Relation between dynamic localization in crystals and trapping in two-level atoms

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It is shown that a recently reported realization of trapping in a two-level system with frequency-modulated fields in quantum optics is intimately related to an earlier demonstration of dynamic localization of charges moving in a crystal under the action of a time-periodic electric field. Size effects on the phenomenon are explored. [S1050-2947(96)50609-8]

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Agarwal and Harshawardhan (AH) [1] have recently commented on the existence of population-trapping states in a two-level system driven by a field that varies in time sinusoidally. Those authors analyze, numerically and analytically, the phenomenon of localization of the population on one state for a substantially long time, followed by an abrupt jump to the other state, if the ratio of the exciting field amplitude to the frequency of the field equals one of the zeros of the J_0 Bessel function.

We show here that this phenomenon is closely related to dynamic localization, i.e., the localization of a moving charged particle under the action of a time-periodic field, reported earlier by Dunlap and Kenkre (DK) [2–4]. Related effects have also been observed in the nonlinear context [5–8].

The crystal Hamiltonian for the transport system considered in Refs. [2–4] is given by

$$H=\hbar V\sum_{m=-\infty}^{\infty}(|m\rangle\langle m+1|+|m+1\rangle\langle m|)$$
$$-\hbar eEf(t)a\sum_{m=-\infty}^{\infty}m|m\rangle\langle m|, \tag{1}$$

where $|m\rangle$ represents a Wannier state localized on a lattice site m, the position operator $\hat{x}$ has been assumed to be diagonal in the Wannier basis, a is the lattice constant, E is the applied electric field, $f(t)$ describes the time variation of the electric field, e is the particle charge, and V is a (nearest-neighbor) matrix element. The analysis of DK gives exact results for arbitrary time dependence of the field [2], specifies the explicit connection of the new phenomenon to Bloch oscillations by exploring the system in momentum space [3], and treats the effects of scattering of the particle due to lattice imperfections by using the stochastic Liouville equation [4]. The key finding of DK was the localization of the particle that occurs when the ratio of the strength of the imposed time-periodic electric field to its frequency becomes equal to certain discrete values, one of the results they obtained being that these discrete values are related to the zeros of the ordinary Bessel function of order 0 when the field is sinusoidal. This dynamic localization phenomenon was studied in the nonlinear context by Konotop *et al.* [5] and by Cai *et al.* [6–8]. Similar effects have also appeared in the work of Holthaus [9], Ignatov and Romanov [10], and Epifanov *et al.* [11], and in the context of avalanches in laser damage in the work of Kenkre *et al.* [12]. In quantum optics, related work on the two-level atom subject to a time-dependent field is due to Eberly and co-workers [13], an early reference being by Shirley [14].

In order to appreciate the connection between the quantum optics treatment of AH [1] and the two-site case of the transport analysis of DK [2–4], note that Eq. (1) above, which is Eq. (1.1) of Ref. [2], has as its special case for two sites ($m=1,2$ rather than $m=-\infty$ to ∞) and sinusoidal field variation,

$$H=\hbar V(|1\rangle\langle 2|+|2\rangle\langle 1|)+\frac{\hbar\mathcal{E}}{2}\cos(\omega t)(|1\rangle\langle 1|-|2\rangle\langle 2|), \tag{2}$$

where $\mathcal{E}=eaE$. Equation (2) is precisely Eq. (7) of Ref. [1]. The parameters V, ω, and $\mathcal{E}$ in Eq. (2) correspond respectively to g, Ω, and $M\Omega$ in Ref. [1]. The condition for trapping derived in Ref. [1], viz., $J_0(M)=0$, is identical to the condition derived in Ref. [2] for dynamic localization, i.e., $J_0(\mathcal{E}/\omega)=0$. Furthermore, the two-level atom treatment [1] of the effect of spontaneous emission on the probabilities of the two atomic levels should be compared to the analysis of the effect of scattering on dynamic localization [4]. For instance, the ‘‘ladderlike’’ formations observed in Fig. 4 of Ref. [1] for the probability p of excitation for different values of spontaneous emission γ appear to correspond to the ‘‘staircases’’ shown for the mean-square displacement for various values of scattering strength α in Fig. 3 of Ref. [4]. It is tempting to conclude that the AH trapping is identical to the DK dynamic localization; however, this is not true. They are, indeed, very closely linked, but describe different aspects of the phenomenon. We will make this important point clear below but first focus on the treatment of finite chains.

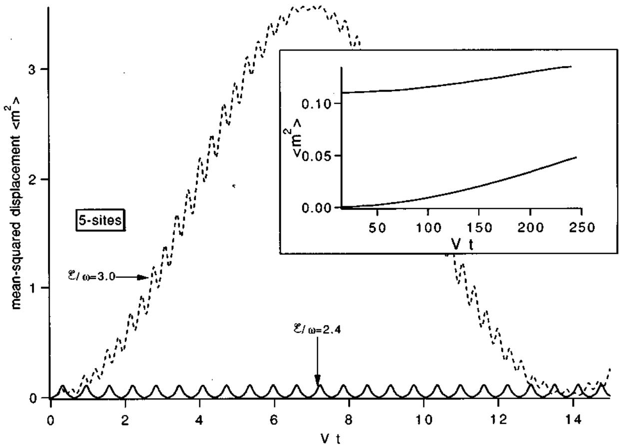

FIG. 1. Dimensionless mean-squared displacement $\langle m^2 \rangle$ for a five-site chain plotted as a function of dimensionless time Vt to show two size effects. In the main figure, short-time behavior is shown. The solid line shows the case where $J_0(\mathcal{E}/\omega)=0$. The dashed line shows the case where $J_0(\mathcal{E}/\omega)\neq 0$. Only when $\mathcal{E}/\omega$ equals the root of J_0 does the particle show localization. Finite-size effects make the mean-square displacement bounded, even for the case when $J_0(\mathcal{E}/\omega)\neq 0$. The two curves in the inset show the evolution of the maxima and minima, respectively, of the solid line in the main figure. We see that, at long times in a finite chain, $\langle m^2 \rangle$ will grow even for $J_0(\mathcal{E}/\omega)=0$.

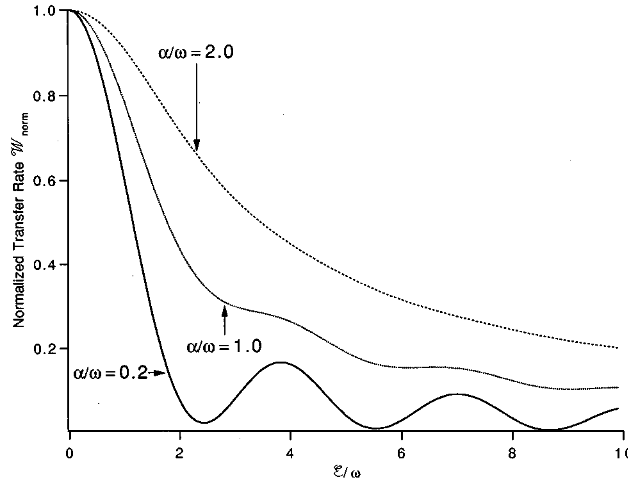

FIG. 2. The quantity $\mathcal{W}$ that represents the effective diffusion constant for a two-state system plotted as a function of $\mathcal{E}/\omega$ for various scattering rates α/ω. Plotted is the normalized transfer rate $\mathcal{W}_{\mathrm{norm}}$ (see text) as a function of $\mathcal{E}/\omega$ for α/ω=0.2,1.0,2.0. Dynamic localization causes the oscillations of the transfer rate. The oscillations are visible for small scattering but are washed out for large scattering.

The explicit calculation presented in DK was for an infinite chain. The AH treatment addresses the two-state system. Of interest in the crystal context is the analysis of the intermediate case lying between these two extremes, viz., a chain of N sites. The primary element introduced in finite chains is boundaries. We will take them to be purely reflecting here. This results in a competition between two characteristic length scales of the system: the confinement length due to the oscillating electric field and the reflection length due to the finiteness of the chain. There are two consequences of this competition, which are apparent when the size of the lattice is not much larger than the confinement length. The first is the return of the particle to its initially localized site after encountering the ends of the lattice. This results in recurrences. The second is that the ratio of the confinement length introduced by dynamic localization (Bessel root condition) to the chain length is not zero, in contrast to the infinite chain case. This results in a slow delocalization even when the Bessel root condition is obeyed. We illustrate both these effects in Fig. 1: the first in the main figure, and the second in the inset. We plot $\langle m^2 \rangle$, the dimensionless mean-squared displacement for a five-site chain as a function of dimensionless time Vt. The solid line denotes the case where $\mathcal{E}/\omega$ is a root of J_0. The dashed line shows the case where $J_0(\mathcal{E}/\omega)\neq 0$. The main figure shows the short-time features of the mean-squared displacement: as the dashed line shows, owing to the fact that the chain is finite, the mean-square displacement grows with time but remains bounded, in contrast to the infinite chain. [cf. Fig. 3(b) of Ref. [2].] The inset shows the occurrence of imperfect localization at long times.

We analyze the effects of scattering through the simple incorporation of a dephasing rate in the density matrix equations, as in Ref. [4]. We treat a finite chain. Its extreme limits correspond to the analysis of the DK system (infinite chain) [4] on one hand and that of AH (two-state system) [1] on the other. We have constructed time evolution plots of $\langle m^2 \rangle$ for finite chains for various values of the dephasing (scattering) rate and seen that, for low and intermediate values of the rate, the ''staircase'' structure seen in Fig. 3 of Ref. [4] is essentially reproduced. Finite-size effects visible clearly in Fig. 4 of Ref. [11] are also recovered. For space reasons we do not exhibit these plots here.

It has been shown in Fig. 1 of Ref. [4] that the *effective* diffusion constant in the presence of scattering, when plotted as a function of $\mathcal{E}/\omega$, exhibits interesting oscillations. In order to stress the connection to the two-level atom case, we plot in Fig. 2(b) the quantity $\mathcal{W}=[\int_0^\infty dt p(t)]^{-1}$, where $p(t)$ is the probability difference between the two atomic levels, which represents the effective diffusion constant for a two-state system. We have plotted the normalized quantity $\mathcal{W}_{\mathrm{norm}}=\mathcal{W}(\mathcal{E}/\omega)/\mathcal{W}(0)$ against $\mathcal{E}/\omega$ for α/ω=0.2,1,2. Note the sharp decrease in the transfer rate when $\mathcal{E}/\omega$ equals one of the roots of J_0, signifying dynamic localization. Note also the fact that, as the scattering rate increases, the oscillations become less noticeable, in agreement with the observation made in Ref. [4] that scattering tends to reduce the effect of dynamic localization.

An analysis of the extent of the overlap between the eigenfunctions of the Hamiltonian with the maximum field applied and those without the field applied results in a criterion measuring the sensitivity to the Bessel root condition. The essential conclusion is that the sensitivity is high when $V/N\mathcal{E}$, the ratio of the intersite transfer to the end-to-end energy mismatch, is small. Figure 3 shows this aspect clearly.

We have seen close similarities between the AH trapping and the DK dynamic localization. We now show that the two phenomena, while related closely, are *not* identical. Figure 4 distinguishes between them. We take $V/\mathcal{E}=0.008$ and plot the evolution of the probability of the initially occupied site

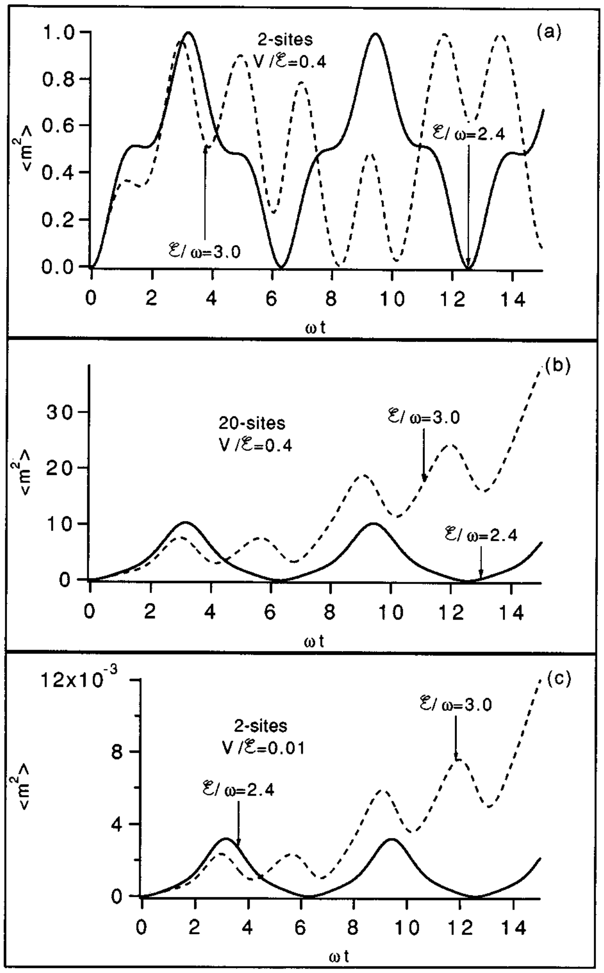

FIG. 3. Effect of the magnitude of $V/N\mathcal{E}$ on the sensitivity to the Bessel root condition seen through a plot of $\langle m^2 \rangle$ vs ωt. Solid lines correspond to $J_0(\mathcal{E}/\omega)=0$ and dashed lines to $J_0(\mathcal{E}/\omega)\neq 0$. In (a) $V/\mathcal{E}=0.4$ and $N=2$. In (b) $V/\mathcal{E}=0.4$ but $N=20$. Comparison of (a) and (b) shows size sensitivity. In (c) $N=2$ but $V/\mathcal{E}=0.01$. Sensitivity is thus restored for the two-site system by reducing $V/\mathcal{E}$.

in a two-site system for the resonance condition ($\mathcal{E}/\omega=30.635$) (solid curve) and a slightly off-resonance condition ($\mathcal{E}/\omega=31.635$) (dotted curve). The fact that, in Fig. 4, the solid curve lies essentially horizontally, while the dotted curve oscillates, signifies dynamic localization. The appearance of the substructure in *both* curves represents the AH trapping. We see little sensitivity of the AH substructure to the Bessel root condition for the parameters considered. The sensitivity of dynamic localization is, however, considerable.

The AH structure, whose source can be traced to Zener's analysis [15], is absent in the DK analysis of the infinite chain. Its gradual disappearance on increasing the size of the chain is seen clearly through a comparison of Fig. 4 with Figs. 5(a) and 5(b). All these three represent $V/N\mathcal{E}=0.016$, the number of sites N being 2, 4, and 16, respectively. The solid (dotted) curve in all three represents the on (off) resonance condition. The substructure is difficult to recognize already for the 16-site system.

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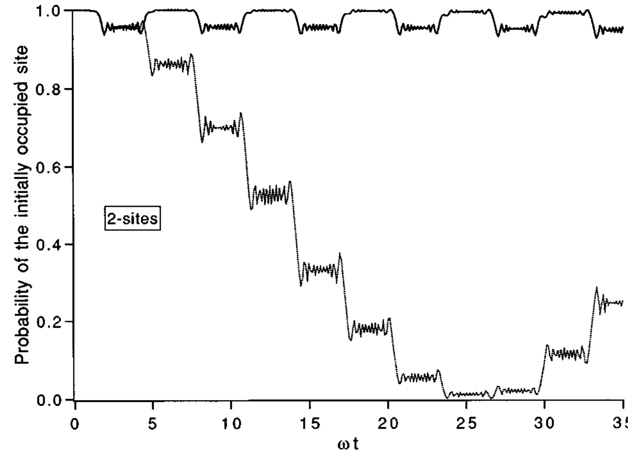

FIG. 4. Coexistence of AH trapping and DK dynamic localization in a two-site system. The probability of the initially occupied site on resonance (solid curve) and off-resonance (dotted curve) is plotted as a function of time. See text for details. The large difference between the two curves represents DK localization. The smaller substructure present in both curves represents AH trapping.

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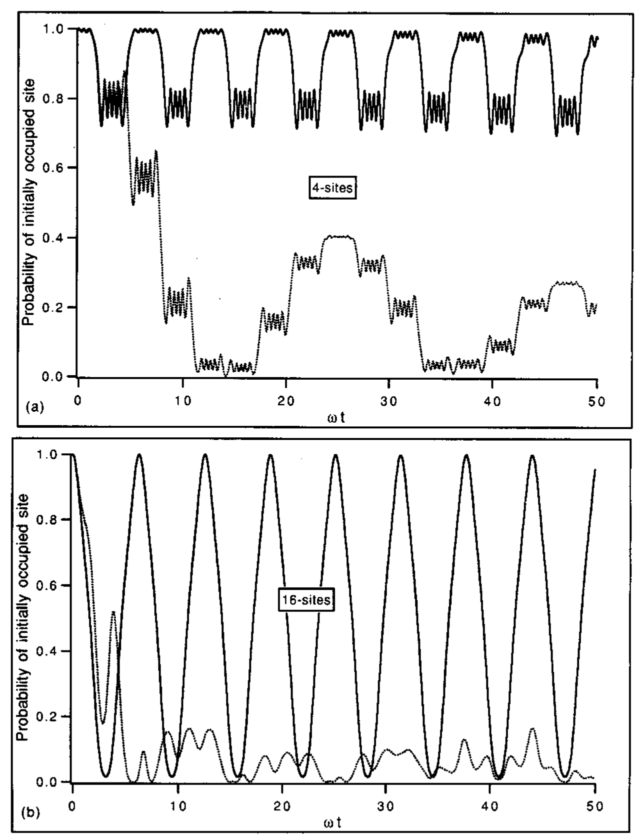

FIG. 5. Disappearance of the AH substructure on increasing the size of the system: The quantity plotted is the same as in Fig. 4, except that the size of the system is increased to four sites in (a) and to 16 sites in (b). The solid (dotted) curves show the on (off) resonance cases. The value of $V/N\mathcal{E}$ is held constant at 0.016.

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