PHYS 554 Advanced Optics

1a) A student misaligns an n=1.5 Fresnel rhomb. Instead of the incident light being polarized at 45° to the plane of incidence, it is polarized at 60° to the plane of incidence. Describe (quantitatively) the output polarization. (Review Chapter 14, matrix treatment of polarization!)

b) Dr. T visits the lab, and notices the error. He corrects it, but unfortunately in so doing he changes the internal angle from the desired 53° to 63°. Describe (quantitatively) the resulting output polarization. (You may neglect the effects of the entrance and exit faces of the rhomb.)

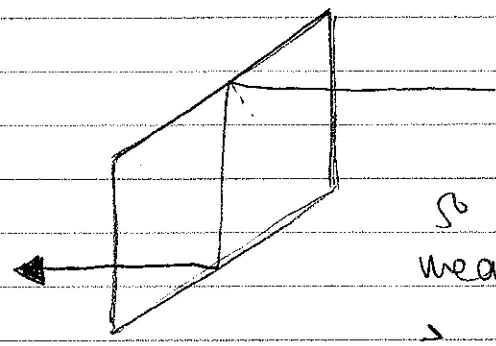
c) By changing the incident polarization plane, is it possible to obtain circularly polarized light with the 63° internal reflection angle?

d) Is it possible to design a "Fresnel prism" using any transparent material in which ONE internal reflection converts linearly polarized light into circularly polarized light?

Pedrotti^3 Chapter 23 Problems 15, 18, 20, 21.

Solutions to Extra Problem on the Fresnel Rhomb

1a.)



Polarized in plane of incidence = TM

So $60°$ to plane of incidence means

$$\vec{E} = E_0 \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix} = E_0 \begin{pmatrix} TE \\ TM \end{pmatrix}$$

the effect of the Rhomb is described by the matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{bmatrix} = \overleftrightarrow{R} \qquad (\text{note } e^{i\pi/2} = i)$$

So the output is $\overleftrightarrow{R} \cdot \vec{E} = \frac{1}{2} \begin{pmatrix} \sqrt{3} \\ i \end{pmatrix}$

Elliptically polarized
TE long axis
CCW rotation
Ratio $\sqrt{3} : 1$.

1b.) From the graph 23-6, the phase shift at $63°$ is 1.07 times as large as at $53°$. Thus, instead of a shift of $-3\pi/4$, the shift will be 2.52 rad.

Two reflections give -5.042 rad $= +1.241$ rad (vs. $+\frac{\pi}{2} = 1.57$ rad)

So $\overleftrightarrow{R} = \begin{bmatrix} 1 & 0 \\ 0 & e^{1.241 i} \end{bmatrix}$ and $\vec{E} = \frac{E_0}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\vec{E}_{out} = \frac{E_0}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{1.241 i} \end{pmatrix}$$

From 14-10, the axis of this elliptical light is $45°$.

The best way to estimate ellipticity is to write the LEFT and RIGHT circularly polarized components (see Klein + Furtak p. 591)

$$a_L = \frac{1}{2}(a_x + i a_y) = \frac{1}{2}(1 + i e^{1.241 i}) = A_L e^{i\phi_L}$$

$$a_R = \frac{1}{2}(a_x^* + i a_y^*) = \frac{1}{2}(1 + i e^{-1.241 i}) = A_R e^{i\phi_R}$$

$$e^{1.241 i} = \cos 1.241 + i \sin 1.241 = 0.32 + 0.95 i$$

$$a_L = \frac{1}{2}(1 + 0.32 i - 0.95) = \frac{1}{2}(0.05 + 0.32 i)$$

$$a_R = \frac{1}{2}(1 + 0.32 i + 0.95) = \frac{1}{2}(1.95 + 0.32 i)$$

thus $A_L = 0.324$

$A_R = 1.976$

the ellipticity is $\dfrac{A_L + A_R}{|A_L - A_R|} = 1.39$

c) No Changing polarization only changes relative amplitudes of TE + TM. ; cannot make the req'd $\pi/2$ phase shift

d) Yes, in Principle,
If you consider eqns 23-36 and 23-37 in the limit as $n \to 0$

$$\tan\left(\frac{\phi_{TE}}{2}\right) \to -\tan\theta \quad \text{and} \quad \tan\left(\frac{\phi_{TM} - \pi}{2}\right) \to \infty$$

$\phi_{TM} \to 0$

so $\theta = \frac{\pi}{4}$ gives $\phi_{TE} = -\frac{\pi}{2}$

So high n internal reflection can work