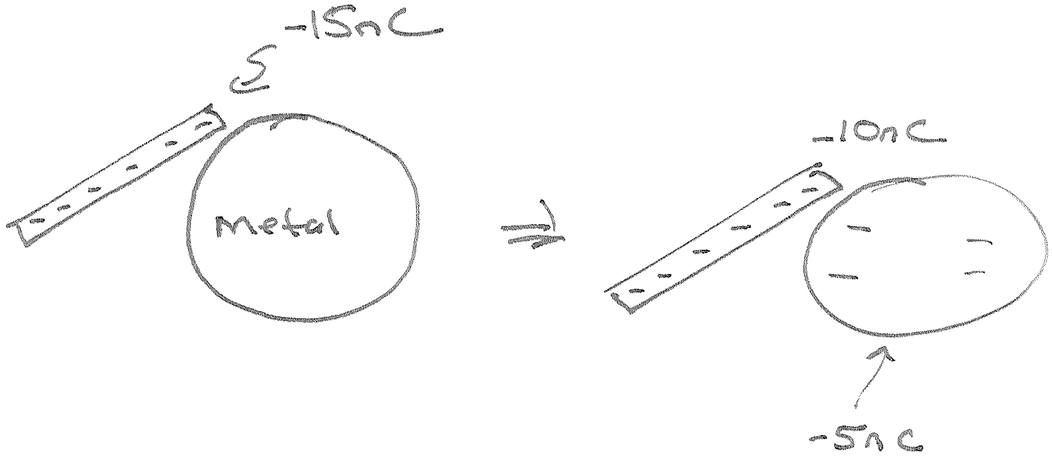
Physics 152, HW #1

MasteringPhysics: 8 problems from chapter 20

One Written Problem

20.4

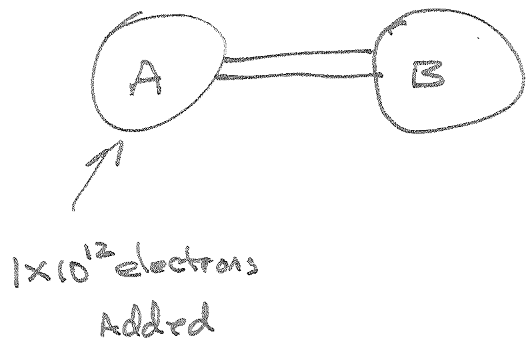


ROD HAD EXTRA electrons AND so transferred them to sphere. Conservation of charge ⇒ −5nC must have been given to sphere.

b) How many electrons transferred. Each electron has -1.6×10^{-19} C of charge. Remember that nano = n = 10^{-9}

$$\frac{-5\,nC}{-1.6 \times 10^{-19}\,C} = \frac{-5 \times 10^{-9}\,C}{-1.6 \times 10^{-19}\,C} = 3.125 \times 10^{10} = 3.13 \times 10^{10} \text{ to } 3 \text{ sig Fig}$$

20.6



$$(1\times10^{+12})(-1.6\times10^{-19}C) =$$

$$-1.6\times10^{-7}C \times \frac{1nC}{1\times10^{-9}C}$$

$$= -160nC$$

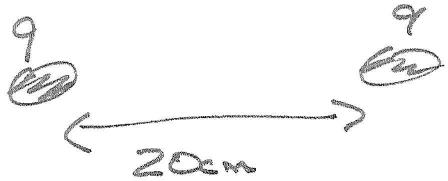
Mastering Physics wants nano Coulomb

The metal connecting rod allows the spheres to share this charge equally $\Rightarrow$ Each get half

$$\Rightarrow Q_A = -80nC, \quad Q_B = -80nC$$

Note: In Real life the metal rod would have some charge too. When we remove it, we'd probably get a little shock as the electrons on it entered our hand. We're supposed to ignore such details here.

Repulsive force between spheres



Assume spheres small enough to be treated as point charges.

$$F = k\frac{|q_1||q_2|}{r^2}$$

$F = 4.57\times10^{-21}N$, $|q_1| = |q_2| = q$, $r = 20cm \times \frac{m}{100cm} = .2m$

$\Rightarrow 4.57\times10^{-21}N = \frac{(9\times10^{9}N\cdot m^2/C^2)(q)(q)}{(.2m)^2} \Rightarrow \frac{(4.57\times10^{-21}N)(0.2m)^2}{(9\times10^{9}N\cdot m^2/C^2)} = q^2$

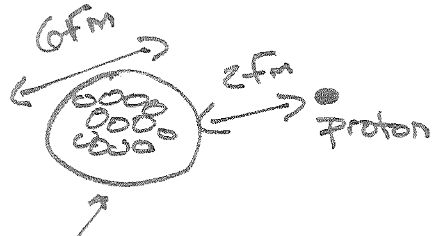
$\Rightarrow q^2 = 2.031\times10^{-32}C^2$ units: $\frac{\cancel{N}\cdot \cancel{m^2}}{\cancel{N}\cdot \cancel{m^2}/C^2} = \frac{1}{1/C^2} = C^2$

$\Rightarrow q = \pm\sqrt{2.031\times10^{-32}C^2} = \pm1.42517\times10^{-16}C$ ← choose negative because extra electrons

Each electron has charge $-1.6\times10^{-19}C$

$\Rightarrow \frac{-1.42517\times10^{-16}C}{-1.6\times10^{-19}C} = 890.73 = 891$ electrons

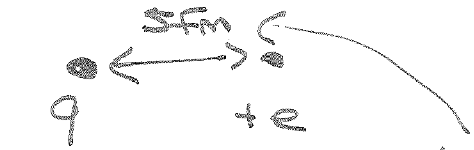
20.41



Nucleus, Charge

$q = +54e$

treat Nucleus as point charge

$\Rightarrow$ 

$\frac{6}{2} + 2 = 3 + 2$
$= 5fm$

$$F = \frac{k q_1 q_2}{r^2} \qquad q_1 = 54e = 54(1.6\times10^{-19}C) = 8.64\times10^{-18}C$$

$$q_2 = e = 1.6\times10^{-19}C$$

$$r = 5fm \times \frac{1\times10^{-15}m}{1fm} = 5\times10^{-15}m$$

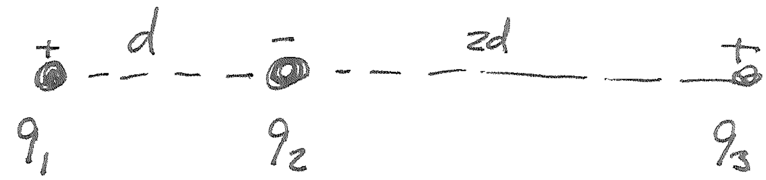
$$\Rightarrow F = \frac{(9\times10^{9}\,N\cdot m^2/C^2)(8.64\times10^{-18}C)(1.6\times10^{-19}C)}{(5\times10^{-15}m)^2} = 497.664N = 500N$$

b) what is proton's Acceleration? $F = ma \Rightarrow a = \frac{F}{m}$

$$a = \frac{497.664N}{1.67\times10^{-27}kg} = 2.98\times10^{29}\,m/s^2\,!!$$

$\leftarrow$ THE electric Force is CRAZY strong at this scale. MAKES you wonder How those 54 protons in the Nucleus stay together!

20.14



$F_{1\,ON\,2} = 5N$, q_3 has 3 times as much charge as q_1

a) What is $F_{3\,ON\,2}$?

Coulomb's Law: $F_{1\,ON\,2} = \dfrac{K|q_1||q_2|}{r_{12}^2}$ and $F_{3\,ON\,2} = \dfrac{K|q_3||q_2|}{r_{23}^2}$

$r_{12} = d$ and $r_{23} = 2d$

The less-mathy way to do this: $\frac{1}{r^2} \Rightarrow$ doubling distance cuts force by 1/4

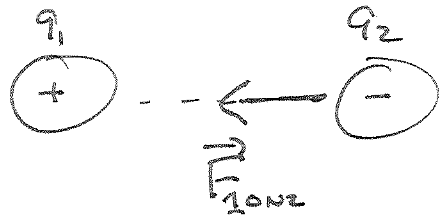
$|q_1||q_2| \Rightarrow$ increasing $|q_1|$ by 3 times but leaving $|q_2|$ the same increases force by 3 times

$\Rightarrow$ overall $F_{3\,ON\,2} = \frac{3}{4} F_{1\,ON\,2} = \frac{3}{4}(5N) = \underline{3.75N}$

More Mathy: $r_{23} = 2r_{12}$, $|q_3| = 3|q_1|$

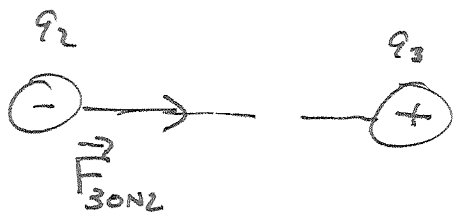
$$F_{3\,ON\,2} = \frac{K|q_3||q_2|}{r_{23}^2} = \frac{K(3|q_1|)|q_2|}{(2r_{12})^2} = \frac{3K|q_1||q_2|}{4r_{12}^2} = \frac{3}{4}F_{1\,ON\,2}$$

b) What direction is $\vec{F}_{1ON2}$?



opposites attract ⇒ Force on q_2 is towards q_1 ⇒ Force on q_2 is to the left

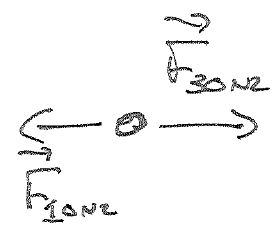
c) WHAT DIRECTION is $\vec{F}_{3ON2}$



Again, opposites attract but now that means that q_2 is pulled to right towards q_3

d) What is the magnitude of net force?

$$\vec{F}_{ON2} = \vec{F}_{3ON2} + \vec{F}_{1ON2}$$



to Right ⇒ only +x component

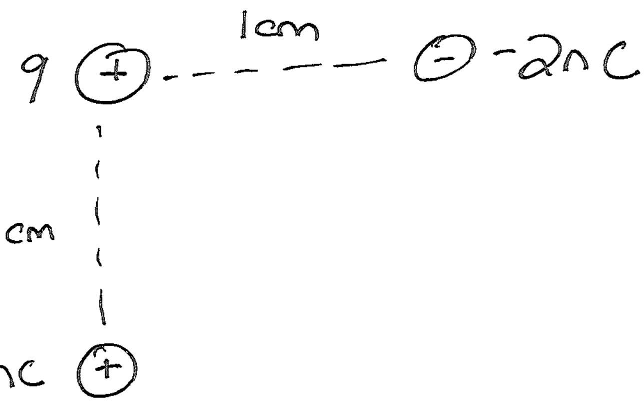
to left ⇒ only −x component

$$F_{ON2,x} = F_{3ON2,x} + F_{1ON2,x} = +F_{3ON2} - F_{1ON2}$$

$$\Rightarrow F_{ON2,x} = 3.75N - 5N = -1.25N \Rightarrow F_{ON2} = 1.25N \text{ to left}$$

$\Rightarrow$ 1.25N for magnitude

20.48



If $q = 1\mu C$, what is the magnitude and direction of net electric force?

$$\vec{F}_{on\,q} = \vec{F}_{+2\,on\,q} + \vec{F}_{-2\,on\,q}$$

due to +2nC   due to -2nC

Coulomb's Law:

$$F_{+2\,on\,q} = \frac{K|2nC||q|}{r_1^2} \qquad r_1 = 1cm$$ (I'll convert the units later!)

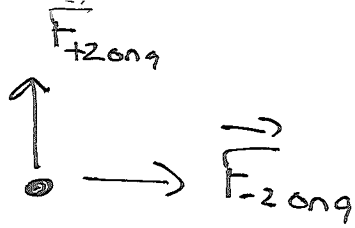
$$F_{-2\,on\,q} = \frac{K|-2nC||q|}{r_2^2} \qquad r_2 = 1cm$$ $\Rightarrow$ two forces have the SAME magnitude.

$2nC = 2\times10^{-9}C$, $1cm = 0.01m$, $1\mu C = 1\times10^{-6}C$

$$\Rightarrow F_{+2\,on\,q} = F_{-2\,on\,q} = \frac{(9\times10^{9}N\cdot m^2/C^2)(2\times10^{-9}C)(1\times10^{-6}C)}{(0.01m)^2} = 0.18N$$

To do vector addition have to next find direction. Opposites attract, like repels $\Rightarrow$

Forces on q:



$\vec{F}_{-2 \text{ on } q}$ to Right $\Rightarrow$ only has +x-component

$\vec{F}_{+2 \text{ on } q}$ up $\Rightarrow$ only has +y-component

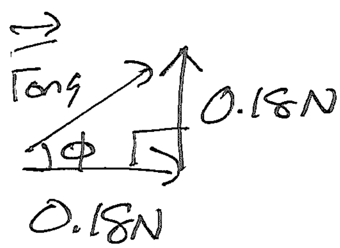
$\Rightarrow F_{-2 \text{ on } q, x} = 0.18N$, $\quad F_{-2 \text{ on } q, y} = 0$

$F_{+2 \text{ on } q, x} = 0$, $\quad F_{+2 \text{ on } q, y} = 0.18N$

$F_{\text{on } q, x} = F_{-2 \text{ on } q, x} + F_{+2 \text{ on } q, x} = 0.18N + 0 = 0.18N$

$F_{\text{on } q, y} = F_{-2 \text{ on } q, y} + F_{+2 \text{ on } q, y} = 0 + 0.18N = 0.18N$

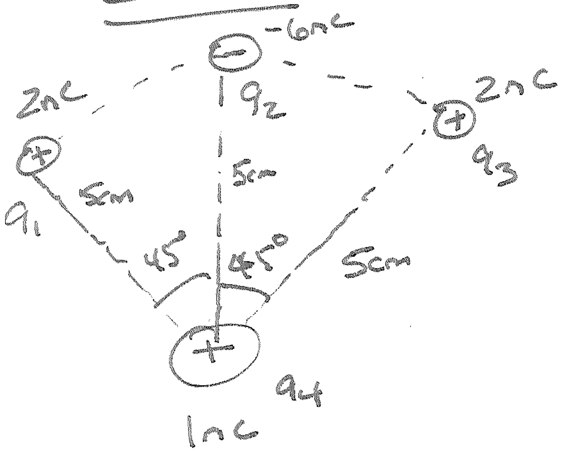
$\Rightarrow$



$F_{\text{on } q} = \sqrt{(0.18N)^2 + (0.18N)^2} = \underline{0.255N}$

$\phi = \tan^{-1}\left(\frac{0.18N}{0.18N}\right) = \tan^{-1}(1) = 45°$

20.51

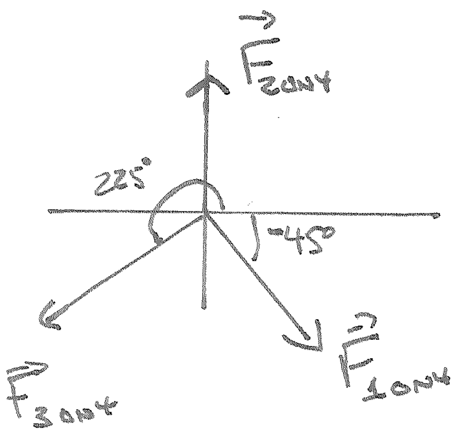


Label charges as shown. Find Net force on q_4

q_1 pos., q_4 pos. $\Rightarrow$ Repulsion at $-45°$ on q_4

q_2 neg, q_4 pos. $\Rightarrow$ attraction up on q_4

q_3 pos, q_4 pos $\Rightarrow$ Repulsion at $180°+45° = 225°$ on q_4



$$\left.\begin{array}{l} q_1 = q_3 \\ r_1 = r_3 \end{array}\right\} \Rightarrow F_{1on4} = F_{3on4}$$

$$F_{1on4} = \frac{K|q_1||q_4|}{r_1^2} \qquad |q_1| = 2nC = 2\times10^{-9}C,\ |q_4| = 1nC = 1\times10^{-9}C$$

$$r_1 = 5cm = .05m$$

$$\Rightarrow F_{1on4} = F_{3on4} = \frac{(9\times10^9 \frac{N\cdot m^2}{C^2})(2\times10^{-9}C)(1\times10^{-9}C)}{(.05m)^2} = 7.2\times10^{-6}N$$

$$F_{2on4} = \frac{K|q_2||q_4|}{r_2^2} \qquad |q_2| = 6nC = 6\times10^{-9}C$$

$$r_2 = 5cm = .05m$$

$$\Rightarrow F_{2on4} = \frac{(9\times10^9 N\cdot m^2/C^2)(6\times10^{-9}C)(1\times10^{-9}C)}{(.05m)^2} = 21.6\times10^{-6}N$$

$$\vec{F}_4 = \vec{F}_{1on4} + \vec{F}_{3on4} + \vec{F}_{2on4}$$

$$F_{4,x} = F_{1\,on\,4,x} + F_{3\,on\,4,x} + \cancel{F_{2\,on\,4,x}}^{\,0}$$ → upwards vector has no x-component

$$= (7.2\times10^{-6}\,N)\cos -45^\circ + (7.2\times10^{-6}\,N)\cos 225^\circ$$

$$= 5.09\times10^{-6}\,N - 5.09\times10^{-6}\,N = 0$$ ← x-components cancel (as we might have guessed from picture)

$$F_{4,y} = F_{1\,on\,4,y} + F_{3\,on\,4,y} + F_{2\,on\,4,y}$$

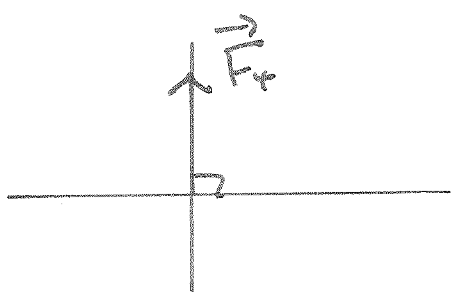
upward vector has entire magnitude in +y.

$$= (7.2\times10^{-6}\,N)\sin -45^\circ + (7.2\times10^{-6}\,N)\sin 225^\circ + 21.6\times10^{-6}\,N$$

$$= -5.09\times10^{-6}\,N - 5.09\times10^{-6}\,N + 21.6\times10^{-6}\,N$$

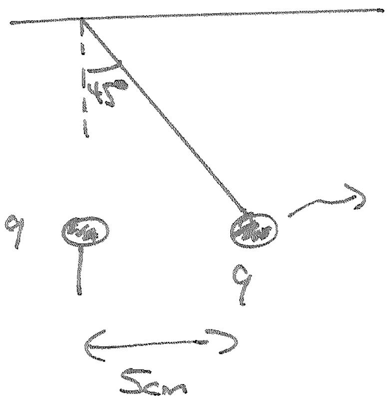
$$= 11.4\times10^{-6}\,N = 1.14\times10^{-5}\,N$$

only y-component



$\vec{F}_4$ at 90° above horizontal

20.62



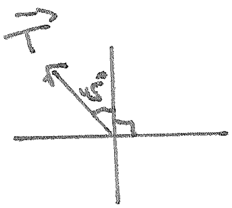
What is q?

$\rightarrow$ Mass, $m = 0.02g \times \frac{kg}{1000g} = 2\times10^{-5}$ kg

There are a total of 3 forces acting on the hanging Bead:

Gravity: $w = mg = (2\times10^{-5}\text{kg})(9.8\text{m/s}^2) = 1.96\times10^{-4}$ N, $\vec{w}$ is down

~~Repulsion~~ Tension: string pull $\Rightarrow \vec{T}$ at

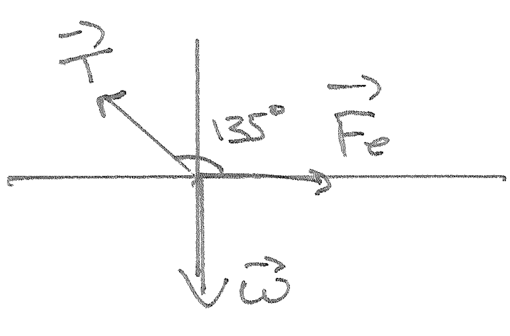


$45° + 90° = 135°$

Electric force: Both ~~spheres~~ beads have same charge $\Rightarrow$ Repulsion $\Rightarrow \vec{F_e}$ to

Right on Hanging Bead.

Bead at rest $\Rightarrow \sum F_x = 0$, $\sum F_y = 0$



Start with $\sum F_y = 0$ since we know $\vec{w}$.

$$\sum F_y = 0 \Rightarrow T_y + \cancel{F_{e,y}}^{0} + w_y = 0$$

$T\sin 135°$

Vector to right has No y-component

-1.96×10^{-4} N

Downward force only has Negative y-component

$$\Rightarrow T\sin 135° - 1.96\times10^{-4}\text{N} = 0$$

$$\Rightarrow T = \frac{1.96\times10^{-4}\text{N}}{\sin 135°} = 0.000277\text{N}$$

Now: $\sum F_x = 0 \Rightarrow T_x + F_{e,x} + \cancel{\omega_x}^{0} = 0$

T_x: $T\cos 135°$

$F_{e,x}$: F_e entire magnitude for vector to right

ω_x: no x-component

$\Rightarrow T\cos 135° + F_e = 0 \Rightarrow 0.000277\,N \cos 135° + F_e = 0$

$\Rightarrow -0.000196\,N + F_e = 0 \Rightarrow F_e = 0.000196\,N = 1.96\times10^{-4}\,N$

~~same~~ same size as ω!

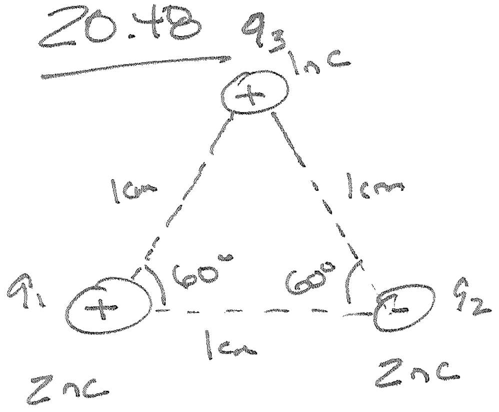
Finally $F_e = K\dfrac{|q_1||q_2|}{r^2}$ $|q_1| = |q_2| = q$

$r = 5\,cm = .05\,m$

$\Rightarrow 1.96\times10^{-4}\,N = \dfrac{(9\times10^{9}\,N\cdot m^2/C^2)(q)(q)}{(.05\,m)^2} \Rightarrow \dfrac{(1.96\times10^{-4}\,N)(0.05\,m)^2}{9\times10^{9}\,N\cdot m^2/C^2} = q^2$

$\Rightarrow q = \sqrt{5.444\times10^{-17}\,C^2} = 7.378\times10^{-9}\,C = 7.4\,nC$

20.48

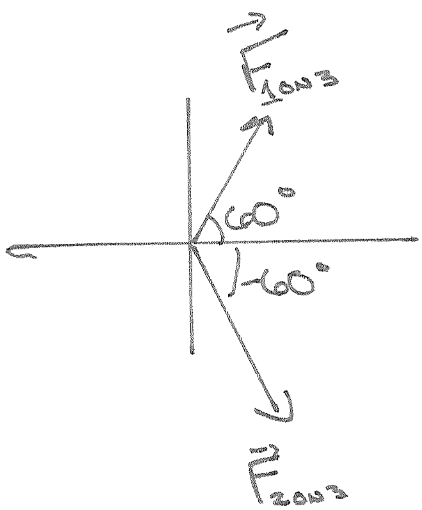


Find magnitude of net force on 1nC.

Label q_1, q_2, q_3 AS SHOWN

q_1, q_3 Both positive $\Rightarrow$ Repulsion at 60° on q_3

q_2 Neg., q_3 positive $\Rightarrow$ attraction at -60° on q_3



$$F_{1on3} = \frac{K|q_1||q_3|}{r_{13}^2} \qquad r_{13} = 1cm = .01m$$

$$F_{2on3} = \frac{K|q_2||q_3|}{r_{23}^2} \qquad r_{23} = 1cm = .01m$$

$|q_1| = |q_2|$ so $F_{1on3} = F_{2on3}$

$$F_{1on3} = \frac{(9\times10^9 N\cdot m^2/c^2)(2\times10^{-9}C)(1\times10^{-9}C)}{(.01m)^2} = .00018N$$

$$\vec{F}_3 = \vec{F}_{1on3} + \vec{F}_{2on3} \Rightarrow F_{3,x} = F_{1on3,x} + F_{2on3,x}$$

$$= (.00018N)\cos 60^\circ + (.00018N)\cos -60^\circ$$

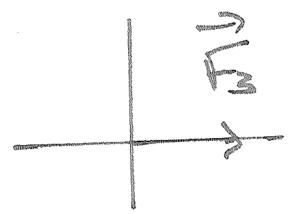
$$= (.00018N)\left(\tfrac{1}{2}\right) + (.00018N)\left(\tfrac{1}{2}\right) = .00018N$$

$$F_{3,y} = F_{1on3,y} = F_{2on3,y} = (.00018N)\sin 60^\circ + (.00018N)\sin -60^\circ$$

$$= .0001559N - .0001559N = 0$$

So as we could have guessed from the free body diagram, the y components cancel

$F_3 = F_{3,x} = 0.00018N$

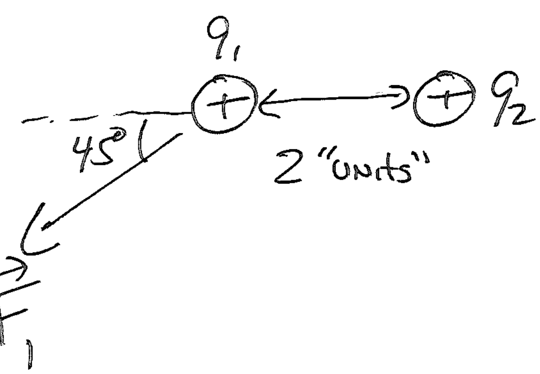


$\vec{F_3}$ at 0°

Written Question #1

$q_1 = q_2 = q_3$ Find Location of q_3 to make Direction of Net Force on q_1 as shown.

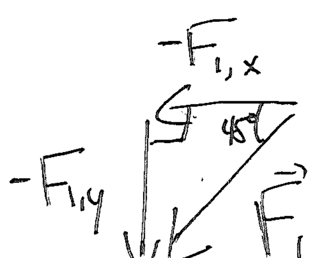
(a)



$\vec{F}_1 = \vec{F}_{2 on 1} + \vec{F}_{3 on 1}$

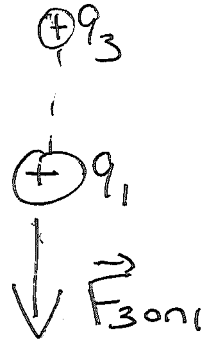
We already know direction of $\vec{F}_{2on1}$. q_1 & q_2 both positive $\Rightarrow$ $\vec{F}_{2on1}$ is to the left. $\xleftarrow{\vec{F}_{2on1}} (+)\, q_1$

$\vec{F}_1$ has an x-component that is to the left!



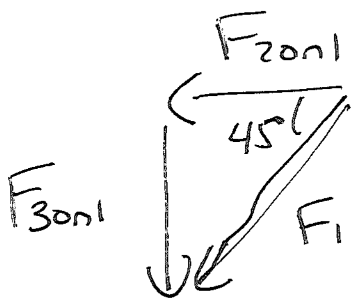
So the easiest way to make $\vec{F}_1$ point in the direction shown is to have $\vec{F}_{2on1}$ create the x-component and have $\vec{F}_{3on1}$ create the y-component.

TO HAVE $\vec{F}_{3on1}$ POINT DOWNWARD, q_3 must be <u>above</u> q_1:



TO FIGURE OUT HOW FAR AWAY TO PUT q_3 USE THE ANGLE GIVEN.

$F_{2on1} = |F_{1,x}|$ AND $F_{3on1} = |F_{1,y}|$ $\Rightarrow$



$$\tan 45^\circ = \frac{F_{3on1}}{F_{2on1}} \qquad \tan 45^\circ = 1$$

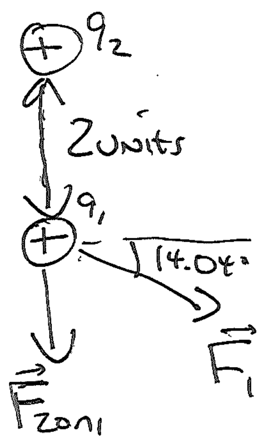
$$\Rightarrow \frac{F_{3on1}}{F_{2on1}} = 1 \Rightarrow F_{3on1} = F_{2on1}$$

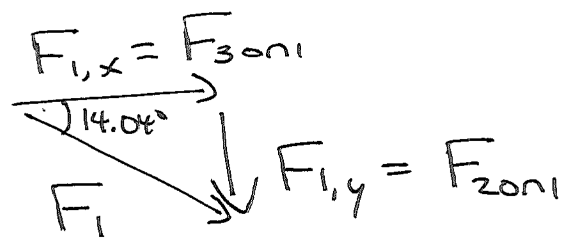
$$F_{2on1} = \frac{k|q_1||q_2|}{r_{12}^2} \qquad F_{3on1} = \frac{k|q_1||q_3|}{r_{13}^2} \qquad q_1 = q_2 = q_3$$

$\Rightarrow r_{12} = r_{13}$ FOR $F_{2on1} = F_{3on1}$ $\Rightarrow$ q_3 must be SAME DISTANCE ABOVE q_1 AS q_2 is to the Right.

$\Rightarrow$ 2 "UNITS" above. (AS SHOWN ON ~~First~~ Last PAGE.)

(b) is pretty much the same as (a) but since the angle isn't 45° anymore $F_{3on1} \neq F_{2on1}$ $\Rightarrow$ different distances.





To make F_{3on1} point to the right, q_3 must be to the left of q_1.

$$\tan 14.04° = \frac{F_{2on1}}{F_{3on1}} \qquad \tan 14.04° = 0.25 = \frac{1}{4}$$

$$\Rightarrow \frac{F_{2on1}}{F_{3on1}} = \frac{1}{4} \Rightarrow \underline{F_{3on1} = 4F_{2on1}}$$

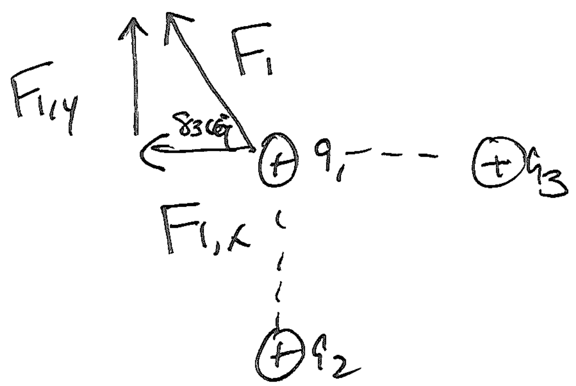
$$F_{3on1} = \frac{k|q_1||q_3|}{r_{13}^2}, \quad F_{2on1} = \frac{k|q_1||q_2|}{r_{12}^2}.$$

$q_1 = q_2 = q_3 \Rightarrow$ only distance can make F_{3on1} bigger. Inverse-square law $\Rightarrow r_{13}$ must be $\frac{1}{2}$ of r_{12} to make F_{3on1} 4x larger.

So $r_{13} = \frac{1}{2} r_{12} \Rightarrow r_{13} = \frac{1}{2}(2\text{ units}) = 1$ unit (as shown)

(c) By now hopefully THE IDEA is CLEAR.

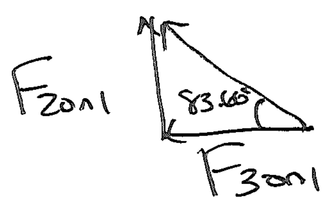
q_2 is BELOW $q_1 \Rightarrow \vec{F}_{2 on 1}$ points up



By putting q_3 to the Right of q_1, $\vec{F}_{3 on 1}$ is to the left.

$$\Rightarrow F_{1,x} = F_{3 on 1}$$

$$F_{1,y} = F_{2 on 1}$$



$$\tan 83.66° = \frac{F_{2 on 1}}{F_{3 on 1}} \Rightarrow 9 = \frac{F_{2 on 1}}{F_{3 on 1}}$$

$\Rightarrow F_{2 on 1} = 9 F_{3 on 1}$. $\Rightarrow F_{3 on 1} = \frac{1}{9} F_{2 on 1}$ INVERSE-SQUARE LAW $\Rightarrow$ INCREASING DISTANCE by 3 cuts FORCE by 9

q_1 & q_2 ARE 1 unit AWAY FROM EACH other $\Rightarrow q_3$ must be 3 units AWAY.

Written Question #1 A positive charge q_1 experiences electric forces from two other positive charges, q_2 and q_3. All three have the same amount of charge ($q_1 = q_2 = q_3$). The three figures below show q_1, q_2, and the direction of the net force on q_1 due to both q_2 and q_3. Add to each figure the location of q_3 such that the net force has the direction given. For full points, you *must* have q_3 at the correct distance from q_1, show how you calculated that distance, and include an explanation of how you determined your answer. *Note:* There are actually many different possible answers to each of these. Give yourself a break and find the easiest solution.