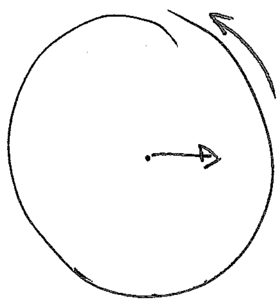
Midterm 1 solutions

1a.



$r = vt$

Ω

$\vec{a} = -vt\omega^2\,\hat{r} + 2v\omega\,\hat{\theta}$

1b. $F_f = ma = \mu m g$

$a = \left[v^2 t^2 \omega^4 + 4v^2\omega^2\right]^{1/2}$

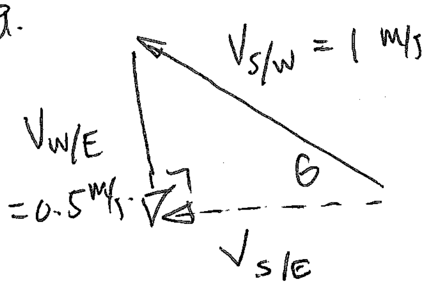
Solve $a^2 = \mu^2 g^2$ for t.

$v^2\omega^2(\omega^2 t^2 + 4) = \mu^2 g^2$

$\omega^2 t^2 + 4 = \dfrac{\mu^2 g^2}{v^2\omega^2}$

$\omega t = \sqrt{\dfrac{\mu^2 g^2}{v^2 \omega^2} - 4}$

2a.

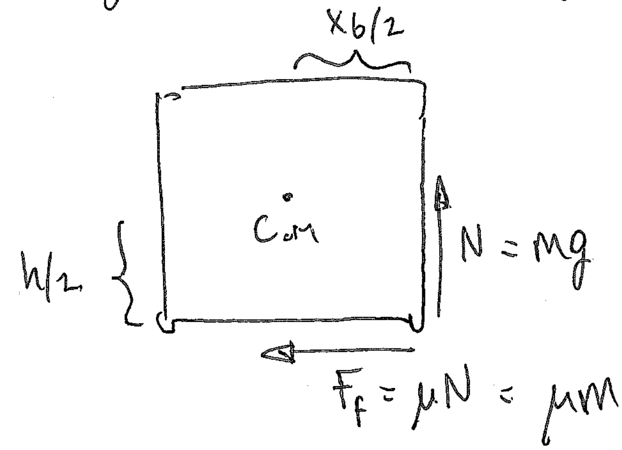


a) $\sin\theta = 1/2 \quad \theta = 30^\circ$ (N of W)

b) $V_{s/E} = \sqrt{1^2 - 0.5^2} = 0.866$ m/s

$t = \dfrac{d}{V} = \dfrac{866}{0.866} = 1000$ s.

3c. Torques about Center of mass just balance, when all weight is on front edge.



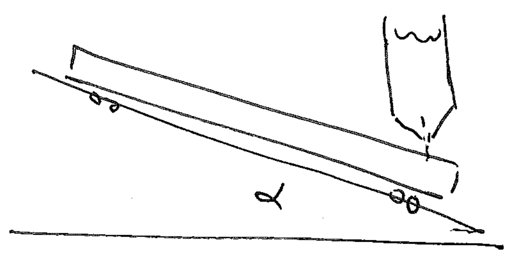
$$\mu mg \cdot \frac{h}{2} = mg\frac{x_b}{2}$$

$$h = \frac{x_b}{\mu}$$

Note: gravity exerts no torque about C.M.

Note: torques about bottom edge do NOT balance, since angular momentum about those points changes even if block does not tip.

4. a)



$F_{grav} = mg\sin\alpha$ down slope

$$F = \frac{dp}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

$$m = M + kt \qquad \frac{dm}{dt} = k$$

so $(M+kt)g\sin\alpha = (M+kt)\dot{v} + vk$

b) $\int F\,dt = \Delta p = p_f - p_i = (M+kt)v$

$$\int_0^t (M+kt)g\sin\alpha\,dt = \left(Mt + \tfrac{1}{2}kt^2\right)g\sin\alpha = (M+kt)v$$

$$v = \frac{M + \frac{kt}{2}}{M+kt}\,gt\sin\alpha$$