Useful Equations

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

PHYC303 Quiz 3 Name: Solutions

1. a.(3 pt)A toy car moves along a diameter of a turntable at constant speed v. The turntable is rotating at constant angular speed ω. The car crosses the center at t=0. Assuming the car does not slip, find an expression for the magnitude of the total acceleration of the car, in terms of t, ω, v.

$$\vec{a} = -r\omega^2\,\hat{r} + 2v\omega\,\hat{\theta}$$

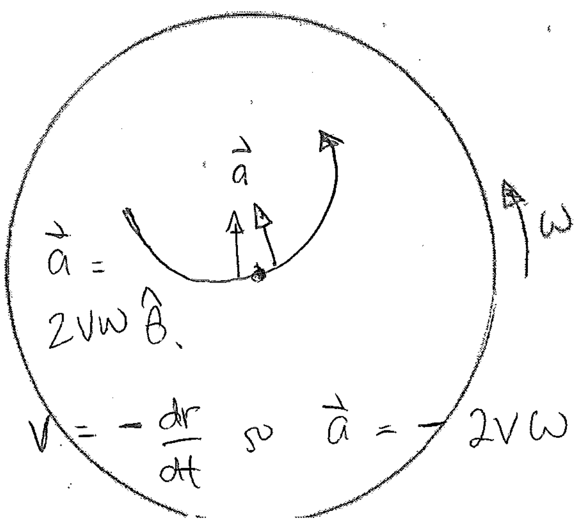
$$= -vt\omega^2\,\hat{r} + 2v\omega\,\hat{\theta}$$

$$|\vec{a}| = \sqrt{v^2t^2\omega^4 + 4v^2\omega^2} = v\omega\sqrt{\omega^2t^2 + 4}$$

b.(2 pt.) What is the acceleration of the car at t=0 (when the car is at the center of the turntable) ?

$\vec{a} = 2v\omega\,\hat{\theta}$ of course, right at $t=0$ $\hat{\theta}$ is undefined! For $t=0^+$, $\vec{a} = 2v\omega\,\hat{\theta}$.

Draw a rough sketch of the motion of the car as seen from above, from t=-1 s to t =+1 s. Does your sketch match your math?



For $t=0^-$, $v = -\frac{dr}{dt}$ so $\vec{a} = -2v\omega\,\hat{\theta}$.

-Turn Quiz Over-

3. (2 pts) A drag force is given by $F = -bv^3$. (v=speed.) No other forces act on the body. Solve for the *time* required to coast from an initial speed of v_0 to a final speed of $v_0/2$. Your answer may include v_0 and b (and numbers, of course.)

$$F = m\frac{dv}{dt} = -bv^3$$

$$\int \frac{dv}{v^3} = \int -\frac{b}{m}\,dt$$

$$+\frac{1}{2}\frac{1}{v^2}\Big|_{v_i}^{v_f} = +\frac{b}{m}t$$

$$\frac{1}{2}\left(\frac{4}{v_0^2} - \frac{1}{v_0^2}\right) = \frac{b}{m}t$$

$$\frac{m}{b}\frac{3}{2}\frac{1}{v_0^2} = t$$

4. (3 pts) Solve for the *distance coasted* in going from an initial speed of v_0 to a final speed of $v_0/2$.

Use $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}\cdot v$

so

$$m v \frac{dv}{dx} = -bv^3$$

$$\int \frac{dv}{v^2} = \int -\frac{b}{m}\,dx$$

$$\left(\frac{1}{v_f} - \frac{1}{v_i}\right) = \frac{b}{m}\Delta x$$

$$\left(\frac{2}{v_0} - \frac{1}{v_0}\right) = \frac{1}{v_0} = \frac{b}{m}\Delta x \qquad \Delta x = \frac{m}{bv_0}$$