Homework 12 Solutions.

1. Raman Line → Temperature

$$k = 360 \text{ cm}^{-1} \qquad c = \frac{\omega}{k}.$$

$$\mathcal{E} = \hbar\omega = \hbar c k$$

$$= 6.6\times10^{-16}\text{ eV}\cdot\text{s}\cdot 3\times10^{10}\ \frac{\text{cm}}{\text{s}}\cdot 360\text{ cm}^{-1}$$

$$\approx 7128\times10^{-6}\text{ eV}$$

$$\approx 7.13\times10^{-3}\text{ eV}.$$

so $$e^{-\mathcal{E}/kT} = \frac{1}{12} \qquad \frac{\mathcal{E}}{kT} = \ln 12$$

$$kT = \frac{\mathcal{E}}{\ln 12} = 2.87\times10^{-3}\text{ eV}$$

$$kT_{RT} = \frac{1}{40}\text{ eV} = 0.025\text{ eV}$$

$$= 25\times10^{-3}\text{ eV}$$

$$T \approx 34.4\text{ K}.$$

2. Mott Transition of excitons.

There are probably different ways to approach this question. I went back to Chapter 10, equation 44, where the Mott transition was first discussed. This potential was for the interaction of an electron with a proton (in hydrogen.) The unscreened potential gives hydrogen eigenstates.

I will modify the unscreened potential to give the observed energy, 14 meV, instead of 13.1 eV (for hydrogen.) There is also a factor of 2 because we take the electron to have its usual mass and the hole the same, so the reduced mass is half what it is for hydrogen.

The bound state energy will vary as $1/\varepsilon^2$, so the effective dielectric constant is

$$\varepsilon \approx \sqrt{\frac{13.1eV}{28\times10^{-3}eV}} = 21.6$$

U(r) in eqn. 14-44 must be reduced by this same factor. We can expect a bound state if the potential well has the same *integrated* depth. (In other words, a wide shallow well has a bound state, as does a narrow deep one.) We just made the well shallower by a factor of 21.6; if we make it wider by the same factor, we should preserve a bound state. $1/k_s$ is a measure of the width of the well, so we will have a bound state when $k_s < (1.19/a_0)/21.6$.

From 14-34, $k_s{}^2 \sim 4n_0^{1/3}/a_0$, where n_0 is the exciton concentration. Combining expressions and taking the Bohr radius as about 1 Å, I find the critical concentration to be about 4×10^{20} excitons/m^3, = 4×10^{14} excitons/cm^3.

Another approach would be to just estimate the radius of the exciton orbit. The Bohr radius of a hydrogenic system depends linearly on the dielectric constant, so the radius should be about 21.6 Å, and the overlap density should be $1/(21.6\text{ Å})^3$ or about 10^{26} excitons/m^3. This is surely an overestimate, however, because the Mott theory gives a conducting system when $a_c = 2.78a_0$, i.e. at a 20-fold lower density than "overlap".

Chapter 14:

7. Eq. (53) becomes $c^2K^2E = \omega^2[\varepsilon(\infty)E + 4\pi P]$, where P is the ionic contribution to the polarization. Then (55) becomes

$$\begin{vmatrix} \omega^2\varepsilon(\infty)-c^2K^2 & 4\pi\omega^2 \\ Nq^2/M & \omega^2 \quad -\omega_T{}^2 \end{vmatrix} = 0,$$

or

$$\omega^4\varepsilon^2(\infty)-\omega^2\left[c^2K^2+\varepsilon(\omega)\omega_T{}^2+4\pi\,Nq^2/M\right]+c^2K^2\omega_T{}^2=0.$$

One root at K = 0 is $\omega^2=\omega_T{}^2+4\pi Nq^2/\varepsilon(\infty)M$. For the root at low ω and K we neglect terms in ω^4 and in ω^2K^2. Then

$$\begin{aligned}\omega^2 &= c^2K^2\ \omega_T{}^2/[\varepsilon(\infty)\omega_T{}^2+4\pi Nq^2/M] \\ &= c^2K^2/[\varepsilon(\infty)+4\pi Nq^2/M\omega_T{}^2] = c^2K^2/\varepsilon(0)\ ,\end{aligned}$$

where ε (0) is given by (58) with $\omega = 0$.