Quiz 15 Name SOLUTIONS

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \qquad U_{cf} = \frac{\ell^2}{2\mu r^2}$$

Orbit Eqn: $\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{\ell^2} F\left(\frac{1}{r}\right)$

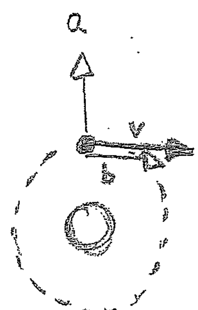
Kepler: $r = \frac{\alpha}{1+\varepsilon\cos\theta}$

$$\left.\begin{array}{l} \alpha = \dfrac{l^2}{\mu k} \\ \varepsilon = \sqrt{1 + \dfrac{2El^2}{\mu k^2}} \end{array}\right\} \quad a = \frac{\alpha}{1-\varepsilon^2} = \frac{k}{2|E|} \qquad b = \frac{\alpha}{\sqrt{1-\varepsilon^2}} = \frac{l}{\sqrt{2\mu|E|}}$$

$k = G\mu M$

$\tau^2 = \frac{4\pi^2\mu}{k} a^3$

1. a) A spaceship is in circular orbit above earth's equator, at speed v. The occupant wishes to escape Earth's gravity by firing the rocket. To minimize the required thrust, should the rocket be fired so as to generate thrust away from the center of the earth (a) or to generate thrust forward, along the direction of the orbit (b)?



It will escape if $E \geq 0$.
Adding speed along existing $\vec{v}$ direction increases energy more than adding it $\perp$. So (b) is most efficient.

1.b) In terms of v, what speed does the rocket need to escape?

Circular orbit $T_i = \frac{1}{2}|U| = \frac{1}{2}\frac{GMm}{r} = \frac{1}{2}mv^2$

Escape $E = 0$ so $T = |U| = \frac{GMm}{r} = \frac{1}{2}mv_f^2 = 2T_i$

$v_f^2 = 2v_i^2$

$v_f = \sqrt{2}\, v_i = \sqrt{2}\, v.$

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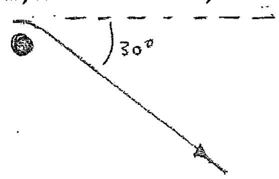
2. Suppose, instead, that the astronaut wants the rocket to leave the Earth's gravitation on a trajectory that (after a long time) takes her at an angle of 30° away from her initial heading. She fires the rockets backward so as to increase her orbital speed. What speed must the spaceship attain, in terms of v, the initial speed?



Hint1: find the eccentricity of the new "orbit".
Hint2: remember that the initial orbit is circular... that tells you something about v.

$r = \dfrac{\alpha}{1+\varepsilon\cos\theta}$ $\quad \theta = 0°$ is straight up in figure,
so the asymptotic angle is $90° + 30° = 120°$

At $\theta = 120°$, $r = \infty$. $1 + \varepsilon \cos 120° = 0$ $\quad \varepsilon(-\frac{1}{2}) = -1 \quad \varepsilon = 2.$

With ε, we find $E = (\varepsilon^2 - 1)\dfrac{\mu k^2}{2\ell^2} = \dfrac{3\mu k^2}{2\ell^2}$

$k = G\mu M$

$\ell = mvr \; (r = r_i)$

$\frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r}$

circular orbit

$(\mu = m)$

$$E = \frac{3m G^2 \cancel{\mu^2} M^2}{2\cancel{\mu^2} v^2 r^2} = \frac{3m G^{\cancel{2}} M^{\cancel{2}}}{2\cancel{G}\cancel{M}\, r^{\cancel{2}}} = \frac{3}{2}\frac{GMm}{r}$$

$$|U_i| = \frac{GMm}{r} = 2T_i$$

so $T = E - U = \dfrac{3}{2}\dfrac{GMm}{r} - -\dfrac{GMm}{r}$

$= \dfrac{5}{2}\dfrac{GMm}{r} = 5T_i$

since $T \propto v_i^2$

$$\boxed{v_f = \sqrt{5}\, v_i}$$