

THEORY OF SUBLINEAR POWER DEPENDENCE OF MICROWAVE HEATING IN SOME CERAMIC MATERIALS

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ABSTRACT

A sublinear dependence of the microwave heating rate on the incident microwave power implying a saturation effect appears to have been observed recently. We present simple model calculations to address this observation on the basis of an idea of spatial confinement of the absorbing charges in grain boundary regions. Two natural lengths exist in this model: the spatial extent of the confining region, and the maximum distance an absorbing charge travels under the combined action of damping and of the oscillating microwave field. We suggest that a mismatch of these lengths results in the observed saturation, more generally, in the observed decrease in absorption efficiency.

INTRODUCTION

Standard calculations of the heating rate of materials subjected to electromagnetic waves, such as microwave radiation, predict a linear dependence on the incident power: the heating rate, equivalently the power absorbed, depends quadratically on the electric field E of the microwave field. This behavior is indicative of standard Joule heating. Even recent theories [1,2] of the phenomenon of thermal runaway, which explain the runaway on the basis of nonlinear feedback of the energy, predict a linear dependence of the heating rate on the incident power. Such a linear dependence is surely observed in many materials such as silica [3,4]. However, a saturation effect, more generally a sublinear dependence of the rate on the microwave power, appears to have been observed in other materials, typified by ferric oxide [4]. No theoretical starting point in the literature known to us, is currently able to address this effect. In the present paper, we describe our recent effort in this direction.

The essential idea behind the calculation we present here is one of *spatial confinement* of absorbing charges in grain boundary regions in a ceramic material. The charges absorb power from an applied oscillating electric field by following the field oscillations and dissipating the absorbed power to a damping reservoir such as the solid lattice. The instantaneous power absorbed is proportional to the square of the velocity of the charge, i.e., to the square of the applied electric field. The time average yields the average power absorbed. This is the situation in a spatially unrestricted medium. However, if the charges are constrained to move in a finite region of space, they may find themselves unable to follow the time dependence of the applied field simply because they may reach the walls of the confining region before the electric field has reversed its direction. The encounters with the wall could result in velocity reversal and, thus, work against efficient absorption of power from the field. Surely, this effect will be negligible in small fields or large regions. However, for a given extent of the confining region, if the input power is increased, the electric field will eventually reach a magnitude consonant with a traversal distance which is larger than the confining length. The consequence will be a sublinear dependence of the absorbed power on the incident power.

Spatial confinement of charges can occur in materials in the proximity of grain boundaries. In many ceramics, particularly those which are clearly ionic in character, atomic

diffusion is thought of as occurring in two parts: diffusion of the cations through the bulk, and the diffusion of larger anions primarily through the grain boundary regions. Let us focus attention on the anions. The grain boundary may be considered as made up of a number of segments along which the anion motion takes place. These linear segments of the boundary provide a possible model for our spatially constraining regions. Another possibility arises from the space charge accumulation region *across* a grain boundary: electrostatic interactions are known [5,6] to lead to the accumulation of space charges on the two sides of a grain boundary and to potentials which are felt by charged interstitials or vacancies. The confining length in the former case could be of the order of microns or tenths of a micron, while in the latter case it could be much smaller, viz. of the order of 200 Å. The following is a brief description of our basic theory and of a recent application to observations on the dependence of the saturation temperature on input microwave power.

BASIC THEORY AND APPLICATION TO EXPERIMENT

For the sake of simplicity, the nature of charge motion in the confined region will be assumed to be free except for damping and the action of the electric field. The equation of motion is

$$\frac{dv}{dt} + \gamma v = \frac{qE}{m} \sin \omega t. \quad (1)$$

where v is the velocity of the charge q with mass m , E is the electric field, and γ is the damping constant. With the notation $\varepsilon = \frac{qE}{m}$, the solution of (1) is

$$v(t) = e^{-\gamma t} \left[v(0) + \frac{\omega \varepsilon}{\gamma^2 + \omega^2} \right] + \frac{\varepsilon}{\sqrt{\gamma^2 + \omega^2}} \sin(\omega t - \varphi). \quad (2)$$

where the phase lag angle φ is given by $\tan^{-1}(\omega / \gamma)$. In an unconfined region, the steady-state heating rate R is given by

$$R = \frac{1}{2} \frac{q^2}{m} \frac{\gamma}{\gamma^2 + \omega^2} E^2. \quad (3)$$

Since the incident power is proportional to E^2 , the rate as given in (3) depends linearly on the incident power. Under the assumption of very large damping, known to be valid for microwave interactions with ceramics, the solution given in (2) simplifies significantly. The result, which can also be obtained directly by taking the large damping limit of (1), is a complete *slaving* of the velocity by the electric field:

$$v(t) = \frac{qE}{m \gamma} \sin \omega t. \quad (4)$$

Results such as (2), (3) and (4) are valid when there are no walls to confine the motion of the charges. Let us now consider the case of a spatially restricted region. The charge is accelerated but hits a confining wall before the oscillating electric field has reversed sign. The charge bounces back, with its velocity reversed in sign. If the collision with the wall is elastic, the magnitude of the velocity is unchanged. The velocity now decreases in magnitude because the electric field acts

against the motion. The charge may return to the wall one or more times before the electric field reverses and forces it to move towards the other restraining wall.

It is possible [7] to show that, if the charge is at rest at one of the walls initially, the average heating rate is given by

$$R = \frac{1}{2} \frac{q^2}{m \gamma} E^2, \quad \frac{L}{L_E} \geq 1 \quad (5)$$

$$R = \frac{1}{2} \frac{q^2}{m \gamma \pi} E^2 \left[\cos^{-1} \left(1 - \frac{2L}{L_E} \right) - 2 \left(1 - \frac{2L}{L_E} \right) \sqrt{\left(1 - \frac{L}{L_E} \right) \frac{L}{L_E}} \right], \quad \frac{L}{L_E} \leq 1 \quad (6)$$

Equation (5) is simply the large damping limit of (3). However, (6) has novel content and involves the quantity $L_E = 2qE/m\gamma\omega$ in addition to the distance L between the walls. The rate depends on the electric field not only through the E^2 term as in (5) but also through the dependence of L_E on the electric field. The overall effect is to lower the heating efficiency. We present Fig. 1 to make this point clear.

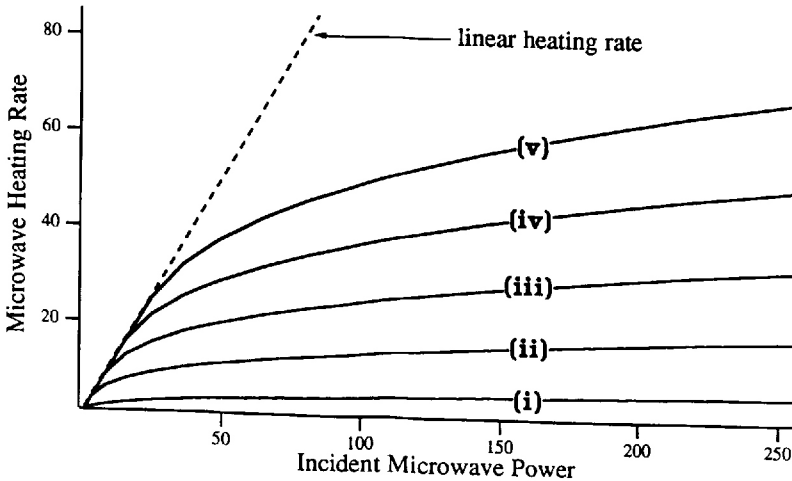


FIG. 1: Theoretical prediction of a sublinear dependence of the heating rate on the incident microwave power, for several values of the confining length L . Values of L for curves (i), (ii), (iii), (iv) and (v) are respectively in the ratio 1:2:3:4:5. The limit of infinite L is approached by all finite- L curves for appropriately small values of the incident power. Units are arbitrary.

An application of these ideas results in a possible understanding of the observed sublinear power dependence in ferric oxide [4]. Thermal runaway in a variety of materials has been explained recently in terms of a microscopic theory [1,2] which results in the following temperature-time dependence:

$$\frac{dT}{dt} = [k_A + k_M f(T)]P - \sigma_1 T^4 \quad (7)$$

where k_A and k_M are related to the absorption coefficients of two kinds of species of absorbers present in the material and $f(T)$ denotes the fraction of the M-absorbers which are free to absorb [8], P is the incident power, and σ_1 is proportional to the Stephan-Boltzmann constant. Equation (7), which has provided a successful description [1,2] of the temperature-time evolution in a large number of materials, when combined with the spatial confinement ideas presented above, results in

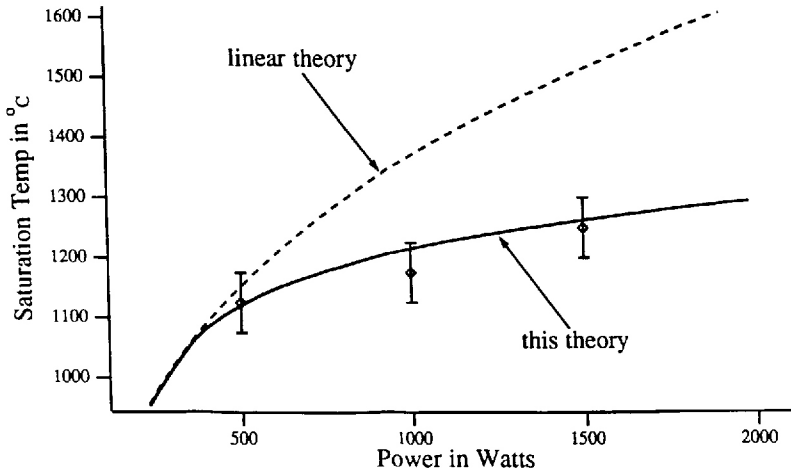


FIG.2: Saturation temperature during microwave heating of ferric oxide plotted as a function of the incident microwave power, along with the results of our theory. Error bars show the extent of uncertainty introduced by the procedure used to estimate values of the saturation temperature from the time-temperature curves reported in [4].

the following relation between the saturation temperature T_s and the input microwave power P :

$$\frac{\sigma_1 T_s^4}{k_A + k_M f(T_s)} = P, \quad \frac{L}{L_E} \geq 1 \quad (8)$$

$$\frac{\sigma_1 T_s^4}{k_A + k_M f(T_s)} = \frac{P}{\pi} \left[\cos^{-1} \left(1 - \frac{2L}{L_E} \right) - 2 \left(1 - \frac{2L}{L_E} \right) \sqrt{\left(1 - \frac{L}{L_E} \right) \frac{L}{L_E}} \right], \quad \frac{L}{L_E} \leq 1 \quad (9)$$

It is important to notice that, in (9), L_E is a function of the field and, consequently, of P .

In Fig. 2 we have shown the saturation temperature T_s as a function of input microwave power with and without the spatial confinement effect along with data deduced from the observations in ref. [4]. There is a probable error of ± 50 deg. C in the saturation temperature, as depicted in Fig. 2. The confining length which corresponds to the solid curve is of the order of a tenth of a micron. At this stage, the comparison of theory and experiment is relatively crude, but we can certainly conclude from Fig. 2 that the spatial confinement mechanism we have proposed is a possible source of the observed sublinear power dependence of the heating rate in ferric oxide.

REMARKS

We have assumed in the analysis presented above that the damping is infinitely large so that the velocity is *completely* slaved by the applied field as in (4). If the damping is large but not infinite, we may follow a method that has been recently used in the context of nonlinear quantum transport in presence of dissipation [9]. The Laplace transform of (1) can be written in the form

$$\tilde{v}(s) = \frac{\tilde{E}(s)}{s + \gamma} = \frac{\tilde{E}(s)}{\gamma} \left(\frac{1}{1 + (s/\gamma)} \right) \approx \frac{\tilde{E}(s)}{\gamma} \left(1 - \frac{s}{\gamma} \right) \approx \frac{\tilde{E}(s)}{\gamma} \exp \left(-\frac{s}{\gamma} \right) \quad (10)$$

where $E(t)$ is the time-dependent electric field, tildes denote Laplace transforms, and s is the Laplace variable. The expansion in powers of (s/γ) that we have made in (10) is obviously valid for large damping and for long times. Using the time-displacement property of Laplace transforms, we can invert (10) into the simple form

$$v(t) = (1/\gamma) E \left(t - \frac{1}{\gamma} \right) = \frac{qE}{m\gamma} \sin \omega \left(t - \frac{1}{\gamma} \right) \quad (11)$$

Finite damping corrections to the theory presented above can be derived easily with (11) as a point of departure. For want of space, we do not show the details here.

We have presented a simple possible explanation of the sublinear power dependence of the microwave heating rate on the basis of spatial confinement ideas. Many of the assumptions made here can be relaxed. These include the complete confinement of the moving charges to segments of the grain boundary, and the assumption that the nature of the motion in the confining region is free. For a fuller discussion of these issues we refer the reader to ref. [7].

ACKNOWLEDGEMENTS

This work was carried out in part with the support of the UNM-NSF Center for Microengineered Ceramics, a collaborative effort of the National Science Foundation (CDR-8800352), the Los Alamos and Sandia National Laboratories, the New Mexico Research and Development Institute, and the Ceramics Industry.

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