g\cos\theta \leq 2g(1-\cos\theta)$

$\cos\theta \leq 2/3. \quad \theta = 48.2^\circ$

5.7.



the force of the ring on the bead has an equal and opposite force of the bead on the ring

$$F_{r/b} = \frac{mv^2}{R} - mg\cos\theta. \quad (\text{see Q6})$$

$$= 2mg(1-\cos\theta) - mg\cos\theta$$

$$= mg(2-3\cos\theta)$$

the total upward force on ring is

$$F_{up} = 2mg\cdot(2-3\cos\theta)\cdot\cos\theta > Mg$$ will lift ring.

$$-6\cos^2\theta + 4\cos\theta - \frac{M}{m} > 0.$$

$$\cos\theta = \frac{-4 \pm \sqrt{16 - 24\,M/m}}{-12} = +\frac{1}{3} \pm \frac{1}{6}\sqrt{4-6\,M/m} = 0$$

Real if $\frac{M}{m} < \frac{2}{3}$. $m > \frac{3M}{2}$.

$$\theta_{crit} = \arccos\left(\frac{1}{3}\right) = 70.5°.$$

If we made $\frac{m}{M}$ twice as large, then

$$\cos\theta_{crit} = \arccos\left(\frac{1}{3} + \frac{1}{6}\sqrt{2}\right) = 55.3°.$$

7.2 "Dropping" sand from inner drum cannot affect its rotation rate so $\omega_a = \omega_a(0)$.

Angular momentum is conserved. Angular momentum of sand transferred $= \Delta M_s \cdot a^2\omega_a$ = angular momentum of outer drum.

$$\lambda t a^2\omega_a = (M_B + \lambda t)\, b^2 \omega_b. \quad \omega_b = \frac{\lambda t}{M_B + \lambda t}\frac{a^2}{b^2}\omega_a.$$

7.14 a) Gravity may be taken to act at C.o.M so

$\tau = Mg\,\ell/2.$

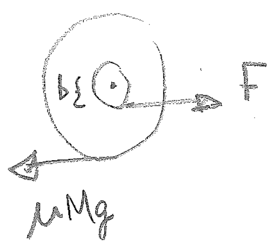
b) $\tau = I\alpha \qquad I = \int_0^{\ell} x^2\,dm = \int_0^{\ell} x^2 \lambda\,dx = \lambda \frac{\ell^3}{3} = \frac{M\ell^2}{3}.$

$$\alpha = \frac{Mg\ell/2}{M\ell^2/3} = \frac{3}{2}\frac{g}{\ell}.$$

c) $a = \alpha r = \frac{3}{2}\frac{g}{\ell}\cdot\frac{\ell}{2} = \frac{3}{4}g.$

d) $F_B = \frac{Mg}{4}.$

7.27 Yo-Yo



$$\tau = I\alpha = \frac{MR^2}{2}\cdot\frac{a}{R} = \mu MgR - F\cdot b$$

$F = ma$

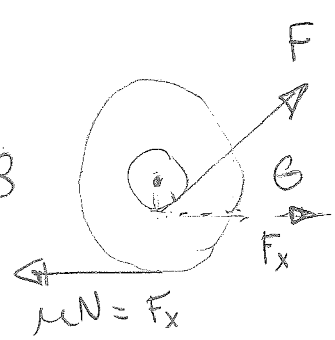
$= F - \mu Mg = Ma$ Solve for a, insert

* Note that + direction for rolling is also + direction for rotation. *

$$\frac{\cancel{M}R^{\cancel{2}}}{2\cancel{R}}\cdot\frac{F-\mu Mg}{\cancel{M}} = \mu MgR - Fb$$

$$F = \frac{3\mu MgR}{R+2b}.$$

7.28



Make torques equal and opposite:

$\cancel{F}\cdot b = F_x\cdot R$

$Fb = \cancel{F}\cos\theta R \qquad \cos\theta = \frac{b}{R}$