Homework 7 Solutions

6. Let E, v vary as e^{-iwt}. Then

$$v = -\frac{eE/m}{-i\omega + (1/\tau)} = -\frac{e\tau E}{m} \cdot \frac{1 + i\omega\tau}{1 + (\omega\tau)^2},$$

and the electric current density is

$$j = n(-e)v = \frac{ne^2\tau}{m} \cdot \frac{1 + i\omega\tau}{1 + (\omega\tau)^2} E.$$

10. For a monatomic metal sheet one atom in thickness, $n \approx 1/d^3$, so that

$$R_{sq} \approx mv_F / nd^2 e^2 \approx mv_F d / e^2 \ .$$

If the electron wavelength is d, then $mv_F d \approx \hbar$ by the de Broglie relation and

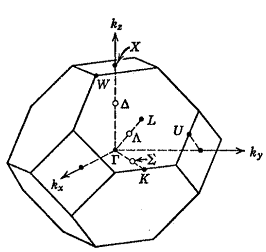
$$R_{sq} \approx \hbar / e^2 = 137/c$$

in Gaussian units. Now

$$\begin{aligned} R_{sq}(\text{ohms}) &= 10^{-9} c^2 R_{sq}(\text{gaussian}) \\ &\approx (30)(137)\,\text{ohms} \\ &\approx 4.1\,\text{k}\Omega \ . \end{aligned}$$

1a. The wavevector at the corner is longer than the wavevector at the midpoint of a side by the factor $\sqrt{2}$. As $\varepsilon \propto k^2$ for a free electron, the energy is higher by $(\sqrt{2})^2 = 2$. b. In three dimensions the energy at a corner is higher by $(\sqrt{3})^2$ than at the midpoint of a face. c. Unless the band gap at the midpoint of a face is larger than the kinetic energy difference between this point and a corner, the electrons will spill over into the second zone in preference to filling up the corner states in the first zone. Divalent elements under these conditions will be metals and not insulators.

2. $\varepsilon = \hbar^2 k^2 / 2m$, where the free electron wavevector k may be written as the sum of a vector K in the reduced zone and of a reciprocal lattice vector G. We are interested in K along the [111] direction: from



Chap. 2, K = (2 π / a) (1,1,1) u, with $0 < u < \frac{1}{2}$, will lie in the reduced zone. The G′s of the reciprocal lattice are given by $G = (2\pi/a)[(h - k + \ell)\hat{x} + (h + k - \ell)\hat{y} + (-h + k + \ell)\hat{z}]$, where h, k, ℓ are any integers. Then $\varepsilon = (\hbar^2/2m)(2\pi/a)^2[(u + h - k + \ell)^2 + (u + h + k - \ell)^2 + (u - h + k + \ell)^2]$. We now have to consider all combinations of indices h, k, ℓ for which the term in brackets is smaller than $6[3(1/2)^2]$ or 9/2. These indices are (000); $(\bar{1}\bar{1}\bar{1})$; $(\bar{1}00)$, $(0\bar{1}0)$, and $(00\bar{1})$; (100), (010), and (001); (111); $(\bar{1}\bar{1}0)$, $(\bar{1}0\bar{1})$, and $(0\bar{1}\bar{1})$; (110), (101), and (011).