Analytic investigation of the motion and capture of excitations in fractal systems

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The motion and capture of excitations on the Sierpinski gasket and on two Koch curves are investigated analytically by solving the master equation explicitly. We obtain the exact time dependence of the average self-propagator and the survival fraction, as well as quantities relevant to the sensitized luminescence such as the time-dependent luminescence intensity and the impurity quantum yield. The capture calculations are particularly applicable in the limit of low trap concentrations. It is seen that, at sufficiently long times, the behavior of the motion and capture is governed by the spectral dimension only, in agreement with results reported earlier on the basis of other approaches.

I. INTRODUCTION

Recent studies have shown that many physical systems exhibit fractal geometry over a range of observable length scales.[1] Systematic studies of critical phenomena[2] and of random walks[3] on fractal structures have been carried out over the past few years. Recently,[4,5] fractal behavior in trapping and reaction has been studied by following the dynamics of continuous-time random walks.

Fractals are self-similar structures that exhibit dilation symmetry and are not translationally invariant. The dimension d of the Euclidean space in which the fractal is embedded is different from the Hausdorff dimension[6] $\bar{d}$ of the fractal. The latter usually has a noninteger value unlike d which is an integer and $\bar{d} < d$. Another dimension, called the spectral dimension,[7] $\tilde{d}$, is also required to describe the transport and trapping phenomena on fractal lattices. It has been shown[3] that various random-walk properties are governed by the spectral dimension and simulation studies[4,5] have verified these results.

In this paper we present a new approach towards the study of motion and capture of excitations in fractal systems. Although our analysis is of general interest, we occasionally use language relevant to the specific physical system consisting of a molecular crystal (also called the host) with impurity molecules (also called traps), the moving excitations being Frenkel excitons.

Our approach[8] is based on the master equation, which describes the motion of excitons in the host augmented by terms representing exciton decay with lifetime τ and capture by traps at rate c whenever the exciton occupies a trap-influenced site. It has been shown[8] that this analysis for translationally invariant systems leads to simple expressions for observables such as the quantum yield or photon count rate in the case of sensitized luminescence. Since the systems under study in the present paper are not translationally invariant, we cannot use all the results of the analysis in Ref. 8. However, we will show that, for low trap concentrations, only small changes are needed in the relevant results[8] in order to describe motion and trapping in fractals.

The plan of the paper is as follows. We consider the motion of excitons without capture or decay in Secs. II and III. The Sierpinski gasket of $d=2$ and $\bar{d}=\ln 3/\ln 2$ (see Fig. 1) is studied in Sec. II. We use the algorithm of Domany *et al.*[9] for the solution of the Schrödinger equation on the Sierpinski gasket to obtain the self-propagator. The long-time behavior of the self-propagator and its dependence on the spectral dimension $\tilde{d}$ is also considered. In Sec. III we analyze two quasi-one-dimensional lattices. These lattices have $d=2$ and $\tilde{d}=1$ but $\bar{d}=\ln 4/\ln 3$ and $\ln 6/\ln 4$ (see Fig. 2). Although the algorithm of Domany *et al.*[9] cannot be used in these cases, it is possible to use the idea of cells as described in Sec. III. The self-propagator and its long-time behavior are analyzed as in Sec. II. At sufficiently long times, the evolution is governed by the spectral dimension. This finding agrees with earlier results.[3,7]

Section IV incorporates the decay and capture terms in the equation of motion. It is here that the absence of translational invariance of the system poses a special difficulty. However, we are able to solve the problem for low trap concentration. The impurity quantum yield and the time-dependent host luminescence for the lattices described in Secs. II and III are the output of our calculation. It is seen again that the long-time behavior is determined by $\tilde{d}$. The discussion of our results is the content of Sec. V.

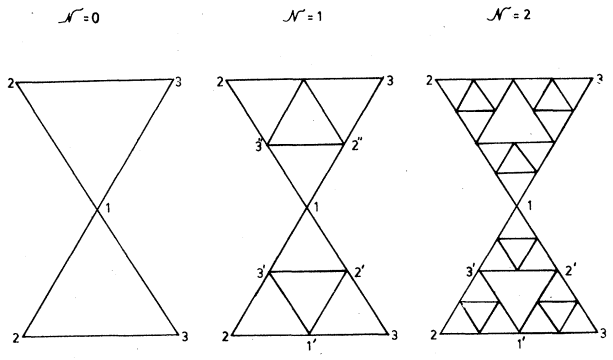


FIG. 1. Construction of the Sierpinski gasket. There are three sites on the largest scale ($\mathcal{N}=0$). First two generations are shown.

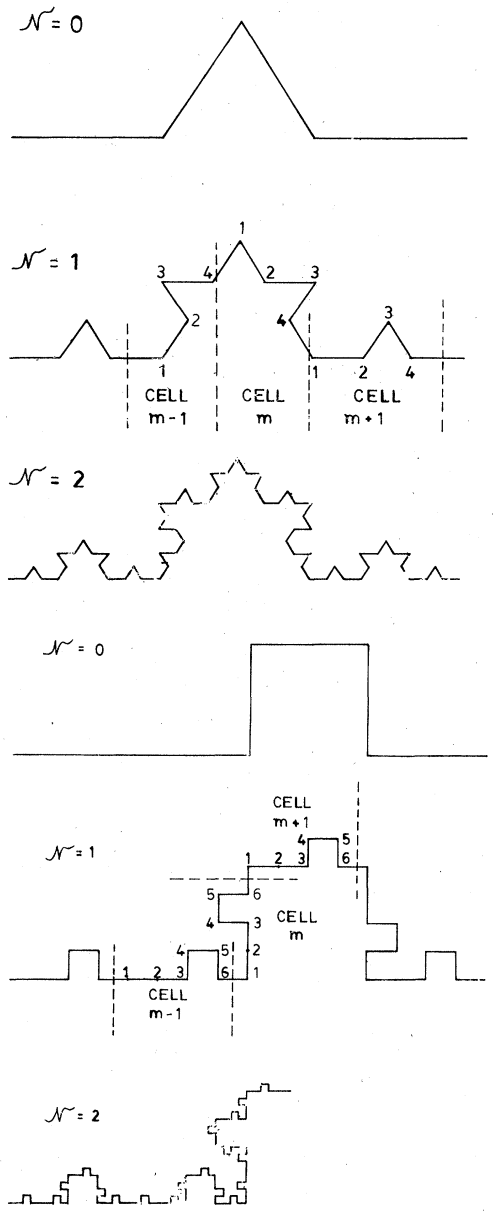


FIG. 2. (a) Construction of the Koch curve for $\bar{d}=\ln 4/\ln 3$. The first two generations are shown. The labeling of the sites within a cell is shown for the first generation. The dotted lines separate the individual cells (see text). (b) Construction of the Koch curve for $\bar{d}=\ln 6/\ln 4$. The first two generations are shown. The labeling of the sites within a cell is shown for the first generation. The dotted lines separate the individual cells (see text).

II. MOTION ON THE SIERPINSKI GASKET

In this section we give a description of the transport of excitations on the Sierpinski gasket[6] (sometimes abbreviated by SG). A sequence of fractal lattices is shown in Fig. 1. Following Domany *et al.*,[9] we use boundary conditions to identify the corners of two triangles on the largest scale as shown in Fig. 1. After any finite number $\mathcal{N}$ of iterations, one has a lattice of N sites where $n=3^{\mathcal{N}+1}$. If $|P(t)\rangle$ is the probability vector whose components $P_m(t)$ are the probabilities that the excitation occupies sites m at time t, the motion of the excitations obeys, by assumption, the master equation

$$\frac{d}{dt}|P(t)\rangle = Fh\,|P(t)\rangle - 4F\,|P(t)\rangle\ , \tag{2.1}$$

where F is the hopping rate between nearest neighbors and the $N\times N$ matrix h has elements h_{mn} which equal 1 if m and n are nearest neighbors and 0 otherwise. If we write

$$|P_\alpha(t)\rangle = e^{-E'_\alpha Ft}\,|P_\alpha\rangle\ , \tag{2.2}$$

we obtain

$$h\,|P_\alpha\rangle = -(E'_\alpha - 4)\,|P_\alpha\rangle = -E_\alpha\,|P_\alpha\rangle\ . \tag{2.3}$$

This constitutes an eigenvalue problem solved by Domany *et al.*[9] In particular, they gave an algorithm for obtaining the eigenvalues E_α and orthonormal vectors $|P_\alpha\rangle$ for a gasket obtained after any number of iterations. This is done by reducing the problem on the $(\mathcal{N}+1)$th generation to the problem on the $\mathcal{N}$th generation. For example, the sites of the lattice after one iteration are grouped into two sets (see Fig. 1). The amplitudes on the sites $1',2',\ldots,3''$ are written in terms of the amplitudes on the sites 1,2,3. Effectively, the eigenvalue equation on nine sites is reduced to a similar eigenvalue equation on three sites. The resulting Hamiltonian has the same form as the earlier one. Equivalently, the eigenvalues and eigenvectors of the $(\mathcal{N}+1)$th generation are written in terms of the eigenvalues and eigenvectors of the $\mathcal{N}$th generation.

A general solution of Eq. (2.1) can be written as

$$|P(t)\rangle = \sum_\alpha \langle P_\alpha\,|P(0)\rangle e^{-(E_\alpha+4)Ft}\,|P_\alpha\rangle\ . \tag{2.4}$$

The sum is over the complete set of eigenvectors. The explicit probability at the (arbitrary) mth site is

$$P_m(t)\equiv\langle m\,|P(t)\rangle = \sum_{\alpha,n'}\langle P_\alpha\,|n'\rangle\langle n'\,|P(0)\rangle e^{-(E_\alpha+4)Ft}\langle m\,|P_\alpha\rangle\ , \tag{2.5}$$

where $\langle n'\,|P(0)\rangle = P_{n'}(0)$ is determined by the initial ($t=0$) distribution of the excitation in the gasket. For the localized initial condition $P_{n'}(0)=\delta_{nn'}$, the probability $P_{m,n}(t)$ that the excitation is at the site m at time t given that at $t=0$ it was at the site n is

$$P_{m,n}(t) = \sum_\alpha \langle P_\alpha\,|m\rangle\langle n\,|P_\alpha\rangle e^{-(E_\alpha+4)Ft}\ . \tag{2.6}$$

As a particular result of (2.6), the self-propagator at the site m, i.e., the probability that the excitation remains at the site of initial occupation, is

$$P_{m,m}(t)=\sum_{\alpha}|\langle P_{\alpha}|m\rangle|^{2}e^{-(E_{\alpha}+4)Ft}. \tag{2.7}$$

In general $P_{m,m}(t)$ is dependent on the site m because of the absence of translational invariance, and therefore is difficult to obtain. However, the macroscopically important quantity is the average of all the self-propagators

$$\psi_{\mathrm{av}}(t)=\frac{1}{N}\sum_{m}P_{m,m}(t) \tag{2.8}$$

and can be evaluated explicitly, since,

$$\langle P_{\alpha}|P_{\alpha}\rangle=1=\sum_{n}\langle P_{\alpha}|n\rangle\langle n|P_{\alpha}\rangle . \tag{2.9}$$

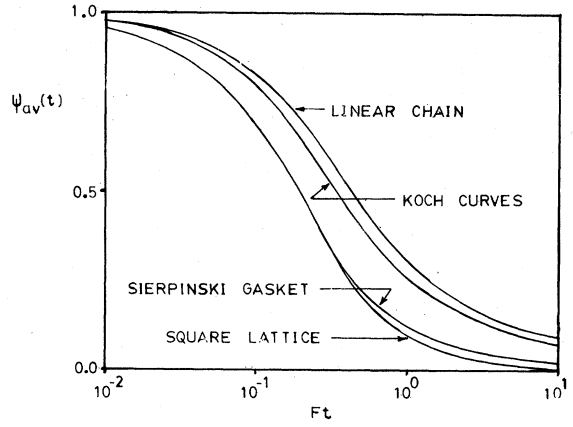
Equation (2.8) yields

$$\psi_{\mathrm{av}}(t)=\frac{1}{N}\sum_{\alpha}e^{-(E_{\alpha}+4)Ft}. \tag{2.10}$$

We rewrite (2.10) as

$$\psi_{\mathrm{av}}(t)=\frac{1}{N}\sum_{\alpha}g(E_{\alpha})e^{-(E_{\alpha}+4)Ft}, \tag{2.11}$$

where $g(E_{\alpha})$ is the degeneracy associated with the eigenvalue E_{α} and the primed summation goes over distinct eigenvalues only. The values of E_{α} and $g(E_{\alpha})$ are obtained from the results of Ref. 9 and the evaluation is therefore complete. We have carried out the explicit evaluation of the energy spectrum up to the eighth generation, in which the number of sites $N=3^{9}=19\,683$. For any large but finite N, $\psi_{\mathrm{av}}(t)\rightarrow 1/N$ as $t\rightarrow\infty$. However, as N increases, $\psi_{\mathrm{av}}(t)$ will approach the limiting value ($N\rightarrow\infty$) very rapidly. We have plotted $\psi_{\mathrm{av}}(t)$ for a large range of t values in Figs. 3(a) and 3(b). From the behavior of $\psi_{\mathrm{av}}(t)$ in the earlier (less that eighth) generations we can state with confidence that for all higher generations $\psi_{\mathrm{av}}(t)$ is not appreciably different from that shown in Figs. 3.



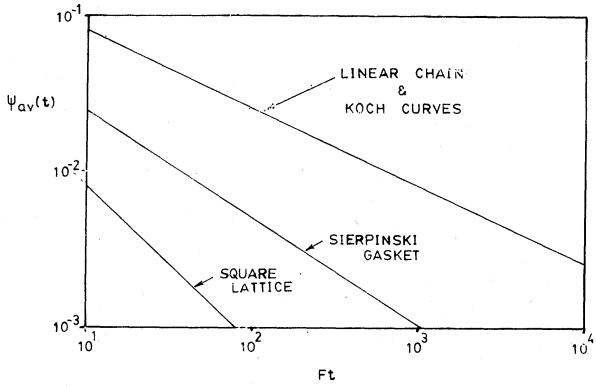


FIG. 3. (a) Average self-propagator $\psi_{\mathrm{av}}(t)$ plotted as a function of the dimensionless time Ft. The propagators for $\bar{d}=\ln 4/\ln 3$ and for $\bar{d}=\ln 6/\ln 4$ are indistinguishable on the scale of the figure. (b) Average self-propagator $\psi_{\mathrm{av}}(t)$ as a function of the dimensionless time Ft on a double-logarithmic scale. The slopes give the long-time behavior. The self-propagators for all $\bar{d}=1$ lattices are indistinguishable on the scale of the figure.

III. MOTION ON QUASI-ONE-DIMENSIONAL LATTICES

We will now analyze the motion of excitations on two lattices which are quasi one dimensional. These lattices have $\bar{d}=1$ and 2 but have different $\bar{d}$ values. Figure 2(a) shows a few generations of a lattice with $\bar{d}=\ln 4/\ln 3=1.262\ldots$.

The method outlined in the preceding section cannot be used to analyze these lattices. However, a much simpler technique will be used here. It utilizes the quasi-one-dimensional nature of the lattices in the sense that if the sites are partitioned into cells, the cells are translationally invariant. We use the boundary condition which identifies the last site of the lattice with the first (note that because of the quasi-one-dimensional and nonbranching[6] nature of the lattice, the sites can be ordered and the first and last terms have meaning when applied to the sites). The four sites of the smallest unit can be considered to form a cell. Figure 3(a) shows how such partitioning can be done for the first generation. These cells can be labeled by m ($m=1,2,\ldots$) and within each cell the four sites can be labeled as shown in Fig. 2(a). If $P_{j}^{m}(t)$ is the probability that the excitation at time t occupies the jth site of the mth cell, and if F is the hopping rate between the nearest neighbors, then

$$\begin{aligned}
\frac{dP_{1}^{m}(t)}{dt}&=F(P_{2}^{m}+P_{4}^{m-1}-2P_{1}^{m}),\\
\frac{dP_{2}^{m}(t)}{dt}&=F(P_{1}^{m}+P_{3}^{m}+P_{4}^{m}-3P_{2}^{m}),\\
\frac{dP_{3}^{m}(t)}{dt}&=F(P_{2}^{m}+P_{4}^{m}-2P_{3}^{m}),\\
\frac{dP_{4}^{m}(t)}{dt}&=F(P_{2}^{m}+P_{3}^{m}+P_{1}^{m+1}-3P_{4}^{m}).
\end{aligned} \tag{3.1}$$

If we exploit the translational invariance of the system of cells and use Fourier transforms

$$P_{j}^{k}=\sum_{m}e^{ikm}P_{j}^{m}, \tag{3.2}$$

we obtain

$$d/dt\,|P^k(t)\rangle = F\mathbb{A}\,|P^k(t)\rangle\ , \tag{3.3}$$

where the vector $|P^k(t)\rangle$ has components $P_j^k(t)$ and the matrix $\mathbb{A}$ is given by

$$\mathbb{A}=\begin{bmatrix} -2 & 1 & 0 & e^{ik} \\ 1 & -3 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ e^{-ik} & 1 & 1 & -3 \end{bmatrix}\ . \tag{3.4}$$

With the localized initial condition

$$P_j^m(0)=\delta_{m,0}\delta_{j,i}\ , \tag{3.5}$$

which implies $P_j^k(0)=\delta_{j,i}$, we solve (3.3) exactly with the use of Laplace transforms

$$\tilde{f}(\epsilon)=\int_0^\infty dt\, e^{-\epsilon t} f(t) \tag{3.6}$$

for all four independent initial conditions (corresponding to the four sites of cell $m=0$). We use tildes to denote Laplace transforms and ϵ to denote the Laplace variable. Thus the resulting $|P^k(\epsilon)\rangle$ can then be Fourier-inverted to obtain $|\tilde{P}^m(\epsilon)\rangle$. The latter give the probability propagators. In particular, the value of the vector $|\tilde{P}^m(\epsilon)\rangle$ at $m=0$ gives the self-propagators

$$\tilde{P}_{1,1}(\epsilon)=(\epsilon+4)(\epsilon^2+4\epsilon+2)/D\ , \tag{3.7a}$$

$$\tilde{P}_{2,2}(\epsilon)=\tilde{P}_{4,4}(\epsilon)=(\epsilon+4)(\epsilon^2+3\epsilon+2)/D\ , \tag{3.7b}$$

$$\tilde{P}_{3,3}(\epsilon)=\frac{1-C/D}{(\epsilon+3)}+\frac{(\epsilon^3+8\epsilon^2+18\epsilon+10)}{D}\ . \tag{3.7c}$$

Here

$$D=[(c+2\epsilon+6)(c-2\epsilon-6)]^{1/2}\ , \tag{3.8a}$$

$$c=\epsilon^4+10\epsilon^3+32\epsilon^2+34\epsilon+6\ . \tag{3.8b}$$

Note that, as a result of the translational invariance of the cell system, the label m is not necessary in Eqs. (3.7): $\tilde{P}_{j,j}(\epsilon)$ is the Laplace transform of the self-propagator for site j in any cell. Equations (3.7) cannot be inverted analytically but a numerical inversion[10] gives all the self-propagators $P_{j,j}(t)$ explicitly in the time domain.

The method outlined above can be used to analyze the lattice of Fig. 2(b). The labeling of cells and the sites within a cell are shown in the $\mathcal{N}=1$ generation [Fig. 2(b)]. The final results for this lattice, which corresponds to (3.7), are

$$\tilde{P}_{1,1}(\epsilon)=\tilde{P}_{2,2}(\epsilon)=(a^3b^2-3a^2b-ab^2-a^3+a+b)/D'\ , \tag{3.9a}$$

$$\tilde{P}_{3,3}(\epsilon)=\tilde{P}_{6,6}(\epsilon)=(a^4b-2a^3-2a^2b+2a+b)/D'\ , \tag{3.9b}$$

$$\tilde{P}_{4,4}(\epsilon)=\tilde{P}_{5,5}(\epsilon)=\frac{(a^3b^2-3a^2b-ab^2-a^3+3a+b)}{D'}+\frac{(1-C'/D')}{a}\ , \tag{3.9c}$$

where

$$a=\epsilon+2\ , \tag{3.10a}$$

$$b=\epsilon+3\ , \tag{3.10b}$$

$$c'=a^4b^2-4a^3b-a^4-2a^2b^2+4ab+4a^2+b^2-1\ , \tag{3.10c}$$

and

$$D'=[(c'+2a^2)(c'-2a^2)]^{1/2}\ . \tag{3.10d}$$

As in the case of the lattice of Fig. 2(a), the self-propagators $P_{j,j}(t)$ for all sites within a cell are obtained from the numerical inversion. The average, $\psi_{\rm av}(t)$, of all self-propagators for these two lattices is shown in Fig. 3(a). The long-time behavior of $\psi_{\rm av}(t)$ is shown in Fig. 3(b).

IV. INTRODUCTION OF DECAY AND CAPTURE IN THE EQUATION OF MOTION

We now introduce the decay and trapping terms in the equation of motion for the quantities $P_m(t)$. As stated in Sec. I, we formulate the problem in the language of Frenkel excitons. For simplicity, we consider a single trap (guest) in the lattice of N host sites. Following Kenkre and Wong[11] we write for the Sierpinski gasket

$$\frac{dP_m}{dt}+\frac{P_m}{\tau}=F\sum_n h_{mn}P_n-4FP_m-c\delta_{m,r}P_r\ . \tag{4.1}$$

This is a generalization of Eq. (2.1). Here τ is the radiative decay time in host, c is the rate at which the single trap is fed by the single trap-influenced host site r, and $\delta_{m,r}$ is the Kronecker δ. All other quantities are defined as in Eq. (2.1). A similar generalization of Eq. (3.1) will be used to analyze capture on quasi-one-dimensional lattices.

One of the observables of the system is the host luminescence, $n_H(t)$, which is given by

$$n_H(t)=\sum_m P_m(t)\ , \tag{4.2}$$

and which, in the absence of the decay represented by $1/\tau$, is the survival fraction of the excitations. The Laplace transform of $n_H(t)$ is

$$\tilde{n}_H(\epsilon)=\frac{1}{\epsilon'}\left[1-\frac{c\tilde{\eta}_r(\epsilon')}{1+c\tilde{\psi}_{r,r}(\epsilon')}\right]\ , \tag{4.3}$$

where $\epsilon'=\epsilon+1/\tau$. $\psi_{r,r}(t)$ is the self-propagator obtained from Eq. (4.1) with $c=0$, $\tau=\infty$, and with localized initial condition; $\eta_m(t)$ is then given by

$$\eta_m=\sum_n \psi_{m,n}P_n(0)\ , \tag{4.4}$$

where $P_n(0)$ is the actual initial condition. For the particular case of uniform initial host illumination $P_n(0)=1/N$ and

$$\tilde{n}_H(\epsilon)=\frac{1}{\epsilon'}\left[1-\frac{(1/N)}{\epsilon'/c+\epsilon'\tilde{\psi}_{r,r}(\epsilon')}\right]\ . \tag{4.5}$$

This result is the same as that for a translationally invariant system with a single trap.[11] If we now assume

that the system has N' traps and that the trap concentration $\rho=N'/N \ll 1$, and that these traps are distributed randomly, we obtain, in analogy to Ref. 11,

$$\tilde{n}_H(\epsilon)=\frac{1}{\epsilon'}\left[1-(1/N)\sum_n \frac{\rho}{[\epsilon'/c+\epsilon'\tilde{\psi}_{n,n}(\epsilon')]}\right]. \quad (4.6)$$

Here the summation extends over all sites of the lattice.

Equation (4.6) can be solved easily for the quasi-one-dimensional lattices because the self-propagators are known for any lattice of sites N. The problem is more complex for the SG. For the results to be independent of N for the observable time scales, one has to consider large lattices. To use Eq. (4.6) one needs to know the complete set of orthonormal eigenvectors $|P_\alpha\rangle$ [see, e.g., (2.4)]. These are extremely difficult to obtain for large lattices. The algorithm of Ref. 9 produces the eigenvectors at any given generation in terms of the eigenvectors of the previous generation. However, they are not orthonormal and have to be orthonormalized at every stage. The difficulty of orthonormalizing a set of N vectors of N components each is obviously considerable for N as large as 3^6.

We considered five iterations and after each we found out the orthonormal set of eigenvectors and subsequently $n_H(t)$ from Eq. (4.6). We also found out numerically that the result was unchanged when Eq. (4.6) was replaced by

$$\tilde{n}_H(\epsilon)=1-\frac{\rho}{\epsilon'/c+(\epsilon'/N)\sum_n \tilde{\psi}_{n,n}(\epsilon')}. \quad (4.7)$$

We therefore assume that Eq. (4.7) which involves the average of the $\psi_{n,n}$'s generally provides an adequate approximation to (4.6). The form for $n_H(t)$ given by (4.7) is much more convenient than that given by (4.6) because it merely requires the average self-propagator $\psi_{\rm av}(t)$. The computation of $\psi_{\rm av}(t)$ is a simple numerical task even for large lattices as we have seen in Sec. II. We have inverted Eq. (4.7) numerically to obtain $n_H(t)$ for a lattice of $N=3^9=19\,683$ sites ($\mathcal{N}=8$). The results are shown in Fig. 4(a).

The passage from (4.6) to (4.7), while necessary for practical reasons for the Sierpinski gasket, is not needed for the Koch curves because the distinct self-propagators for the latter are only three in number and are furthermore known analytically. It is interesting to note, however, that we have seen from a numerical calculation that the above assumption is valid for Koch curves also.

We exhibit the results of the preceding analysis in Fig. 4. The host luminescence is shown as a function of t in Fig. 4(a) for a fixed concentration $\rho=10^{-4}$ for all the lattices considered in this paper. We have set $1/\tau=0$ in these plots to eliminate the exponential decay which is important to excitons in molecular crystals but which is not of particular relevance to behavior in fractals. Several features are observed. For all times the curve for $\bar{d}=\ln 6/\ln 4$ lies below that for the linear chain ($\bar{d}=1$) and above that for $\bar{d}=\ln 4/\ln 3$.

The survival fraction—or in the language of Frenkel excitons and molecular crystals, the time-dependent luminescence intensity—$n_H(t)$ discussed above requires (numerical) Laplace inversions. In the physics of molecular crystals there is another quantity, viz., the quantum yield, which is of interest and which requires no Laplace inversion.[8] The guest quantum yield, ϕ_G, for our system is defined as the fraction of the initial excitation that is trapped by the guest molecules. Therefore,

$$\phi_G=1-(1/\tau)\int_0^\infty dt\, n_H(t), \quad (4.8)$$

we display ϕ_G as a function of Ft in Fig. 4(b).

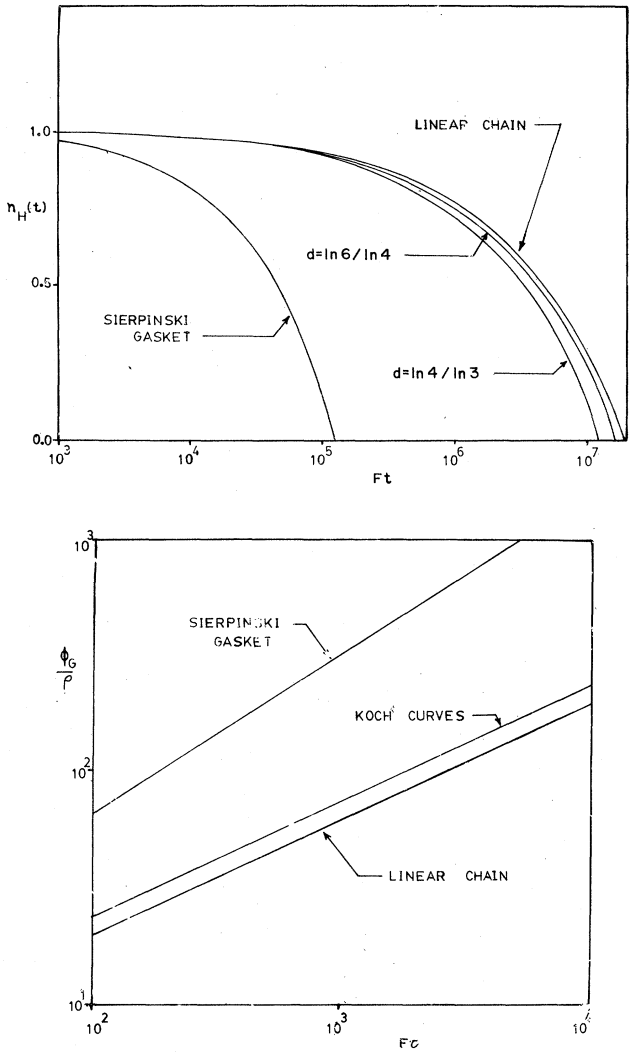


FIG. 4. (a) Survival fraction or the host luminescence $n_H(t)$ plotted against the dimensionless time Ft. The capture parameter value c is arbitrarily set to $10F$. The concentration ρ is 10^{-4}. (b) Ratio ϕ_G/ρ of the guest quantum yield to the trap concentration is plotted as a function of Ft on a double-logarithmic scale. The value of C/F is arbitrarily set to 10.

V. DISCUSSION

We have analyzed two kinds of fractal systems in this paper: the quasi-one-dimensional lattices (Koch curves) and the Sierpinski gasket. To treat the former we used

the simple-cell idea based on the translational invariance of the groups of sites in the Koch curves. To treat the Sierpinski gasket we used the algorithm of Domany *et al.* which gives the eigenvalue spectrum of the master-equation matrix. The average self-propagator, which is a measure of the motion alone, is shown in Fig. 3(a) for the three fractal systems along with the self-propagator for the linear chain ($d=\bar{d}=\tilde{d}=1$) and for the square lattice ($d=\bar{d}=d=2$) for comparison. We see that for $1<\bar{d}<2$ the average self-propagator seems to be bounded by the self-propagators on the Euclidean lattices for one and two dimensions. We note that at a given time the self-propagator does not monotonically decrease with increasing $\bar{d}$. This is not clear on the scale of Fig. 3(a) since the curves for $\bar{d}=\ln 4/\ln 3=1.262\ldots$ and $\bar{d}=\ln 6/\ln 4=1.292\ldots$ appear identical. The differences are, however, evident in the numerical calculations. Note also that the $\bar{d}=1.292\ldots$ propagator remains higher than the $\bar{d}=1.262\ldots$ propagator, but lower than the linear-chain propagator for the range of time shown in Fig. 3. This nonmonotonic dependence on $\bar{d}$ is perhaps unexpected and makes the study of Koch curves interesting.

One observes from Fig. 3(b) that the long-time behavior is dependent only on $\tilde{d}$. For all $\tilde{d}=1$ lattices the slope of the straight lines is $-\frac{1}{2}$. Thus $\psi_{\rm av}(t)\sim t^{-1/2}$ for large t. The straight line for the Sierpinski gasket has slope -0.682 while the Euclidean two-dimensional case has slope -1. These results can be therefore written as $\psi_{\rm av}(t)\sim t^{-\tilde{d}/2}$ since $\tilde{d}$ for the Sierpinski gasket is $1.365\ldots$. This conclusion has been arrived at earlier.[3,7] For the gasket this result is transparent from a consideration of Eq. (2.12) and the results of Ref. 9 concerning the eigenvalue spectrum. From the latter[9] one sees that $0\le E_\alpha+4\le 8$ and that, at $E_\alpha=-4$, the number of states $N(\Delta E)$ in the interval ΔE is proportional to $(\Delta E)^{\tilde{d}/2}$. It thus follows from Eq. (2.12) that a significant contribution to $\psi_{\rm av}(t)$ for large t comes from terms corresponding to $E_\alpha+4$ within ΔE of zero. Hence one obtains the result $\psi_{\rm av}(t)\sim t^{-\tilde{d}/2}$.

Our trapping analysis is analytical and its results are shown in Fig. 4. The numerical calculation shows that at a given time the survival fraction for $\bar{d}=1.292\ldots$ is always lower than the survival fraction for the one-dimensional chain and higher than the survival fraction for the $d=1.262$ lattice.

If we set the ratio C/F to zero in our trapping analysis, we can compare our results with those of Blumen *et al.*[4] They have shown that the survival fraction $n_H(t)$ obtained from a numerical simulation of random walks on the Sierpinski gasket can be fitted very well to Eqs. (51) below,

$$n_H(t)=e^{-Ax+Bx^2}, \tag{5.1a}$$

$$x=\rho(Ft)^{\tilde{d}/2}. \tag{5.1b}$$

The important point is that it is the spectral dimension $\tilde{d}$ that governs the time dependence as seen from Eqs. (5.1). We find that Eqs. (5.1) give an excellent fit to our results for the survival fraction too. For the low concentrations considered in our analysis x is small and hence the term in x^2 in Eq. (5.1a) can be neglected. We find, however, that our value of A for the Sierpinski gasket is higher than that of Ref. 4. The linear chain and the two Koch curves have different Hausdorff dimensions but they all have the same time dependence, according to Eqs. (5.1) for the survival fraction.

Our analysis thus supports the results that trapping properties are governed by the spectral dimension $\tilde{d}$. Figure 4(b) shows the guest quantum yield as a function of Ft and one sees that the slope equal $\tilde{d}/2$. We would like to mention here some recent work[12,13] that has been brought to our attention which uses approaches different from ours and analyses capture at higher trap concentrations.

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[1]R. Orbach, in *Organic Molecular Aggreates,* Vol. 49 of *Springer Series in Solid State Sciences,* edited by P. Reineker, H. Haken, and H. C. Wolf (Springer, Berlin, 1983), and references therein.

[2]Y. Gefen, A. Aharony, and B. Mandelbrot, J. Phys. A **16**, 1267 (1983); **17**, 435 (1984); **17**, 1277 (1984).

[3]R. Rammal and G. Toulouse, J. Phys. Lett. **44**, L13 (1983).

[4]A. Blumen, J. Klafter, and G. Zumofen, Phys. Rev. B **28**, 6112 (1983).

[5]J. Klafter and A. Blumen, J. Chem. Phys. **80**, 875 (1984).

[6]B. Mandlebrot, *The Fractal Geometry in Nature* (Freeman, San Francisco, 1982).

[7]S. Alexander and R. Orbach, J. Phys. Lett. **43**, L625 (1982).

[8]V. M. Kenkre, in *Exciton Dynamics in Molecular Crystals and Aggregates,* edited by G. Höhler (Springer, Berlin, 1982).

[9]E. Domany, S. Alexander, D. Bensimon, and L. Kadanoff, Phys. Rev. B **28**, 3110 (1983).

[10]H. Stehfest, Commun. ACM **13**, 47 (1970); **13**, 17 (1970).

[11]V. M. Kenkre and Y. M. Wong, Phys. Rev. B **23**, 3748 (1981); see also V. M. Kenkre and P. E. Parris, *ibid.* **27**, 3221 (1983).

[12]J. Klafter, G. Zumofen, and A. Blumen, J. Phys. Lett. **45**, L49 (1984).

[13]G. Zumofen, A. Blumen, and J. Klafter, J. Phys. A **17** L479 (1984).