Physics 570

Homework #10

Due Thursday, 12 April, 2007

1. Consider the manifold corresponding to the spherically-symmetric, interior Schwarzschild solution, corresponding to a fluid with constant density, and look at the 3-dimensional slices of constant time, where we know the metric is given by

$$\underset{3}{\mathbf{g}} = \frac{dr^2}{1 - 2\frac{\mathcal{M}(r)}{r}} + r^2\, d\Omega^2 = \frac{dr^2}{1 - 2m\frac{r^2}{R^3}} + r^2\, d\Omega^2 \ .$$

Please calculate the conformal tensor for this 3-dimensional manifold, and show that it vanishes. Such a space is referred to as conformally flat. As well show that the remaining ingredients in the curvature are simply constant, so that this is indeed a 3-manifold of constant curvature.

There is a coordinate transformation that preserves spherical symmetry and is such that the new (3-dimensional) metric in those coordinates is proportional to

$$\underset{3}{\mathbf{g}} \propto d\chi^2 + \sin^2\chi\, d\Omega^2 \ ,$$

which is proportional to the metric of a (closed) Robertson-Walker cosmology at a constant time slice. Can you find this transformation between the two?

2. One can write various sorts of divergences of tensorial quantities in a simplified way which does not appear to need any covariant derivatives. To see this, show that for an arbitrary tangent vector $\widetilde{T}$ with components relative to some **coordinate basis** as T^α, show that the following equality is true:

$$T^\alpha{}_{;\alpha} = \frac{1}{\sqrt{-g}}\left(\sqrt{-g}T^\alpha\right)_{,\alpha} \ ,$$

where $-g$ is the negative of the determinant of the metric, and therefore a positive quantity.