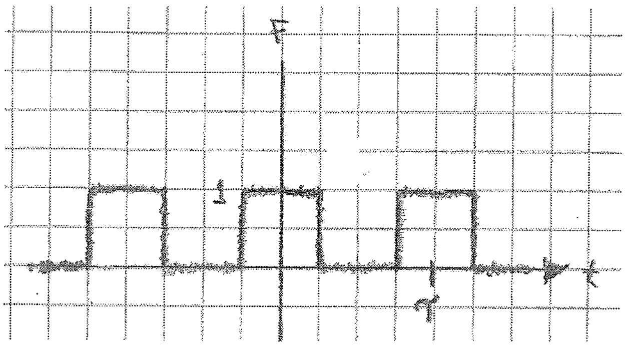
Name SOLUTIONS

1. What are the first two (non-zero) terms in the Fourier series expansion of the square wave function shown? The function is zero for half a cycle and 1 for the other half. It's symmetric about t=0.



$$a_0 = \frac{2}{T}\int_0 F(t)\cos \cancel{0}\omega t\,dt = \frac{2}{T}\cdot 1\cdot \frac{T}{2} = 1$$

$$a_1 = \frac{2}{T}\int_0 F(t)\cos \omega t\,dt = \frac{2}{T}\int_{-T/2}^{T/2} F(t)\cos\omega t\,dt = \frac{2}{T}\int_{-T/4}^{T/4}\cos\omega t\,dt$$

$$= \frac{2}{T}\cdot\frac{1}{\omega}\sin\omega t\Big|_{-T/4}^{T/4} = \frac{2}{T}\,\frac{1}{\omega}\cdot 2 = \frac{4}{\left(\frac{2\pi}{\omega}\right)\omega} = \frac{2}{\pi}$$

$$F(t) \approx \frac{1}{2} + \frac{2}{\pi}\cos\omega t$$

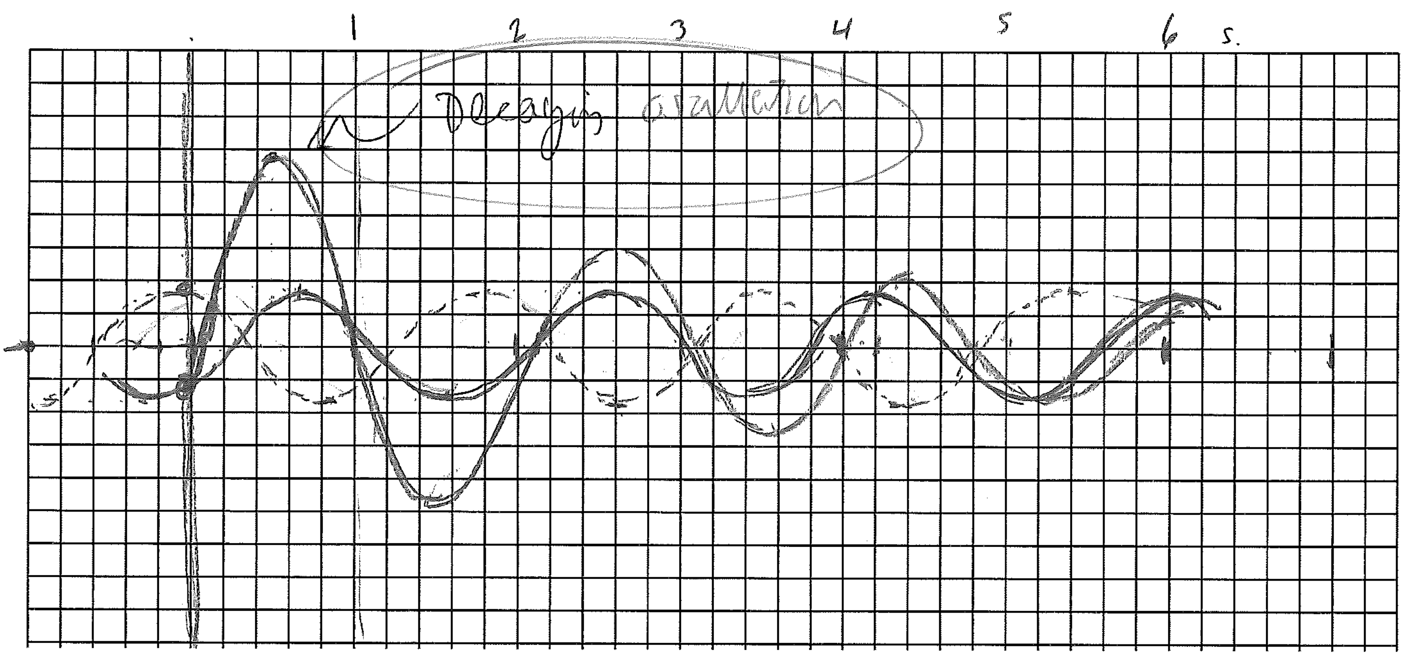
2. A linear harmonic oscillator is driven by a force F = mAcosωt for all time.
A = 1 m/s², m = 1 kg. The undamped resonant frequency is ω_0 = 3 s⁻¹, the damping β = 0.5 s⁻¹.
The driving frequency is ω = √13 s⁻¹.

a) What is the equation of motion x(t)? mA → f_0 on equation sheet = 1

$x(t) = D\cos(\omega t - \delta)$ $\quad D^2 = \dfrac{1}{(3^2-13)^2 + (2\cdot\frac{1}{2}\sqrt{13})^2} = \dfrac{1}{29}$ $\quad D = \dfrac{1}{5.385} = 0.186$

$\delta = \arctan\left(\dfrac{2\cdot\frac{1}{2}\sqrt{13}}{-4}\right) = -42^\circ \quad 138^\circ$ $\quad \omega = \sqrt{13} = 3.6 \quad \tau = 2\pi/\omega = 1.74\ s$

b) An impulse of 2 kg-m/s is applied to the oscillator at t=0. (The force in part a continues unabated.)
What is the motion for t>0?
Sketch the motion x vs. t for t>0.



$\omega_1 = \sqrt{\omega_0^2 - \beta^2} = \sqrt{3^2 - 0.5^2} = 2.96\ s^{-1}$

response to impulse $x(t) = \dfrac{2\ kg\ m/s}{1\ kg \cdot 2.96\ s^{-1}}\, e^{-\beta t} \sin(2.96t)$ $\quad t>0.$

$= 0.676\, e^{-t/2} \sin(2.96t)$ $\quad \tau = \dfrac{2\pi}{2.96} = 2.12\ s.$

$$\boxed{x(t) = 0.186\,m \cos(\omega t - \delta) + 0.676\, e^{-t/2} \sin(\omega_1 t)}$$

Grading: if you tried to find amplitude, phase in part a, 2 pts.
if you added Green's function to part a for b, 2 pts.
If your plot shows a decaying ~~LAPA~~ sinusoid, 2 pts.